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Inequality, new directions: full normative foundations and their consequences

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Abstract

The innovation in the measurement of unjust inequalities of forty-five years ago was a derivation from social evaluation and a rich set of equivalent ethical-technical properties. Extending the foundations to the whole of social ethics provides similarly meaningful, rich, and yet basically simple structures, with policy applications. Equal-equivalence is the basic concept of ethical inequality analysis. It is extended from income to utility functions and to multidimensional manifolds. The former gives the concept of comparable individual welfare, cleaned from irrelevant differences in hedonic capacities and tastes. The latter defines the measures of multidimensional inequality. This is furthered by the definitions of equality and inequality in liberty. Justice, liberty, efficiency and incentive compatibility are reconciled by equalizing the proceeds of a given labour and letting people free to work and earn for the rest, thus associating incomes for desert and for merit. Implementability is proven by an actual policy. This proposes the degree of equalization as the relevant measure.

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1. Introduction: taking ethics seriously

1.1 From the ethical turn to full axiological rationality: from Schur to ELIE

Frank Cowell and Ravi Kanbur organized a conference in September 2009 in Ithaca, to celebrate the 40th anniversary of a few works and entitled *Inequality, new directions*. The relation so established between a past research breakthrough and presently needed advances is particularly cogent, not as an answer to the platitude "what do we do next?" but, much more interestingly, because the nature of the initial innovations seems to still be the best guide for finding out the most useful, important and necessary new directions. To show this – and because there has been a tradition of misrepresentation – the next section briefly recalls the genesis and the actual content of the relevant sections of the article *The optimal production of social justice*, presented at the 1966 International Economic Association Conference on Public Economics in Biarritz. The initial innovations were both and jointly technical and ethical, and in both aspects philosophical advances. First, ethical epistemology demands that a principle be considered and evaluated from all its angles, according to all its properties and

implications – see, for instance, Plato's "dialectics" in the *Republic* and, with regard to consequences, Rawls's "reflective equilibrium". Could one find a better example of this epistemological principle than the twenty or so properties different in intuitive meaning but mathematically equivalent that include the transfer principles, concentration-curve and Lorenz-curve dominances and the Schur-Ostrovski-Birkhoff-von Neumann structures (rectifiance, isophily, preferences for averages, mixtures and concentrations, etc.)?[2] Second, the theory of unjust inequalities was derived from an overall evaluation of the social state. The key concept for this derivation was the *equal-equivalent income*, i.e. the income such that, if everybody had it, the situation would be as good as the actual one – this measure both is a social evaluation and has the nature and metric of an income, comparable with the average. We will see that extensions of this idea to the *equal-equivalent utility function* and to the *equal-equivalent manifolds* for multidimensional inequality, permit to solve basic questions in inequality measurement, social ethics and distributive policy. As for the multiple manifestations and meanings of deep principles – which may be the case of all basic scientific laws –, what occurred for the unidimensional "transfer principle" will be seen to also occur for the bi-dimensional "equal-labour income equalization" (ELIE) which replaces the wasteful partial equalization of all earned income by the orthogonal total equalization of the part of income earned by a given labour, and hence the measure of income inequality by this degree of equalization (this extends the "transfer principle" for two persons into a linear "concentration" of all total incomes – leisure included – or, equivalently, a series of transfers proportional to the differences in productivity).[3]

Inequality deemed unjust is the main cause of social strife. Its definition is, consequently, the essence of ideological conflicts. It also is, in the sphere of thinking, the basic concern of social ethics ("political philosophy") and normative economics. Actually, these disciplines and the required definition of inequality constitute largely the same topic since most theories in these disciplines are either the justification or the assumption of some kind of equality in one or a few things – thus blaming the corresponding inequalities. This remark suggests a large field of important new directions for inequality studies. Both sides of this analysis will benefit and – in a sense and to some extent – will eventually fuse. This will imply an extended variety of the formal properties of the inequality material: one-

[2] Some of the relevant formal relations and properties were known to mathematicians but many were not; some mathematicians' works were also influenced by discussions with economists about distributions and their transformations (Claude Berge, Paul Lévy, André Lichnerowicz, Benoît Mandelbrot, etc.).
[3] It will be seen to also amount to equal freedom, general labour reciprocity, basic income financed by equal labour, etc.

dimensional, multidimensional, of orderings, of various types of rights and liberties, multicriterial, sequential, conditional, ad so on. A frequent basic situation is that philosophy and normative economics (or the part of the latter which is not inequality studies) define relevant equalities only,[4] whereas real life is with inequalities and demands their comparison, which adds new technical and moral questions. Some common problems will be found in various guises, such as defining Pareto-efficient second-best justice (for instance, find the least unequal Pareto-efficient multidimensional allocations – the dimensions can be goods, Rawls's "primary goods", Weber-Walzer's "spheres of justice", basic needs, Nussbaum-Sen's "capabilities", etc. –; a solution based on multidimensional inequality theory leads to "super-equity" or "convex no-envy", that is: no individual prefers an allocation in the convex hull of individuals' allocations to her own[5]; another example extends the ordinally egalitarian equity-no-envy principle – no individual prefers another's allocation to her own – into a comparative inequality structure and defines an efficient maximin[6]; still another example is the noted division of incomes into an equal part and a freely earned part which permits Pareto efficiency and this demanded liberty, and will be shown to amount to efficient equal freedom). Indeed, this broadening of the view of inequality studies will often permit them to find deeper structural solutions for superior policies.

The definition of income inequality from a social ethical evaluation function was just a first and primitive application of this integration with social ethics. Economics usually values individuals' utilities, and their differences break the symmetry in incomes down ("to each according to her needs, tastes or intensity of desire"), unless individuals have identical functions representing their "welfare", which then need to be defined (a solution is shortly provided). One should warn here that a common present-day external view that social ethics is torn down by a feud between so-called "welfarism" (Hicks 1959) and "non-welfarism" is obsolete and mistaken. For most actual questions, people agree about whether welfare and which welfare is a proper criterion of distribution. Suffering and basic needs are relevant. Tastes, desires and capacities to enjoy are often relevant for distribution in small face-to-face empathic groups (such as families). But their differences turn out not to be considered relevant for overall distributions in large societies, by anybody including voters and officials who choose the distributive policy (see section 2.2). There, lower income inequality, supported in part by some particular concept of comparable welfare for lower incomes, and a

[4] A survey can be found in Kolm 2010.
[5] Kolm 1971, 1973b, 1996b.
[6] Kolm 1999b.

right to the fruit of one's labour and earning capacities, happen to be the two competing challengers.

This paper is organized as follows. Section 1.2 recalls the genesis and characteristics of the initial work in question, and section 1.3 presents an outline of the questions and results. Section 2, about welfare and inequality, shows which aspect of "welfare" may be deemed relevant for overall income distribution ("macrojustice"), defines the uniform individual welfare function cleaned for irrelevant differences, applies it to multidimensional inequality measurement, applies this to inequality-averse optimum taxation of earned income, and shows that the inefficient ideal equality can be corrected by a demanded and efficient untaxed free part of labour. This result is studied in section 3 which shows that it amounts to efficient equal liberty (with different domains of choice), has a number of other remarkable egalitarian and efficiency properties, and is easily implementable by actual efficient policies. Section 4 extends this equality to the comparison and measures of freedom inequality by purchasing power. Section 5 outlines the theory of multicriterial inequality and applies it to complementing consequentialist inequality by valuing allocation according to deserts, or to partial merit which secures efficiency. Finally, a conclusion shows the main issues and challenges faced by inequality analysis that takes ethics seriously.

1.2 The turn of the last third of the 20th century in income inequality comparison and measurement

1.2.1 History

In **1966**, two things relevant for our purpose occurred.

1) Publication of the book **Monetary and Financial Choices (Modern Theory and Techniques)**, Paris: Dunod, 1966 (in French). Much of it is on choice in uncertainty. It includes in particular:

- What was later called *second order stochastic dominance* (and third order and variants too), with all the various *equivalent properties*.
- The "linear uniform *concentration* to and *expansions* from the mean", i.e. what Rothschild and Stiglitz later called "decreasing risk" and "increasing risk".
- Various uses of the *certainty equivalent*.
- *Measures* of risk, notably all those using the certainty equivalent.

And so on.[7]

[7] A number of concepts and results presented in this volume and interesting for income (or other) distribution have not yet been applied to this topic.

2) The 1966 **Biarritz International Economic Association Conference on Public Economics**, with the paper **The Optimal Production of Social Justice** (jointly in English and French).

The book and the paper were written simultaneously in the previous years. Hence the logic common to risk and to distribution (e.g. of income) was applied simultaneously to both topics. The content of the book was the object of seminars and working papers and rested on previous theoretical and applied work.[8]

The 1966 *Collected papers and Proceedings of the Conference on Public Economics*, edited by H. Guitton and J. Margolis in English and French, later appeared in *book form* in 1968 in French (Economie Publique, Paris, CNRS) and finally in English in 1969 (Public Economics, London, Macmillan).

1.2.2 Remarks on formal landmarks

On formal grounds, a few facts can be noted from the beginning of these studies, *in the Biarritz paper*. The first ones concern inequality in the distribution of a quantity such as income.

(1) The comparisons between the **equal equivalent income** $\bar{\bar{y}}$ and the mean $\bar{y}$ of an income distribution of y_i to n individuals i are crucial. By definition

$$W(\{y_i\}) = W(e\bar{\bar{y}})$$

where W is a social ethical evaluation function and e is a vector of n ones.

In particular the case of $W=\Sigma f(y_i)$ additive and symmetrical, and hence *all the generalized means* as equal equivalents $\bar{\bar{y}} = f^{-1}[n^{-1}\Sigma f(y_i)]$ are considered, including explicitly $f(y)=y^{\alpha}$, Log y, $-e^{-\beta y}$.

All the 6 comparisons between the equal equivalent $\bar{\bar{y}}$ *and the mean* $\bar{y}$ *using ratios and/or differences have a specific important meaning*: relative $(\bar{y}-\bar{\bar{y}})/\bar{y} = 1-(\bar{\bar{y}}/\bar{y})$), absolute $\bar{y}-\bar{\bar{y}}$, total $n\cdot(\bar{y}-\bar{\bar{y}})$, yield $\bar{\bar{y}}/\bar{y}$, unit cost $\bar{y}/\bar{\bar{y}}$, excess unit cost $(\bar{y}/\bar{\bar{y}})-1=(\bar{y}-\bar{\bar{y}})/\bar{\bar{y}}$ (they also measure the social waste of inequality).

Further concepts for non-symmetrical W are presented.

(2) The basic comparisons are between income distributions with **different total amounts and means**. The case of same total income or mean is a particular case (the "constant-sum case"). The sum of the m lowest as a function of m or of m/n was called the

[8] This refers to a research report at the Ecole Polytechnique and to administration studies of multidimensional underdevelopment in West Africa.

concentration curve in statistics textbooks. The corresponding **concentration-curve dominance** is considered. Lorenz-curve dominance is too.

(3) Both measures that are **intensive** (i.e. invariant to scale) and **equal-invariant** (i.e. invariant under equal additions) are considered, with the corresponding specific measures (**including the intensive**). Note that "intensive" is the sciences' term for homogeneity of degree zero for a proper reason – i.e. not for unit invariance, which is dealt with by contravariant transformations of the functions.

(4) For each inequality measure I, its **absolute form** I^a and its **relative form** $I^r = I^a / \bar{y}$ (sometimes relative to the equal equivalent $I^a / \bar{\bar{y}}$) are *a priori always* **considered jointly**.

• Hence the interest of *synthetic measures* of inequality which have an intensive relative form and an equal-invariant absolute form.

• The effects of the aggregation of populations and of the addition of various income distributions on inequality are analyzed.[9]

• The concept of egalitarian-equivalence as solution for multidimensional inequalities.

Other works added:

-*Multidimensional* inequalities.[10]

-The principles of *diminishing transfers* (third order stochastic dominance and variants).[11]

-*Intermediate measures* between intensive and equal-invariant measures, by considering inequalities in "augmented incomes" y_i+c where $c \in [0, \infty]$, with an intensive relative measure for $c=0$ and an equal-invariant absolute measure for $c \to \infty$ (if an equal-invariant measure were a relative form, the corresponding absolute inequality would increase under an equal addition to all incomes; and if an intensive measure were an absolute form, the corresponding relative inequality would decrease under an equiproportional increase in all incomes – if all incomes were multiplied by a number, this inequality would be divided by this number; these effects are unlikely).[12] Bossert and Pfingsten (1990) took the augmented-income idea but failed to understand the full concept because they mistook the

[9] The 1966 Biarritz paper also contains the analysis of distribution as a public good with people concerned about others' welfare (hence the importance of the properties of opinions about the comparisons of distribution). In 1970, in a remarkably didactic paper, Atkinson considers $\bar{\bar{y}}$ and focuses on the relative measure $(\bar{y} - \bar{\bar{y}}) / \bar{y}$, $f(y)=y^\alpha$, the constant-sum case and Lorenz dominance. See also Dasgupta, Sen and Starett (1973). Surveys of forms and properties of inequality indexes based on the equal-equivalent $\bar{\bar{y}}$ can be found in Blackorby, Bossert and Davidson (1999) and Weymark (1999).

[10] 1973a (1975, 1977).

[11] 1972 (1973c, 1976b).

[12] 1976b.

relative/absolute distinction for the intensive/equal-invariant distinction. As a result, their alternative proposal, always relative in the foregoing sense, has an impossible absolute form when it is equal-invariant. And as a consequence of their lack of distinction, this proposal lacks consistency: it cannot be derived from an additive social evaluation function of the income distribution whereas this structure is the basic justification of the transfer principle which they insistently want.[13]

-Application of the multidimensional case to the least unequal Pareto-efficient allocation and consequences. Relation with the theory of equity-non-envy.

-Inequalities in *liberty*, with several concepts.

-Inequalities due to desert, merit, and in opportunities.

-Applications to optimum distribution.

1.3 Outline of questions and results

However, both initial inputs – the ethical derivation and the set of equivalent properties – were wanting for related reasons which still affect the foundations of present-day conventional wisdom in inequality studies. Let us outline the rationality of the issue.

1.3.1 Inequality as irrationality or arbitrariness

The basic and first question should probably be: why inequality? Why is inequality sometimes a relevant issue? It could be because of its effect on another issue, such as poverty, or more indirect effects on growth or social unrest, or again some sociologically relevant structures or comparative sentiments which do not exist with equality. Then efficient studies should consider directly these effects. If inequality in itself is a concern, the answer to the question "why inequality?" is probably, in Freudian fashion, "why equality?" in the first place. The answer to this more direct question is that equality is a requirement of rationality in the most common sense of the term (not in economics): providing a reason, a justification, or their possibility. Indeed, if someone should have something for a reason, this reason makes use of some characteristics of this person, and, hence, any other person who has identical such characteristics should have an identical item.[14] This holds *prima facie*, that is, in the absence of overpowering reasons such as impossibility or interference of other relevant principles. This is simply "equal treatment of equals". Then, inequality is stigmatized because it is

[13] See Kolm 1996c and 1999a, page 77.

[14] The full logic of this question is much richer and can be found in Kolm 2010. Kolm 1994 derives consequences for the logic of justified distributions.

irrational or arbitrary. Of course, ethics comes in the choice of the nature of the items and characteristics deemed relevant. This issue is justice (Aristotle: "justice is equality, as everybody thinks it is"). The allocated items that are the basic and direct objects of ethical judgment by a conception of justice are its "end-values".

1.3.2 Income, utility or "welfare"

An unqualified direct concern for lower income inequality reveals that incomes are taken as end-values. This is a priori at odds with "welfare economics" which takes "utility" levels as end-values, since people have different capacities to enjoy (each of us knows someone who has both a higher income and a lesser happiness than someone else). This is $u_i(y_i)$ where u_i is individual i's utility function. However, for the issue of the overall income distribution in a large society, called *macrojustice* – including the effect of the income tax –, people actually hold that differences in tastes, hedonic capacities (capacities to enjoy) and the desires they induce are irrelevant, and, since these people include voters and officials, this idea underlies actual fiscal choices (see sections 2.2 and 2.7). If this leads to discard functions u_i (e.g. Rawls, 1971), incomes y_i may in fact be end-values and their inequality is the problem.

Then, a "progressive transfer" from a higher income to a lower one of less than the difference (or no larger than half of it) may be favoured. It certainly reduces the inequality between these two incomes. But since it also stretches the intervals between the decreased income and equal and higher ones, and between the increased income and equal or lower ones, its effect on the overall inequality for $n>2$ is not unambiguously obvious.[15] However, this does not affect another different but common sentiment favouring this transfer: it would augment the "welfare" of the poorer more than it diminishes that of the richer (at least if the former's income is sufficiently low – transfers between two differently rich people are ambiguous in this respect), and that this is socially beneficial on the whole. This implies first a notion of personal welfare as the same concave function of income for everybody, $u(y_i)$, and, second, an overall evaluation by $\Sigma u(y_i)$, or, perhaps, by any Schur-concave function $W[\{u(y_i)\}]$. For this reason, Bentham (1789) and Pigou (1912) argued that utilitarianism favours income equality. However, utiitarianism is the highest $\Sigma u_i(y_i)$ and individuals have different utility functions u_i. This comparison of variations is a concept of variational local justice: the end-values are the $\Delta u(y_i)$. The progressive transfer of incomes also makes $u(y_i)$

[15] This may explain the reasons of the opinion surveys of Amiel and Cowell (1999) – see a discussion in Kolm (2001a). The logical reasons that can be mobilized for suggesting that such a transfer could nevertheless diminish the overall inequality are gathered in Kolm (1999).

and $u(y_j)$ less unequal by a shrinking of their interval, but the relations with the other $u(y_k)$ are as for the y_i with the same unclear overall conclusion.

However, the first question is the relation between the uniform individual welfare function u and the different utility functions u_i. This will be solved by the remark that the noted macrojustice opinion actually discards *differences* in individuals' tastes and hedonic capacities, which leads to taking as individual welfare function u the technical concept of the *equal-equivalent utility function* (section 2.4). This permits to make sense of one of the noted equivalent conditions, that saying that a distribution gives a higher value than another to $\Sigma u(y_i)$ for all concave functions u, whereas the other proposed justifications of this property are found to be wanting (section 2.1). Moreover, since this definition of function u applies whatever the arguments of functions u_i, for instance when they are $u_i(y_i,\ell_i)$ where ℓ_i is individual i's labour, it defines the uniform functions $u(y_i,\ell_i)$ often used in scholarly studies, for instance by Mirrlees (1971) with the perceptive argument that "differences in tastes raise different kinds of problems" (than those relevant for the optimum income tax).

1.3.3 Equal-labour income equalization

However, actual policy possibilities ignored by these taxation studies(section 3.2), unanimous preference and Pareto efficiency, and a demand for freedom lead, rather, to complement some basic equality of income and labour by an extra freely earned income (sections 2.7 and 2.8). This solution also turns out to amount to an efficient equal freedom of choice for necessarily different domains. It is incentive compatible, and it associates equality and liberty in many remarkable ways (balanced labour reciprocity, basic income financed by equal labour, etc.) (section 3).

If individuals' actual utilities are no longer end-values, indeed, Pareto efficiency has to be reached by some efficient free exchange, and it is desired for several reasons: the classical non-waste property should remain and, in addition, a remaining possibility of unanimous improvement may violate a collective freedom and may threaten the stability of the social state in question and, hence, its actual possibility.

1.3.4 The equal-equivalent manifolds and multidimensional inequalities

As a general rule, discarding utility from the ethical evaluation leaves, as end-values, a bundle of goods or a domain of choice. Multidimensional inequality[16] replaces the unique equal-equivalent income by the *equal-equivalent manifolds* for functions u_i and u, which are iso-welfare u manifolds and define relative and absolute multidimensional inequality measures (section 2.6). Efficiency is secured either by free exchange from an equal allocation or by the noted second-best egalitarian "convex no-envy" (which, however, when each agent consumes some of each good, boils down to equal income with efficiency prices).[17] Moreover, Michael Walzer (1983), following Max Weber (1962), points out a common ethical preference for equality in each specific "sphere of justice".

1.3.5 Freedom from equality or from variously unequal entitlements

Equal freedom of choice for a priori different domains extends to the measure of inequality in this freedom by that of purchasing power (section 4). Moreover, besides valuing freedom from an irrelevance of utility or for Pareto efficiency, there are two further possible twin reasons for taking as social end-value – hence for preferring lower inequality in – the domains of free choice or the liberty they offer, rather than the resulting individual choices (a multidimensional allocation). They are attaching value to freedom in itself (in spite of the costs and anguish of choice and a possible aversion for responsibility), and holding people accountable for what they are responsible for. Responsibility requires free choice, and conversely if one finds responsibility to be relevant. Actually, one classical egalitarian allocative principle consists in proposing an ideal equal sharing of what is given to society and no interference with the allocation of what individuals do, choose and are responsible for, hence in being concerned with inequality in the former items only.[18]

Other ethical views hold that individuals are a priori entitled to or accountable for some given items. These items and the reason for this allocation are varied. However, they often are some personal characteristics and hence their consequences. The great classical polar distributive ethics are distinguished along this criterion. Classical liberals advocate full self-ownership. Full welfarism accepts to redistribute the values of both earning capacities and, by compensations, capacities to enjoy, through income redistribution. Income

[16] See Kolm (1973a, 1975, 1977), Atkinson and Bourguignon (1982), Maasoumi (1986), and a very abundant and outstanding further literature reviewed, notably, in Weymark (2006) and Savaglio and Vannucci (2009).
[17] Kolm 1996b.
[18] This is a main reason for Aristotle's distinction between "arithmetic equality" (equal sharing) and "geometric equality" (according to merit). In recent discussions, see for instance G. Cohen 1989. However, Cohen follows Rawls and Dworkin in holding people fully accountable for their capacities that can be described by a utility function. For logical consequences, see Fleurbaey (2008).

egalitarians (e.g. Rawls) wish to redistribute the product of earning capacities but hold capacities to enjoy to be self-owned. Societies, however, are more subtle and cut within each type of capacities, with a part self-owned and the other being the base of distributive policies. They object to inequality in the distribution of this second part only.

The skill of the equality or inequality theorist consists in interpreting adequately these social ethical positions. Consider a person *i* who can earn a given wage rate of w_i and is free to choose to work ℓ_i which produces the income $w_i \ell_i$. Assume she is entitled to the ownership of a share of her earning capacities only. Moreover, any tax or subsidy including ℓ_i in its base induces Pareto inefficiency. One mistaken interpretation of the right is to equally share the fraction $tw_i \ell_i$ of the income $w_i \ell_i$ ($0<t<1$). Another is to be satisfied with some progressive taxation of the incomes $w_i \ell_i$ and a corresponding decrease in a measure of their inequality. The skillfull policy, by contrast, is to share equally the product of some smaller labour *k* and leave the extra earnings to the person who so is free to use the available capacities by choosing ℓ_i. This is also the genuine interpretation of the noted ethics: individual *i* is entitled to a given share not of her earnings but of her potential earnings and she freely chooses which part of this share to use. The person's income is then

$$y_i = k\overline{w} + (\ell_i - k)w_i$$

where $\overline{w} = \Sigma w_i / n$ if *n* is the number of individuals. The first part is according to *desert* for labour *k*, and the second to *merit*, that is labour plus the capacities it uses. Coefficient *k* is the degree of income egalitarianism of the solution. A country successfully similarly bases its income tax on wage rates by exempting overtime labour earnings over a rather low benchmark (see section 3.2).

1.3.6 Multicriterial inequalities

Finally, the equal-equivalent theory can help solve the problem of *multicriterial* inequality – for instance, what is the measure of the objectionable income inequality if there is some value in the ethics that income should be according to needs and according to work, effort, desert or merit (section 5).

2. Welfare and inequality

2.1 What does "welfare" mean?

2.1.1 Properties

I have to clarify the meaning of a basic property of the theory of unjust inequalities proposed in the Biarritz article. The issue of this paper that has attracted attention the most is a number of equivalent properties for ranking income distributions according to their inequality. Some of these formal relations were known to mathematicians and many others were not but, at any rate, social scientists are primarily interested in the social meanings of these properties, including ethical meanings. This clarification turns out to have theoretical and social consequences that extend far beyond the focussed issue of the theory of income inequality comparison or measurement and are deeper and more important. First, indeed, it justifies a common but as yet unjustified or ill-founded practice in the theory of optimum distribution and taxation, the assumption that the individuals have identical "utility functions". Second, however, it also leads one to complement this theory for values demanded by society, such as equality in liberties.

This property is that $\Sigma f(y_i)$ is higher for one income distribution than for another for all concave functions f. Why? What does this mean? This amounts to the transfer principle which, however, cannot be said a priori to diminish overall inequality. Should this transfer be favoured because it augments the poorer's welfare more than it diminishes the richer, or should we favour a poorer's extra dollar more than a richer's one for the same reason? This is probably the reason of the noted view of Bentham and Pigou that utilitarianism favours equality. However, utilitarianism maximizes $\Sigma u_i(y_i)$. What is a u without an i? We are not helped by the fact that *any* concave function u yields this property, since the u_i are a priori different. What if the receiver is a sedate person fully satisfied with her average income and the other is a greedy or sybaritic character who relishes any extra dollar or knows how to make the best of it? Stating that favouring the highest $\Sigma u(y_i)$ is the view of an "ethical observer" does not help much in itself: why would this person hold this view?

2.1.2 Explanations by uncertainty?

Uncertainty may be called upon in order to try to solve the problem. This may be done in two dual ways: the u_i for each income y_i may be uncertain, or the income of each individual may a priori be uncertain.

A possible answer is that we take function $u(y_i)$ because we do not know the actual utility functions $u_i(y_i)$.[19] In order to obtain $\Sigma u(y_i)$ with an increasing concave function u, one has to start from a utilitarian $\Sigma u_i(y_i)$ with increasing concave functions u_i (these u_i are those relevant for comparing differences in happiness, a comparison which is in itself problematic, but this is the fate of utilitarianism).[20] Assuming a probabilizable uncertainty, and a "rational" risk evaluation, the corresponding social maximand would be E $F[\Sigma \tilde{u}_i(y_i)]$ where $\tilde{u}_i$ is a stochastic function, $F[\Sigma u_i(y_i)]$ is a specification of the risk-relevant, von Neumann-Morgenstern, cardinal specification of the maximand, and E denotes the mathematical expectation. The only general way for this to be ordinally equivalent to a form $\Sigma u(y_i)$ requires two assumptions, each of which is not a priori and in general satisfied. (1) Function F would be affine (and increasing), that is $\Sigma u_i(y_i)$ would be a specification of the risk-relevant cardinal family. (2) Functions u_i would be independent identically distributed and then one would take $u(y)=\mathrm{E}\,\tilde{u}_i(y)$. Moreover, further assumptions are required for the final result. (3) This distribution would itself be unknown. (4) The comparison of the income distributions would be demanded to hold for all possible cases, this being assumed to imply all increasing concave functions u. However, in any application in which more is known about utility functions (even if they remain uncertain) this would have to be taken into account.[21]

Another proposal would be that preferring a higher $\Sigma u(y_i)$ for all concave functions u expresses this preference for any risk-averse individual who considers that he could have any of the actual incomes with the same probability.[22] Such an individual would indeed have a concave risk-relevant VNM utility function u and this would be her "rational" preference in this uncertainty. However, a social ethical judgment about just distribution has no reason to have the same form as an individual selfish preference in uncertainty. These are different questions. The social ethical judgment is accountable towards society, other people and moral.[23] Moreover, a number of individuals are actually not risk-averse in some range of incomes.

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[19] This derivation is worked out in the most meaningful possible way in Kolm 1999a.
[20] See Kolm 1996a, pages 360-366.
[21] In Amartya Sen's presentation at the Biarritz conference (of which I was the discussant), in order to clarify the famous proposition of Abba Lerner and related discussions by Samuelson, Friedman and Harsanyi, the first crucial hypotheses are unambiguously presented as explicit assumptions: "Assumption 3 (Additive Probable Welfare): Probable social welfare is the unweighted sum of the Planner's mathematical expectation of individual utilities", and "Assumption 4 (Equal probability)" assumes the same probability distribution for all u_i.
[22] Vickrey (1945) suggests the idea of an observer considering that he could have any of the actual individual incomes with the same probability.
[23] This objection also applies to all theories in the family of the "original position" (a full discussion is provided in Kolm 2005, pages 358-360).

2.1.3 Other possibilities

In fact, the actual reasoning assumes more simply that there exists something called individual welfare, representable by an increasing concave function of the individual's income, which is the same function for everybody, and such that the quality of the income distribution can be appraised by the sum of the values of this function for all individuals (the sum can be replaced by any Schur-concave function of these values). However, we do not actually know this individual welfare function, and hence we are interested in properties that hold for all such functions. What is, however, the relation between this welfare function and the individuals' different utility functions? What is the actual meaning of this function? Actually, "welfare" is a most ambiguous term (which exists in English only – contrary to "well-being"). It is probably most commonly understood as income or as consumption goods. However, economists focus on "psychological welfare" (Rawls), and the noted intuition of this welfare as an impersonal or transpersonal strictly concave function of income exists (for sufficiently low incomes).

Another possible answer is that function f may not refer to individual properties at all. Maximizing $\Sigma f(y_i)$ with an increasing strictly concave function f may just describe attaching some value to equality in the y_i since this is the outcome of this maximization for sharing a given Σy_i. This would be so because the y_i are taken as end-values. This means that, for this distributive justice, the individuals are deemed accountable for all the psychological and physiological characteristics that utility functions represent.[24] When the y_i are not all equal, the loss in the evaluation induced by this unequal sharing of Σy_i depends on the specific choice of function f. Hence, if all we know is this preference for equal y_i in the sharing, we are a priori interested in properties that hold for any such function f.[25]

However, since the transfer principle and its equivalent properties (including "concentration" and Lorenz curves dominances – see section 1.2) happen to occupy such a large place in the studies of inequality with a normative connotation, it is important to relate it to direct judgments about people's situation. And this is still much more important given the applications such as optimal taxation and distribution. Then, since the attempted theoretical derivations from combinations of utilities with some uncertainty are so shaky, and since the

[24] The Biarritz paper writes "responsible", as Ronald Dworkin (1981) and John Rawls later said about "tastes", but responsibility implies possibility to influence, which is only limited in this case. Influencing one's own desires is a central topic of the volume Kolm 1982.

[25] The additive form amounts to comparisons such that the variation of a subset of the y_i augments or diminishes inequality irrespective of the levels of the fixed, unchanged y_i.

justifications of the comparison use a concept of welfare, one has to define this relevant concept. However, this concept is not clearly defined by psychology or physiology alone. Welfare is somewhere between goods or income and happiness. Then, a way of understanding better what is meant by this interpersonally compared entity consists in finding out its complement, that is, what people think *not to be relevant*, in individuals' psychology or physiology, when comparing the effects of variations of individuals' incomes. This would be the complement, in "utility" encompassing all effects, of the comparable welfare deemed relevant.

However, simple observation shows that the psychological (and physiological) items deemed to be relevant or irrelevant in interpersonal comparisons of the effects of distribution depend on the context. They are different, in particular, for the distribution between members of a large community not more specifically related – for instance in national distributions such as by the income tax – or for distribution within a family or between suffering people. Our concern here is general income distribution, hence the former case, called the issue of "macrojustice". Note that the present question is the nature of the items used or not in the comparison – e.g. happiness, welfare, tastes, etc. – rather than the structure of their use – for instance as the corresponding egalitarianism, utilitarianism or something in between. Then, considering how one answers, or, more importantly, how people in general answer, questions such as the following raises the crucial and central issue of social ethics.

2.2 Questions and distinctions

Should you pay a higher income tax than someone else because she likes dollars more than you do, notably those taken away and one is utilitarian, or less than you do, notably the remaining ones and one is egalitarian (in utility)? Should you pay more or less than the other because the other (or you) has a cheerful character (which may lead one to enjoy a euro more or to regret its absence less – opposite effects again)? Or are these psychological characteristics or differences thought to be irrelevant for this issue, that is, people would be deemed entitled to their benefits and accountable for their shortcomings (as enforcing money transfers or modulating the income tax to compensate for differences in physical beauty is neither practiced nor – it seems – advocated)?

Should you finance someone else's beverage because she only likes expensive wines? This classical "expensive tastes" argument extends in two ways. The other person may have to compensate you for your inability to experience such delicate gastronomic pleasures. And utilitarians meet "cheap tastes": should you finance the other's beverage because she likes

cheap beer, and hence generates low-cost utility? Differences in tastes raising "different kinds of problems" is precisely the reason Mirrlees gives in 1971 for taking identical individual utility functions for determining the income tax.

Rawls (1982) takes up still another possible meaning of "utility" when he notes that, for "social justice" (the present macrojustice), "Desires and wants, however intense, are not by themselves reasons in matters of justice. The fact that we have a compelling desire does not argue for its satisfaction any more than the strength of a conviction argues for its truth".

Finally – since income is mostly earned – should I take the 10 dollars you just earned because I like them more than you do (or more than you dislike the labour with which you earned them)?

When everybody shares the same opinion, this includes people who chose a policy such as voters and officials, and a policy based on the opposite opinion cannot be implemented. This is also respect for democracy. This property is the unanimity aspect of "endogenous social choice", i.e. finding the social choice criteria in society itself.

Of course, as we have noted, tastes and capacities to enjoy or hedonic capacities are prominent when allocating within a family or, more generally, small groups with mutual information and empathy between members. Such capacities are also unanimously taken into account when the issue is the relief of suffering. These are the proximity-empathic and painfull welfarisms ("familism" and "dolorism"). This may indeed be altruism motivated by empathy or pity rather than issues of justice. If we relate them to justice they would be cases of microjustice concerned with allocations specific with regard to people, reason, goods or circumstances. This opposes the question of *macrojustice* concerned with the general rule of society and the allocation of the value of the main resources to everybody in general purchasing power. (It is also useful to consider a domain of mesojustice concerned with goods that are specific but particularly important and concern everybody, such as education and health).

The foregoing remarks may be thought of as condemning welfarism for macrojustice (as, for instance, Rawls did). However, we will see that the opposite is the case: they save welfarism, by permitting to define "strict welfarism" using a universal individual welfare function u – different from "utility-welfarism" that considers functions u_i – and which can both describe the "welfarist intuition" noted at the onset and justify using such a function in various studies.

2.3 End-values of macrojustice theories

The foregoing questions may lead one to discard individual tastes and hedonic capacities, or, more thinly and sufficiently, differences in them, to determine socially possible and desired macrojustice (which includes the income tax and main transfers). Moreover, fully discarding individual utility functions leads to two possibilities. If this function just represents the satisfaction, pleasure, happiness, etc., the individual derives from consumption, this discarding leaves consumption goods. If, rather, this function intends to describe the individual's choice by its maximization, discarding it leaves the domain of the individual's free choice.

Let us denote $x_i \in X \subseteq \Re^m$ a vector of quantities of goods for individual i. They may be or include final consumption goods or other goods. In particular, they may be individual i's income y_i or (y_i,ℓ_i) or (y_i,λ_i) where ℓ_i and λ_i denote individual i's labour and leisure respectively. Let also $u_i(x_i)$ denote a "standard" utility function of individual i; $u(x_i)$ the interindividually identical "individual welfare function"; and L_i a domain of free choice of individual i. Some freedom of choice is also implicit in the allocation of goods that are not final consumption goods. Income y_i is notably such a case: it may represent either an aggregation of consumption goods or purchasing power (a kind of freedom).

The individual end-value of macrojustice social ethics can be $u_i(x_i)$, $u(x_i)$, x_i or the liberty offered by L_i. If the noted general opinions about hedonic capacities and tastes lead to discard utility functions u_i, the resulting end-values are x_i or L_i. If one considers these opinions as objecting to the relevance of interindividual *differences* in hedonic capacities and tastes only, the end-value may be a function $u(x_i)$, the same for all individuals, if one can derive such a function from the functions u_i by erasing the effects on them of different hedonic capacities and tastes.

From the rationality of equality, the fact that an item is such an end-value is manifested by a preferred, ideal, *prima facie* equality of this item across individuals. When this equality finds, as obstacle, impossibility or the joint relevance of some other principle, the solution is often described by the maximization of some social ethical maximand which would yield this equality if these obstacles did not exist – hence, observing this maximand reveals the underlying end-value.

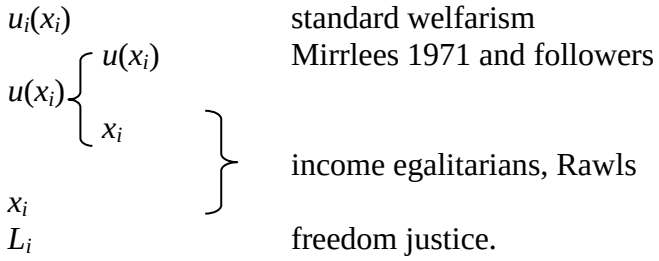
This individual end-value is, for instance, x_i (in particular y_i or (y_i, ℓ_i)) for standard "egalitarians". It is also for Rawls who concludes, from the observation that distribution is never achieved by maximizing a $W[\{u_i(x_i)\}]$, that functions u_i have to be discarded altogether for what he calls "social justice" – which is our macrojustice (he says "not micro" and, once,

"macro", but his term may be ambiguous since, for instance, it might be understood as including the care of abnormal handicaps whereas this is an issue in microjustice). However, Rawls emphasizes individuals' freedom of choice from and with means allocated to them which are his "primary goods" (plus classical basic liberties). They are his end-values which should ideally, *prima facie*, be equal. There is one economic primary good in 1971, income y_i (or wealth), to which he adds leisure in 1974 at the instigation of Richard Musgrave (this may better be called free time, as time free from labour, for a primary good, in contrast with the consumption-good flavour of the concept and term of leisure).

Standard "welfarist optimum income tax" studies use a function $u(x_i)$ with $x_i=(y_i,\ell_i)$. Most of them say that it is because they do not know functions u_i. However, as we have recalled, the initiator Jim Mirrlees (1971) happens to be more profound and a keener observer by stating that he takes the same function u for all individuals for discarding irrelevant "differences in tastes". Yet this raises a problem for the theory since individuals maximize functions u_i rather than function u, and classical Pareto efficiency is also with functions u_i. Mirrlees, then, in 1986, reverts to functions u_i and rejects function u. This raises the vast information problem of knowing the u_i, but this is not actually an obstacle since, at any rate, society discards differences in tastes and hedonic capacities for the choice of the income tax, it does not determine it by comparing marginal variations (or overall levels) of individual utility functions u_i, and hence by maximizing any function $W[\{u_i(y_i,\ell_i)\}]$. Moreover, we shortly remark that the standard maximization of $W[\{u(x_i)\}]$, with the same function u and often $x_i=(y_i,\ell_i)$, can be read in two ways: the social individual end-value may be seen as either $u(x_i)$ or x_i.

However, if the bundles of goods x_i include several goods, their ideal multidimensional equality fails in general to be Pareto-efficient (individuals' preferences are a priori different). This problem is not raised by income taken by itself (income egalitarians, Rawls 1971), but it is if leisure or labour is added (e.g. Rawls 1974) – yet this case is particular because the individual prices, the wage rates w_i, are a priori different. One solution consists in letting individuals freely exchange from their equal allocation – this will be suggested shortly for income and labour.

Finally, this discussion of the end-values of a just overall income distribution leads one to consider five general cases – two of which amount to the same, but which will have to be added a further one – for the nature of the social individual end-value, with generally $x_i=(y_i,\ell_i)$:



What is, however, this interpersonal function *u*? Mirrlees and others provide no clue to its determination or, indeed, to its precise meaning.

2.4 The equal-equivalent utility function as the individual welfare function

The psychological (or physiological) distinction between hedonic capacities and tastes on the one hand and individual welfare on the other would be an interesting piece of analysis, but it is not a necessary one here. Indeed, what matters to represent the noted common opinion about the irrelevance of individual differences is erasing these differences in a social ethical evaluation. Therefore, given a standard welfare function $W[\{u_i(x_i)\}]$, define function $u(x)$ as, for each $x \in X$,

$$W[\{u_i(x)\}] = W[e\ u(x)] \qquad (1)$$

where e is a vector of n ones and n is the number of individuals i. Adopting the standard assumption that function W is *non-decreasing and increasing in at least one argument at each point*, function $u(x)$ is well-defined. This operation "averages away" the differences in functions u_i. However, for this averaging to be "balanced", function W has to be *symmetrical*, which we assume. And for this property to be meaningful, functions u_i have to be comparable by more or less, that is at least *co-ordinal, the only requirement for defining function u*.[26] This function W is used as "averaging function".

By an analogy with the *equal-equivalent income* of inequality theory, function *u* can be called the **equal-equivalent utility function**. It can be taken to represent the common *individual welfare function* when the interindividual differences in tastes and in hedonic capacities are averaged away.

If individual utility functions u_i are uncertain (as they are) and representable by stochastic variables $\tilde{u}_i$, function *u* can also average uncertainty away by choosing for W a specification $\hat{W}$ of the corresponding von Neuman-Morgenstern evaluation function and defining $u(x)$ by

[26] Hence there is a "fundamental utility" in the sense of *Justice and Equity* (1971).

$$\mathrm{E}\hat{W}[\{\tilde{u}_i(x)\}] = \hat{W}[e\,u(x)].$$

where E denotes mathematical expectation, for each x.

Calling $w(a) = W(e\,a)$, form (1) also gives function u as

$$u(x) = w^{-1} \circ W[\{u_i(x)\}],$$

and similarly with uncertainty.

Then, the strictly welfarist maximand is

$$W[\{u(x_i)\}] = W\left(\{w^{-1} \circ W[\{u_j(x_i)\}_j]\}\right). \qquad (2)$$

If $W = \Sigma g[u_i(x_i)]$, then $u(x)$ is the generalized mean of the $u_i(x)$ with function g,

$$u(x) = g^{-1}[n^{-1}\Sigma g \circ u_i(x)]$$

and

$$W[\{u(x_i)\}] = n^{-1}\Sigma_{i,j}\, g \circ u_i(x_j)\,.$$

In particular, for the utilitarian $W=\Sigma u_i(x_i)$, $u(x)= n^{-1}\Sigma u_i(x)$, and the comparison is of $\Sigma_{i,j} u_i(x_j)$.

With $W=\min u_i(x_i)$, $u(x)=\min u_i(x)$, and the comparison is of $\min_{i,j} u_i(x_j)$.

Form (2) is a function of all individuals' evaluations of all individuals' allocations $u_j(x_i)$. One can show that, in the two polar cases of "equity" defined as $u_i(x_i) \geq u_j(x_i)$ for all i, j (sometimes problematically called "no envy")[27] and of "adequacy" defined as $u_i(x_i) \geq u_j(x_i)$ for all i, j – the u_i being comparable – [28]

$$W[\{u(x_i)\}] \leq W[\{u_i(x_i)\}]$$

with < if at least one of the defining inequalities is <. The difference between both values means that some part of the individuals' satisfaction – say that which is due to their *sui generis* tastes or hedonic capacities rather than to their individual welfare as defined here – is left to the accountability of the individuals rather than counted in the purely welfarist social evaluation. The relation is easily seen. With adequacy, the replacement of all functions u_i by the "average" u for evaluating x_i lowers the value left for the social evaluation. And, with equity-no-envy, replacing in form (2) the $u_j(x_i)$ by $u_j(x_j)$ to obtain $W[\{u_i(x_i)\}]$ augments (does not reduce) the social value.

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[27] The description of the sentiment of (strong) envy requires the consideration of other utility functions, of the form $U_i(x_i, x_j)$ with $U_i(x_i, x_j) < U_i(x_i, x_i) \equiv \tilde{U}_i(x_i)$ expressing the painfulness of this sentiment and where $\tilde{U}_i(x_i)$ so defined represents individual i's "envy-free preferences" (see Kolm 1995). Equity occurs for instance when the individuals choose their x_i on the same domain, for instance when they are (y_i, ℓ_i) with a uniform income tax.

[28] Kolm 1971.

The definition of function u has two consequences, one for the theory of unjust inequality and the other, more important, for the theory of social optimality.

For the theory of inequality, consider a utilitarian evaluation function $\Sigma u_i(x_i)$ with functions u_i having the required metric. If differences in tastes and hedonic capacities are irrelevant, the equal-equivalent utility function $u(x)=n^{-1}\Sigma u_i(x)$ replaces functions u_i and the evaluation function becomes $\Sigma u(x_i)$. One readily sees that function u is concave if functions u_i are. For income $x_i=y_i$, the functions are $u_i(y_i)$, $u(y)$ and $\Sigma u(y_i)$. If functions u_i are not known but are known to be concave, one may want an evaluation by $\Sigma u(y_i)$ for all concave functions u. This uncertainty is used as in Lerner's theory, but there is no assumption that Σu_i is the von Neumann-Morgenstern social welfare function. Moreover, if not all functions u_i are concave, function u may nevertheless be concave, provided that a sufficiently large fraction of the u_i have this property. The result uses the utilitarian sum and the possible ignorance of individual utility functions u_i. However, it is *a consequence of the formulation of the general opinion of irrelevance of differences in hedonic capacities and tastes for judging the overall income distribution*. It rests, basically, on the endorsement of this general moral stance, which is practically unavoidable for implementing resulting policies.

2.5 The welfare-relevant measure of inequality

Consider the case of income distribution, $x_i=y_i$ for all i. An evaluation-consistent measure of inequality is based on the equal-equivalent $\bar{\bar{y}}$ defined with an evaluation function $U(\{y_i\})$. This $\bar{\bar{y}}$ is utility-consistent if $U(\{y_i\})=W[\{u_i(y_i)\}]$. However, for macrojustice discarding differences in hedonic capacities and tastes, functions u_i have to be replaced by the individual welfare function u. We thus have a utility-consistent and a welfare-consistent equal equivalent, $\bar{\bar{y}}_u$ and $\bar{\bar{y}}_w$ defined by

$$W[\{u_i(y_i)\}]=W[\{u_i(\bar{\bar{y}}_u)\}]=W[eu(\bar{\bar{y}}_u)]$$

and

$$W[\{u(y_i)\}]=W[eu(\bar{\bar{y}}_w)]=W[\{u_i(\bar{\bar{y}}_w)\}]$$

respectively. A priori $\bar{\bar{y}}_w\neq\bar{\bar{y}}_u$ if the y_i (or the functions u_i) are not all equal. If $u_i(y_i)\geq u_i(y_j)$, or $u_i(y_i)\geq u_j(y_i)$, for all i, j with at least one strict inequality, then, from a previous remark, $\bar{\bar{y}}_w<\bar{\bar{y}}_u$ and the inequality measures $(\bar{y}-\bar{\bar{y}})/\bar{y}$ and $\bar{y}-\bar{\bar{y}}$ are larger for the welfare-relevant inequalities than for the utility-relevant ones.

2.6 The equal-equivalent manifolds and measures of multidimensional inequality

2.6.1 Inequality in welfare or well-being: truncations

If inequality matters for its effects on people's welfare or other well-being only (at least by its direct effects), the inequality in the individuals' bundles of goods $x_i \in \Re^m$ can be reduced to the inequality in the corresponding values of functions $u(x_i)$ or $u_i(x_i)$. Function u provides an ordering of the x_i but nothing more specific with the foregoing assumptions since it is ordinal. Functions u_i provide the same structure since they have been assumed to be co-ordinal. Consider, then, either u or the u_i, and denote $v_i=u(x_i)$ or $v_i=u_i(x_i)$, respectively. Assume that the judgments are invariant under permutations of the i in each situation. The available structure permits the following inequality comparisons. For n=2, inequality can be said to be lower for (v'_1, v'_2) than for (v_1, v_2) if $[v'_1, v'_2] \subset [v_1, v_2]$. The former pair is said to be "inclusion more equal" than the latter. For any n, this generalizes in the *truncation* comparison: inequality decreases if there exist two numbers a and b with $a \leq b$ such that any $v_i<a$ increases to a and any $v_i>b$ decreases to b, the others v_i do not change ant at least one v_i changes. A truncation can be realized by a sequence of transformations in which each pair of values is unchanged, or is transformed into an inclusion more equal pair, or is such that the two values are equal and vary by remaining equal.[29] However, more specific comparisons, and measures, are a priori favoured.

2.6.2 Reduction to income

If individuals are deemed accountable for their ordinal preferences, or responsible for their choice, and buy the goods with their income, the multidimensional inequality reduces to the one-dimensional income inequality. If the goods are marketed at a uniform market price for all, if p is the m-vector of the prices and y_i is the individual i's income, the dual (Roy) utility function of individual i is $\hat{u}_i(y_i, p) = u_i(x_i)$, function of income for given prices. If some prices are not the same for all individuals – an example is provided by the case in which leisure is one of the goods, with the wage rate as buying price – the standard solution to resort to the comparison and measure by the purchasing power or real income with a linear price index is studied in section 4.

2.6.3 The equal-equivalent manifolds

[29] See Kolm 1999.

In other cases, the multidimensional inequality may have to be faced more directly. In the unidimensional case, the definition of the equal-equivalent income has permitted the derivation of measures of inequality from a social evaluation function. For the general multidimensional inequality between the *n* bundles $x_i \in \Re_+^m$, the evaluation is a function $U(\{x_i\})$: $\Re^n \times \Re^m \to \Re$. However, for $n \geq 2$, the equal equivalent is no longer a single quantity but any bundle $x \in \Re_+^m$ of the $m-1$ dimensional **equal-equivalent manifold** $E \subset \Re_+^m$ defined as

$$x \in E \Leftrightarrow U(\{x_i\}) = U(ex) = U(x,...,x)$$

where *e* denotes the *n*-duplication.

For evaluation functions that respect individuals' utilities or welfares, the equal-equivalent manifold is a iso-equal-equivalent utility, that is, iso-welfare, manifold. Indeed, these two cases are, from the definitions of *E* and *u*, with the corresponding $E=E_u$ and E_w,

$$U(\{x_i\}) = W[\{u_i(x_i)\}] = W[\{u_i(x)\}] = W[eu(x)] \Leftrightarrow x \in E_u$$

and

$$U(\{x_i\}) = W[\{u(x_i)\}] = W[eu(x)] = W[\{u_i(x)\}] \Leftrightarrow x \in E_w,$$

respectively. If the allocation $\{x_i\}$ satisfies equity-no-envy or adequacy with at least one strict inequality, E_w corresponds to a lower *u* than E_u (figure 1).

Figure 1. The equal-equivalent manifolds

2.6.4 Evaluations-consistent relative and absolute measures of multidimensional inequalities

Denote $\bar{x} = (1/n)\Sigma x_i$.

The only possible measure of inequality in the x_i without additional property or information seems to be the relative inequality *I* defined by $(1-I)\bar{x} \in E$.

However, one can also define absolute measures of inequality *J* by defining a reference bundle $a \in \Re_+^m / \{0\}$ and having $\bar{x} - Ja \in E$. Bundle *a* is a standard relevant basket of goods chosen as are the analogous baskets of the coefficients of linear price indexes, by statistical, economic or social considerations (it may in particular be a quantity of only one "numéraire" good, for instance income if this is one of the goods).

Hence,

$$U(\{x_i\}) = U[e \cdot (1-I)\bar{x}] = U[e \cdot (\bar{x} - Ja)].$$

It is assumed that U is increasing and that there is at least some amount of some good. Hence I and J are well-defined. If $a=\bar{x}$, $J=I$. If we had $x_i=0$ for all i, then $\bar{x}=0$ and $J=0$. Since there is some quantity of some good, $U>U(0)$ and $\bar{x}\neq 0$, hence $1-I>0$ and $I<1$. If all x_i are equal, $x_i=\bar{x}$ for all i, $I=J=0$. If U satisfies the strict generalized concavity property $U(\{x_i\})<u(e\bar{x})$ if the x_i are not all equal, then $1-I<1$ and hence $I>0$, and $J>0$, if the x_i are not all equal. For given $\bar{x}$, $-I$ and $-J$ can be taken as U, hence have the corresponding properties of function U.

Among these properties, U, I and J may be symmetrical in the x_i – expressing that the x_i are the only relevant properties of the individuals. U may be concentration-increasing, and I and J concentration-decreasing, meaning that they increase or decrease under a "concentration" of the x_i, that is, $U[(1-\lambda)x_i+\lambda\bar{x}]$ is an increasing function of $\lambda\in[0,1]$.[30] They are pair-wise concentration-increasing (for U) and concentration-decreasing (for I and J) when they increase (vs. decrease) under similar concentrations of pairs of x_i (a multidimensional possible extension of the transfer principle). U may increase, and I and J decrease, when the allocations of each good become more equal, for instance in a commodity-wise concentration (concentration for one commodity only), or in a transfer principle for each good; this expresses good-specific inequality aversion, a sentiment that Max Weber holds to be general and emphasized in Michael Walzer's "spheres of justice" – however, this would not hold for very specific goods substitutable with others but may hold for larger classes of goods.[31]

However, these reductions of inequality between individuals' commodities or bundles of commodities in the multidimensional context neglect people's preferences (the standard, non-moral ones, i.e. the $u_i(x_i)$). In particular, multidimensional equality is a priori not Pareto efficient with these different utility functions u_i. Consider, then, utility-respecting and welfare-respecting evaluation functions, that is, $U(\{x_i\})=W[\{u_i(x_i)\}]$ and $W[\{u(x_i)\}]$. The corresponding inequality measures are the relative and absolute utility-consistent and welfare-consistent inequality indexes I and J, I_u, J_u and I_w, J_w respectively, defined as

$$(1-I_u)\bar{x}=\bar{x}-J_u a\in E_u \text{ and } (I-I_w)\bar{x}=\bar{x}-J_w a\in E_w.$$

With equity-no-envy or adequacy with at least one strict inequality, $I_u<I_w$ and $J_u<J_w$. Differences in the x_i imply that $W[\{u_i(x_i)\}]$, which is symmetrical in the u_i, is not in the x_i,

[30] See Kolm 1966a.

[31] Actually, one of Walzer's spheres is general income and its spending. Walzer should have noted that it is much larger in volume than the other, whatever the de facto extent of the market in modern societies. This is the field of macrojustice. Other spheres are domains of mesojustice (e.g. education, health) and microjustice.

and hence the free distribution of given $n\bar{x}$, or redistribution transfers, that maximize it generally lead to unequal x_i and hence to some optimal positive inequalities I_u and J_u ("to each according to her tastes, needs or preferences"). Equality in the u_i (which are co-ordinal) is provided by W=min which gives a highest u_i (or min u_i) with some allocation inequalities I_u and J_u.[32] However, when one considers broad classes of goods rather than overly specific goods, there is generally a strong correlation between the distribution of each good: the same people have more or less of each (as Rawls says of his "primary goods", for instance, but this holds with more numerous types of goods) and the correlation is also with income. Hence the conflict between material equality and differences in tastes seems less serious in fact than in theory.

2.6.5 Egalitarian-equivalents

A classical concept of multidimensional allocation is that of egalitarian equivalence (Kolm 1966b, Pazner and Schmeidler 1978).

Definition

The allocation $\{x_i\}$ is *egalitarian-equivalent* if there exists an individual allocation x such that $u_i(x_i)=u_i(x)$ for all i.

This x will be denoted the *equivalent* (individual allocation).

Theorem 1.

The equivalent of an egalitarian-equivalent allocation is on the utility equal-equivalent manifold of this allocation for any evaluation function W.

Indeed, if $u_i(x_i)=u_i(x)$ for all i,

$W[\{u_i(x_i)\}]=W[\{u_i(x)\}]=W[eu(x)]$

by definition of function u, which implies $x\in E_u$ by definition of E_u, whatever the function W. Hence, all manifolds E_u for all functions W intersect at the equivalent x.

[32] If, however, differences in individuals' tastes and hedonic capacities are morally irrelevant, only the remaining uniform "welfare" u remains. Moreover, classical studies that use identical functions u (e;g; for "optimum taxation") assume them to be strictly concave and assume W to be a symmetrical concave function of the $u(x_i)$ (which then have a quantity metric); this implies $W[eu(\bar{x})] > W[\{u(x_i)\}]$ for unequal x_i and hence an optimality of $I_w=J_w=0$. With these assumptions, the concept of welfare may reconcile egalitarianisms in allocations and in individuals' evaluations. The outcome is not Pareto efficient with the different utility functions u_i (utility Pareto-efficiency), but it is with the functions $u(x_i)$ (welfare Pareto-efficiency). The common moral opinion that discards differences in tastes and hedonic capacities may advocate the latter. If the u_i or u are assumed to have properties of quantities, then the one-dimensional average, equal-equivalent and the derived inequality measures can be expressed for utilities. Bosmans, Lauwers and Ooghe consider explicitly a fundamental utility (Kolm 1971) but co-cardinal, and derive a "multidimensional" transfer principle with it.

The set of equivalents to Pareto-efficient egalitarian-equivalent allocations constitute a m–1 dimensional manifold in the space of quantities of goods. For instance, there is one on each ray from the origin (Kolm 1996a, page 252).

2.6.6 Optimization and Pareto efficiency

When the allocation $\{x_i\}$ varies and augments $W[\{u_i(x_i)\}]$ or $W[\{u(x_i)\}]$, manifolds E_u or E_w shifts towards higher quantities, respectively. In a free allocation of given total quantities $n\bar{x}$, $\bar{x}$ remains fixed and I_u and J_u, or I_w and J_w, decrease, respectively. The optimum $\{x_i\}$, which maximizes $W[\{u_i(x_i)\}]$ under the constraint gives the limiting E_u. For given $\bar{x}$, this gives the lowest such inequalities I_u and J_u, the optimum inequalities. Then, when function W varies, this optimum describes all Pareto efficient allocations. For given $\bar{x}$, one (in general) of them gives the smallest I_u. This is, in this sense, *the least unequal Pareto-efficient allocation* (a similar concept can be defined for J_u for given a).

Another remarkable egalitarian Pareto-efficient allocation is that which is egalitarian-equivalent with an equivalent proportional to $\bar{x}$.

2.7 Strict welfarism and beyond

Strict (restricted, weak, pure) welfarism is obtained by replacing utility functions u_i by the individual welfare function u. Its social maximand is, therefore,

$$W[\{u(x_i)\}]. \qquad (2)$$

Studies that use such a maximand, notably with $x_i=(y_i,\ell_i)$ for "welfarist optimum income taxation", always take a weakly concave W (for allowing for the "utilitarian" sum) and a strictly concave function u.[33]

A maximand of form (2) can a priori have, as end-value relating to individuals i, either the $u(x_i)$ or the x_i. If such end-values are all that relevantly distinguish the individuals, the maximand should first be symmetrical in them, and, second, prefer an equalization of these values. The symmetry is the case for both the $u(x_i)$ and the (x_i) since symmetrical function W is symmetrical. For the x_i, however, this would justify having replaced the u_i by u, since symmetrical function $W[\{u_i(x_i)\}]$ is a priori not symmetrical in the x_i. Five reasons suggest preferring the x_i to the $u(x_i)$ as end-values.

[33] Therefore they assume u to have the metric of a quantity.

(1) Empirically, Bourguignon and Spadaro (2008) find that the progressivity of the actual income tax schedules cannot be derived from a maximand of the form $W[\{u(y_i, \ell_i)\}]$, which suggests that this is not the ethics followed implicitly or explicitly by governments.

(2) Theoretically, the end-value is the item the equality of which is preferred *prima facie* (ideally, intrinsically). With a strictly concave SWF W, this can be "pure welfare" $u(x_i)$. Indeed, if the $u(x_i)$ are not all equal,

$$W[\{u(x_i)\}] < W[e\, n^{-1} \Sigma u(x_i)].$$

However, one can do better for "society's welfare". Indeed, if the x_i are not all equal, with a weakly concave W (allowing for the utilitarian sum) and a strictly concave function u,

$$W[\{u(x_i)\}] \leq W[e\, n^{-1} \Sigma u(x_i)] < W[e\, u(\, n^{-1} \Sigma x_i)].$$

That is, if we take as end-value the x_i rather than the $u(x_i)$, equality gives a socially better situation. In this sense, the equality of the x_i is a better egalitarian ideal than that of the $u(x_i)$. This may imply that the x_i constitute a priori a deeper end-value than the $u(x_i)$.

(3) In the noted general opinions for choosing the income tax (section 2.2), the differences between the individuals' enjoyments discarded as irrelevant may a priori be due to differences in the x_i and not in the functions u_i, and then they could remain with the $u(x_i)$. Then the noted opinions may discard function u also, or at least its concavity (with $x_i = y_i$) (but egalitarianism in y_i, in $u(y_i)$, or the highest $\Sigma u(y_i)$ would elicit the same answers).

(4) Selecting the x_i as end-values joins the solution directly obtained by egalitarians (and by Rawls) which interpret the common view of irrelevance as discarding individual utility functions u_i altogether, or as a result of this discarding.

(5) The equality of the x_i, applied to earned income and labour and completed as indicated shortly, will turn out to have many meaningful properties. Then, it amounts in particular to the last of the rationales noted above (section 2.3), the equal liberty of domains of choices. Now, a noted central tenet of ethical epistemology is that a principle has to be evaluated from all its angles and possible meanings, with all its properties.

With equal $x_i = x$ for all i, maximand (2) takes form (1) which has two remarkable consequences. First, it amounts to maximizing $W[\{u_i(x)\}]$, with the actual individual utilities u_i, with their full actual differences. Second, it also amounts to maximizing $u(x)$. This determines the best equality, but depends on the corresponding constraints.

2.8 Distributing earned income

The main question is the distribution of earned income. This is the largest part of income, by very far in an intertemporal view in which capital, which is itself produced by definition, is taken as the result of the natural resources, one of which is individuals' given productive capacities used by labour.

Then, $x_i=(y_i,\ell_i)$, or the equivalent (y_i,λ_i) for post-1974 (post-Musgrave) Rawls. The equality is $y_i=y$ and $\ell_i=\ell$ for all i. If w_i denotes individual i's wage rate (unit productivity), the constraint on the distribution is $ny=\Sigma w_i\ell$ or $y=\ell\,\overline{w}$ where $\overline{w}=(1/n)\Sigma w_i$ is the average wage rate. Hence the best choice is $\ell=k$ that maximizes $u(y,\ell)$ under this constraint or $u(\ell\,\overline{w},\ell)$. For a differentiable function and an interior solution, k satisfies $\overline{w}\,u_1+u_2=0$. This is the choice of labour and earnings by the "average individual" with the average utility function u and the average productivity $\overline{w}$. Figure 2 representing labours and incomes shows the maximum of function u under the constraint represented by the line of slope $\overline{w}$ from the origin, reached at point $K=(k,\overline{w}\,k)$.

Figure 2. The two-part income: equality and liberty

This solution, however, has three defects.

1) Liberty. The solution generally violates individuals' freedom since they prefer, from this allocation, to work more (this will be the relevant case) and keep the extra earnings.

2) Pareto efficiency. For the same reason, this solution is generally not Pareto efficient.

3) Self entitlement. One of the questions of section 2.2 suggests that people attach some value to being entitled to one's earnings. Since these earnings depend on the individuals' given productivity, this implies valuing some self-ownership of these productive capacities.

The same answer solves these three problems: from the obtained egalitarian solution, let people free to work more and to keep the extra earnings. Then, if individual i freely chooses to work ℓ_i, she earns the extra $(\ell_i-k)w_i$, and her resulting disposable income is

$$y_i=k\,\overline{w}+(\ell_i-k)w_i. \qquad (3)$$

This is a two-part income, sum of an egalitarian income $k\,\overline{w}$ and of a "classical liberal" income $(\ell_i-k)w_i$. The egalitarian income is the same for everybody, and results from the redistribution of individuals' earnings with the same labour k, the "equalization labour" (the informational possibility is shown shortly). Free exchange without redistribution is a property

advocated by "classical liberalism". It is applied here to the extra labour $\ell_i - k$. Coefficient k is a degree of equalization. The particular case $k=0$ is the ideal of full classical liberalism. Figure 2 shows the individuals' budget lines from the common point K with slopes w_i, and individuals' choices on these lines.

2.9 Summary of theories

The question of the end-value and objective of the various theories, and the way they derive from one another, can now be summarized (figure 3). From the individual utility functions $u_i(x_i)$, used in a SWF $W[\{u_i(x_i\}]$, of basic economics, general opinion about income distribution in macrojustice leads one to discard either the utility functions (as Rawls does, for instance) or the differences in hedonistic capacities and tastes. The latter solution leads to "strict welfarism" with an individual "welfare function" $u(x_i)$, the same for all individuals. This gives meaning to the highest $\Sigma u(y_i)$ with concave u of Pigou and Bentham and of the classical property of income inequality theory. However, the quantity of goods x_i is still preferred as an equalizand, which joins the solution of directly discarding utility functions from the $u_i(x_i)$. This is the solution of income egalitarians and of Rawls (1971) who later adds leisure (1974) or, equivalently, labour. From this equality, adding free exchange of labour permits liberty, Pareto efficiency and some demanded self-ownership of productive capacities.

Figure 3. Four theories

3. ELIE, equal liberties

3.1 Equal-labour income equalization

This distributive scheme is "equal-labour income equalization" (ELIE). We shortly see that it amounts to equal liberties (with different domains of choice). Form (3) also writes

$$y_i = w_i \ell_i - T_i \qquad (4)$$

where

$$T_i = k \cdot (w_i - \overline{w}) \qquad (5)$$

is a tax or a subsidy of $-T_i$ if $T_i<0$. That is, each hour of person i's labour k is taxed by $w_i - \overline{w}$ if $w_i > \overline{w}$ and subsidized by $\overline{w} - w_i$ if $w_i < \overline{w}$. This de facto implies $k \leq \ell_i$ since taxing leisure is generally not accepted and providing a wage supplement for hours that produce no wage

seems absurd. In fact, ELIE schemes that diminish any reasonable measure of income inequality as much as actual national redistributions have an equalization labour k of 1 to 2 days per week (from the USA to social-democrat Scandinavian national communities). Hence $k<\ell_i$ for normal full labours ℓ_i. Cases of $\ell_i \leq k$ are reported to the general case by particular theories and devices, such as a theory for involuntary unemployment[34] or, for part-time labour contracts or second wages in families, as in the French tax law presented in the next section. The remaining extremely rare cases of particularly low labours of productive people have a number of possible solutions which are cases of microjustice. They range from a universal basic income permitting non-earning activities (van Parijs) to drafting people whose labour can save lives, passing through just demanding these able people to pay their consumption – the solution of both Rawls and Saint Paul ("he who does not work does not eat").

A consequence is that $y_i \geq k\,\overline{w}$, a minimum income determined along with coefficient k. In rather homogeneous societies, there is often a rough consensus about a norm of minimum income which entails a similar opinion about the distributive coefficient k.

Individual i's "total income", her income plus the value of her leisure $w_i\lambda_i$, is, taking $\ell_i+\lambda_i=1$ as the measure of total time,

$$v_i=y_i+w_i\lambda_i= k\,\overline{w} +(1–k)w_i, \qquad (6)$$

or, if $v=\{v_i\}$ and $w=\{w_i\}$ denote the vectors of the v_i and w_i,

$$v= k\,\overline{w}\, e+(1–k)w. \qquad (6')$$

That is, the v_i are a **concentration** (a linear uniform concentration towards the mean) of the w_i with coefficient k.[35] This structure of transformation is one of the two simple ones that diminish inequality the most for a given amount of transfers.[36] It amounts to transforming the w_i by a proportional decrease (in proportion k) and an equal increase that restores the total sum. A *synthetic inequality index* is an index the absolute form of which $I(x)$ for $x\in \Re_+^n$ is *equal-invariant* ($I(x+\mu e)=I(x)$) and the relative form of which $I^r=I/\overline{x}$ with $\overline{x}=n^{-1}\Sigma x_i$ is *intensive* ($I^r(\lambda x)= I^r(x)$), from which the absolute form is *extensive* ($I(\lambda x)=\lambda I(x)$). Then, for the absolute form of any synthetic index, $I(v)=(1–k)I(w)$, and $k=[I(w)–I(v)]/I(w)$ in which I^r can equivalently replace I. Examples of synthetic indexes are the Gini absolute and relative coefficients, the variance and the standard deviation, and $\Sigma(|x_i–\overline{x}\,|)$ for $I(x)$.

[34] Kolm 2005, chapter 12.
[35] See Kolm 1966a for application to risk.
[36] The other is bitruncation noted above.

This distributive structure has other important meanings. Its transfers are *from each according to her capacities, or equally in labour, to each equally* (in income). It also amounts, for its participants, to an *equal universal basic income* ($k\overline{w}$) *financed by an equal labour of all* (k) *or according to capacities*. Moreover, it amounts to a *general balanced labour reciprocity*: each individual yields to each other the product of the same labour (k/n). This distribution also amounts to an *equal sharing of the value of productive capacities* when the fraction k is measured in *income value* (output) and the rest in *labour-leisure value*. Each individual receives according to *desert* or to her work for labour k and to *merit*, i.e. to her work and her capacities, hence to her works, for the rest.

These transfers are those of the distributive branch or function of public finance. Other taxes finance other public expenditures. If distribution is optimum in this way, these other financings should be neutral in this respect, that is they should be according to benefit taxation. There are, however, other classical principles of public finance. One is paying according to capacities, which, for earned income, should be capacities to earn, the w_i. Another is by "equal sacrifice" which could be by an equal effort or labour. Both come to the same. Individual i finances bw_i of budget $B=n\,b\,\overline{w}$. This is how ELIE finances the basic income of $k\overline{w}$ and, on the whole, individual i pays $(k+b)w_i$, the product of her labour $k+b$. Then, on figure 1, individual budget lines are translated towards higher ℓ by b, and they all pass through the same point $\ell=k+b,\ y=k\,\overline{w}$. Individuals may also pay an equal amount a (which translates the budget lines down by a and moves their common point to $y=k\,\overline{w}-a$, $\ell=k$). They may also make payments of both types, $a+bw_i$, which provide the budget $n(a+b\overline{w})$ and give both the foregoing translations to the budget lines, moving their common point to $y=k\,\overline{w}-a$, $\ell=k+b$. Principles of all types may exist jointly.

This simple, core distributive theory is completed in various ways. The first extends it to multidimensional labour (duration, intensity, education-formation-training, etc.). These extensions are the object of two volumes: *Macrojustice* (2005) and a collective volume edited by C. Gamel and M. Lubrano (2010).[37]

3.2 Informational possibility: the example and experience of the French tax system

Tax T_i of (5) can also be written as

$$T_i=(k/\ell^o)w_i\,\ell^o-k\,\overline{w} \qquad (7)$$

[37] See also the contributions of Fleurbaey and Maniquet, and Trannoy and Simula in Fleurbaey, Salles and Weymark (2010).

where ℓ^o is a benchmark labour. This shows that it amounts to two bonuses, an exemption of the earnings of overtime labour over ℓ^o, and a uniform tax credit or rebate $k\overline{w}$, the same for all individuals, from a linear income tax.

Mirrlees (1971) suggests that the tax base is earned income $w_i\ell_i$ because the government cannot know the wage rates w_i, and a vast literature starts with this assumption. He ends the same article, nevertheless, by noting that labour duration ℓ_i can also be observed, which yields the wage rate w_i, and that we have other means of estimating a person's earning capacities which however – he thinks – would induce much hiding and evasion.

However, the French tax system has the two bonuses of form (7), including the exemption of overtime labour over a low official labour duration.[38] It amounts to basing the tax on the wage rate w_i. This duration can be reduced so that most working people do some overtime labour. This applies to wage income which is 9/10 of labour income. There is practically no cheating because this could not be done without the tax administration being informed about it.[39] Inputs of labour other than duration are also taken care of. Education is public and free and financed by the income tax (hence choosing more education elicits higher public costs financed with the future higher tax on the increased wage rate). Productivity premia (for labour intensity) and formation premia will be truncated. In non-wage labour (1/10 of total), self-employed people, professionals and farmers often pay a lump-sum tax. Productivity can be estimated by comparison with wage labour of the same type. All the routine of tax administration with statements, checking, various estimates, verification, penalties, etc. can be and is used.[40] Tax authorities find that, on the whole, evasion is very much lower than when the base is total earned income.[41]

The gain in revenue efficiency and administration costs is matched by gains in overall economic efficiency, justice and liberty. Marginal labour is not taxed, inframarginal labour units only are. The exemption is both for the income tax and for the "contributions" financing social security, and the marginal wedge so suppressed can be as high as 65%. The tax base consists in given productive and earning capacities (when all dimensions of labour are taken

[38] Expressed in hours per week and in days per year for executives whose daily hours are not clear. For part-time labour contracts, the exemption is of the "complementary hours".

[39] This is due to the reporting by employers. Tax authorities are very clear about this result and explain why employers cannot de facto hide their pay accounting and cheat on it (except, perhaps, when there is no more than a couple of employees). This situation is being integrated in economic analysis (see Kleven, Kreiner and Saez, 2009).

[40] There are minor attempts of people who work more intensely than standard to present the extra earnings as due to overtime labour. But since intensity is another input of labour, this is perfect for the theory and instructions are to accept their claim.

[41] It is easier not to declare overtime labour than to declare false overtime labour because, in the latter case, the unit wage should also be falsified and it is related to the category of labour.

into account) that is, items individuals are not responsible for. There is no marginal interference with free exchange of labour and, as shortly noted, ELIE schemes induce people to work with their most valuable skills (and thus to reveal them), they respect basic social liberty even for people who pay a tax, and they secure equal real freedom.[42]

3.3 Social liberty: full "formal" freedom (the ELIE paradox: the higher the tax paid, the larger disposable income and de facto freedom of choice are)

The two standard kinds of liberties (apart from mental freedom) are relevant for income distribution. One is social liberty, freedom from forceful interference, also more or less described as negative freedom (Kant, J.S. Mill, Berlin), civic freedom (Mill), basic rights or liberties, or "formal freedom" (Marx). The addition of other means of various possible kinds provides "real freedom" (Marx). In a society with social liberty, individuals are only constrained not to use force against others – insofar as they do not voluntarily abstain from it. When individuals' intentions are incompatible, this is solved by the allocation of the relevant means, often rights and in particular property rights, resulting from the distribution and exchanges. Hence social liberty can be full for all, and it is then equal.

Social liberty implies that people have the right to act (without forceful interference and forcefully interfering), hence to use their capacities: they have the use-rights in their capacities, here the free choice of labour ℓ_i. Classically, this liberty is conceived from an allocation of given resources (as with Pareto's model of markets). This is the case with ELIE. However, with $k>0$ there is some redistribution of the value of the earning power of given productive capacities – i.e. of the rent of these capacities. Then, some people receive transfers but some others pay a tax. However, the following property holds.

Theorem 2 (the social liberty theorem)

With an ELIE structure of transfers, someone who pays a tax, or a higher tax than someone else, has a higher disposable income for the same labour and more de facto *freedom of choice (she can both work less and consume more*).

Proof

[42] A welfarist (with function *u*) optimum tax study (Kolm 1974) optimized for both the tax schedule and the tax base. It then proposed ways of basing the tax on wage rates, including the exemption of overtime labour, and analyzed the resulting tax structure. However, the present French tax policy resulted from public presentations and discussions of the proposals of the volume *Macrojustice.* The present overtime exemption has been the central policy proposal of a presidential candidate.

With $k>0$, individuals i with $w_i<\overline{w}$ receive $k\cdot(\overline{w}-w_i)>0$. However, individuals with $w_i>\overline{w}$ yield $T_i=k\cdot(w_i-\overline{w})>0$. Such an individual i with a higher w_i pays a higher T_i, but her disposable income $y_i=k\overline{w}+(\ell_i-k)w_i$ is also higher for each $\ell_i>k$. This implies a higher freedom of choice of the pair (ℓ_i, y_i), by inclusion of domains of possible choice. The individual can in particular work less (ℓ_i) and consume more (y_i). Moreover, this amount is globally transferred to individuals with $w_i<\overline{w}$, which augments their freedom of choice. The basic reason for the proposition is that, with an ELIE scheme, someone who pays a higher tax T_i has a higher earning power w_i, and the tax takes only part of this advantage. □

This possibility shown by ELIE transfers has a major importance for political philosophy. Classical liberalism opposes all non-voluntary transfers. It defines itself by full self-ownership or by social liberty and thinks that they imply each other. Full self-ownership certainly implies social liberty, would it only be from the concept of ownership. The converse rests on the idea that a forced transfer – say a tax – forces the person to work more or to consume less. With an ELIE scheme, however, a tax, and a higher tax, go with the possibility of the taxed person to work less and to consume more. Therefore, classical liberalism has better justify self-ownership of the value of one's given productive capacities otherwise, which it can do with its second kind of justification, the concept of natural right. That is, an individual's capacities fully belong to her (property) because they belong to her (being part of); they are hers because they "are" her; and earned income results from the free exchange of their services. Classical liberalism advocates full self-ownership, which is the particular case of ELIE with $k=0$.

3.4 Equal total freedom

Identical domains of choice is doubtlessly a case of equal freedom. This is not achieved by ELIE schemes with different given wage rates w_i. Moreover, it is possible to define identical domains from which the individuals choose their ℓ_i and y_i, but if the result has to be Pareto efficient and to respect social liberty in the sense of section 3.3 and if the w_i are not all equal, then the construction of such sets has to use information about specificities of people's preferences.[43] Yet these characteristics are assumed to be irrelevant for this problem. However, it is possible to define equal economic freedom that respects Pareto efficiency and social liberty with different w_i and does not consider people's preferences, but the

[43] Kolm 2008, pages 9-10.

corresponding domains of choice are not identical. ELIE distributive schemes correspond to this case, with two different and independent definitions of equal freedom.

One definition, which is a classical simple principle, is *equally free exchange from an equal allocation*. Figure 2 illustrates this structure. The equal "initial" allocation is of income $k\overline{w}$ and of leisure 1–*k* or labour *k*. It is represented by point *K*. From it, the individuals freely exchange labour for earned income with the wage rate they can obtain w_i.

However, the following result also holds.

Theorem 3 (the total economic freedom theorem)

The ELIE distribution corresponds to different budget sets providing equal freedom of choice.

Proof

The standard way of defining equal consists in deriving it from a definition of more and less. This leads to order domains of choice according to the freedom of choice they provide. This order is sufficiently described by an ordinal "freedom function" $F(D)$ such that $F(D)>$, = or < $F(D')$ expresses this freedom order for two domains of choice D and D'. An individual's choice of labour ℓ and income (consumption) y can be described by her choice of y and of leisure $\lambda=1-\ell$ in her budget set $Py+W\lambda\leq V$ where $P>0$ is the price of consumption goods and $W=Pw$ and $V=Pv$ her wage rate and her total income expressed in the same nominal units, respectively (figure 4 for $P=1$). This domain of choice is fully described by this total income V and the prices P and W. Hence one can write $F=F(V; P, W)$. If the prices are classically represented by a linear price index $\alpha P+\beta W$ with $\alpha>0$ and $\beta\geq 0$, $F=\phi(V, \alpha P+\beta W)$. Since multiplying P,W and V by the same positive number does not change the equation of the domain, and the domain, function F is homogeneous of degree zero, and so is function ϕ. Hence $\phi=\varphi[V/(\alpha P+\beta W)]$. Since F is ordinal, so are ϕ and φ, and φ is increasing since F is increasing in V. Hence the ranking is according to $V/(\alpha P+\beta W)$. This is the classical "purchasing power" or real (total) income. The foregoing amounts to the "axiom" that freedom of choice in budget sets is ranked according to purchasing power. Note that linear price indices are the standard use (for instance the Paasche and Laspeyre indices). They represent the value of a bundle of goods the quantities of which are the coefficients. The only other meaningful price indexes are those derived from a given utility function taken in its indirect (Roy) form. They are not relevant here since the reference is not utility but liberty. The linear price index is neutral in the choice of units of consumption goods and of labour/leisure since each price is multiplied by a quantity of the corresponding good, and

these obtained money values are meaningfully added – since prices intervene in the same direction (a higher price of any good restricts freedom of choice by inclusion of domains). Any other aggregation of the money values of the goods can hardly have actual meaning.[44]

Then, since $V/(\alpha P+\beta W)=v/(\alpha+\beta w)$, equal liberty means that $v=(\alpha+\beta w)\,\gamma$ for some constant γ. That is, for all i, $\beta\gamma w_i+\alpha\gamma=v_i=w_i-T_i$ or $(1-\beta\gamma)w_i-T_i=\alpha\gamma$. Since $\Sigma T_i=0$, this implies $(1-\beta\gamma)\,\overline{w}=\alpha\gamma$ and, denoting $1-\beta\gamma=k$, $T_i=k\cdot(w_i-\overline{w})$, that is ELIE. □

All budget lines $y_i+w_i\lambda_i=v_i=w_i-T_i$ pass through the same point $K(\ell_i=k,\ y_i=k\,\overline{w})$.

Figure 4. Equal-freedom budget sets

3.5 Incentive compatibility

ELIE transfers do not depend on labour ℓ_i and, therefore, do not induce the corresponding wasteful disincentives. However, they depend on wage rates and this could induce people to work with capacities that are not their most remunerated ones. Indeed, if w_i denotes the highest wage rate individual i can obtain, this individual can also generally earn various rates $w'_i<w_i$ by not using her best (most highly paid) skills at work.[45] She may make such a choice if she thinks that the fiscal authority bases her taxes and subsidies on this actual and observed w'_i, in order to diminish the tax or transform it into a subsidy if $w_i>\overline{w}$, or to augment the subsidy if $w_i<\overline{w}$ (hence she would benefit whatever $\overline{w}$ if $k>0$, and therefore she need not know $\overline{w}$ to behave this way). The individual may think that the government would take the observed w'_i as base either because it deems the actual wage rate to be the appropriate basis so as not to tax or subsidize unused capacities of value $w_i-w'_i$ (just as it chooses $k\leq\ell_i$), or because it mistakes it for the value of capacities w_i, or for any mixture of these reasons. However, the following property holds.

Theorem 4 (incentive compatibility)

With ELIE transfers, individuals choose to work with their most remunerated capacities.

Proof

Individual i thus chooses both labour ℓ_i and skills that earn $w'_i\leq w_i$, that maximize some increasing ordinal utility function

[44] See also Kolm 2009.
[45] See Dasgupta and Hammond (1980).

$$u^i[1-\ell_i,(\ell_i-k)w'_i+k\overline{w}'],$$

where $\overline{w}'=(1/n)\Sigma w'_j$.[46] Variables ℓ_i and w'_i are independent. The derivative $\partial u^i/\partial w'_i$ has the sign of ℓ_i-k+k/n if individual *i* takes the w'_j for $j\neq i$ as given (no collusion), but whatever they are. Therefore, individual *i* chooses $w'_i=w_i$ if $\ell_i>k\cdot[1-(1/n)]$. This is the case for macrojustice in which $\ell_i\geq k$. □

Hence, *the individuals choose to work with their best skills and thus to "reveal" their capacities and to exhibit their economic value*. The government can understand this (it need not know individuals' utilities, but only that individuals prefer higher disposable incomes for given labour). Hence, it need not raise questions about basing taxes and subsidies on the actual values of capacities w_i or on the observed wage rates w'_i since using the latter as base makes them be the w_i. And the individuals can in the end know this conclusion.[47]

3.6 The coefficient of equalization

The directly meaningful parameter with respect to inequality is the coefficient or degree of equalization *k*. This equalization labour describes an equalization of proceeds of unequal earning capacities from the result of free exchange alone. Its complement 1–*k* is a degree of remaining inequality due to the inequality in earning capacities (which the individuals are free to use for earning or not by being responsible for this choice of labour). As we have seen, *k* is also the degree of concentration of the distribution of total incomes (labour included), equal to the relative decrease in synthetic measures of inequality under the distribution policy. It is also the fraction of the average wage rate turned into a minimum income and the equal labour that provides the basic income. Socially, this degree of redistribution is a degree of solidarity with respect to the endowment of earning powers, common ownership of the value of productive capacities and balanced labour reciprocity. This is a highly significant figure. Similarly, for any redistribution, the duration such that a complete equalization of incomes during it reduces a measure of inequality as much as this redistribution does, is a particularly

[46] Choosing a more remunerated but more painful or disagreeable activity, or the contrary, is considered as working more or less, and a corresponding full analysis has to consider, in a framework of multidimensional labour, the relevant dimension(s) that affect both the productivity and the painfulness or intrinsic attractiveness of labour.

[47] If the government used the w_i if it could know them, with $T_i=k\cdot(w_i-\overline{w})$, and each individual *i* could choose her skills used and $w'_i\leq w_i$, her income would be $\ell_i w'_i+k\cdot(\overline{w}-w_i)$, and she would also choose $w'_i=w_i$ if she chooses to work at all ($\ell_i>0$) and hence when $\ell_i>k$.

meaningful measure of the intensity of the equalization achieved by these transfers. This is the coefficient of the "equivalent ELIE". It provides a particularly clear and intuitive meaning of the degree of equalization achieved by a policy. Its use in public statistics, comparative or not, would be quite enlightening (expressing values in duration of earning or spending is, for this reason, a rather common practice in popular presentations). Moreover, we have seen that this index is particularly and richly meaningful on the grounds of ethics and theory.

In the foregoing, coefficient *k* has been derived from a SWF *W*. However, one should determine this function, as well as any such function used otherwise, and the meaningfulness of coefficient *k* also permits a more direct determination of its level. A number of methods have been developed for determining the degree of compromise between equality and self-ownership desired by a society and by the impartial judgments of its members.[48]

4. Inequality in economic freedom of choice or purchasing power

After inequality in a quantity and multidimensional inequality, the next step is inequality in domains of choice. Actually, income is often considered for the freedom of choice in buying goods it provides (e.g. by Rawls), although, if it is earned income, leisure or labour are to be added, as in the previous section. Besides the straightforward comparison by inclusion which notices that a domain that includes another does not offer a lower possibility and freedom of choice, one would have in general to define different domains providing equal freedom, a complete (weak) ordering of domains according to the freedom of choice they provide, and if possible a measure of this freedom. The previous section has shown an example of equal freedom for different domains. Actually, this is a particular application of a common practice for a particular but also particularly important case. This is the basic economic case where the choice is buying with an income and the domains are budget sets. If prices are the same for each agent, the issue is simply comparing incomes – the sets are related by inclusion. The problem occurs when the prices also differ (different places, dates, conditions, or wage rates when leisure or labour are among the goods, etc.). The common practice, then, is to compare the freedom of choice by the *purchasing power*, the income divided by a price index or "real

[48] See Kolm 2005, part 4. Note that for individuals *i* with $w_i = \overline{w}$, T_i=0 whatever *k*, hence their self-interest is not affected by the level of *k* and therefore their opinion about it expresses their social ethical view only.

income". The inequality can then be any unidimensional income inequality with these magnitudes.[49]

If $y \in \Re_+$, $x \in \Re_+^m$, $p \in \Re_{++}^m$ and $a \in \Re_+^m / \{0\}$ denote income and vectors of quantities of goods, prices and coefficients of the price index, respectively, and *px* and *ap* denote scalar products, a budget set is $B(y, p) = \{x : px \le y\}$, a price index is *ap*, and the purchasing power or real income is $\eta = y / ap$. The price index *ap* is the value of the bundle *a* (this price index for purchasing power does not refer to a utility). The relation $\eta ap = y$ shows that the point ηa is on the budget hyperplane $\{x : px = y\}$. Hence η is the largest number of vectors *a* that can be added within the budget set. The η of budget sets $B(y, p)$ and hence their comparison depend on the choice of *a*, and this choice of the relevant price index is a classical object of economists' and statisticians' discussions and of social choices and contracts it may concern.

This structure of the measure of the possibility offered by budget sets can be derived from two basic formal axioms. Denote a priori a measure of the freedom of choice offered by $B(y, p)$ by the continuous function

$$F(y, p) = \Phi(\pi)$$

where $\pi = p/y$ is the income-normalized vector price (function *F* is homogeneous of degree zero).

Theorem 5 (the purchasing power theorem)

The freedom of choice F is proportional to the purchasing power η if and only if:

1) $\Phi(\pi) > \Phi(\pi') \Rightarrow \Phi(\pi + q) > \Phi(\pi' + q)$ *for all admissible* $q \in \Re^m$.

2) $F(\lambda y, p) = \lambda F(y, p)$ *for all admissible* $\lambda \in \Re_+$ *and all p.*

Proof[50]

Condition 1 holds if and only if there is a $a \in \Re_+^m / \{0\}$ such that $\Phi(\pi) = \varphi(a\pi) \equiv f(\eta)$ with $\eta = y/ap = 1/a\pi$. Then condition 2 yields the result. □

49 Ranking or measuring the possibility or freedom of choice by the volume of these domains has the advantage of putting some weight on all possibilities. It has, however, a number of diriment vices. (1) The volume is zero when any one good is excluded (quantity zero). (2) The volume ranking in *m*-dimensional space differs from the ranking of the volumes in *m*–1 dimensional space of the boundary hypersurfaces constituted by the points of the possibility sets that dominate others (for all other points there is one point of this hypersurface that offers more of at least one good and less of none). Either preference for the goods or free disposal require considering these boundaries. The different ranking is seen, for instance, for budget sets and *m*=2. (3) The volume ranking of budget sets does not change if the prices of goods are permuted. (4) The volume ranking of two budget sets does not change if the same number multiplies the prices of one good in one and of another in the other (consider goods very different in importance). See Kolm 2009.

50 See Kolm 2009.

5. Multicriterial inequality and the method of egalitarian equal-equivalent, desert and partial merit

Inequality is unfair *from a point of view* if individuals' (or other relevant entities') corresponding characteristics are identical and hence can *prima facie* justify equal items only. Inequality, however, may be not only multidimensional but also multicriterial. Let us denote the end-values as individuals' incomes for illustration and because it is a very important case, but they may be any other items (perhaps individual welfare), in any group or situation. To each according to her needs or to her work (or effort) are different criteria that underlie classical contending philosophies. Unfair inequality may be different incomes for the same needs or for the same work or effort. One may choose one principle, but political compromise or moral synthesis may also lead to consider both. And there may be a larger number of criteria (which may for instance complement or specify these two). How can this multiplicity of criteria be taken into account in description and – this is what matters in the end – in policy? An answer is the method of "egalitarian equal-equivalents" or "equalizing averages". It consists in replacing each income of a set of individual incomes that should ideally be equal for a reason by the same income which is an "egalitarian equal-equivalent" or "equalizing average" of these incomes, in a sequential way shortly described.

Given a subset of ν incomes $y=\{y_i\}$, an "egalitarian equal-equivalent" of these incomes is an income y^e defined as

$$F(y)=F(ey^e)$$

where e is a vector of ν ones and F is an "equalizing" inequality-averse function: an increasing symmetrical $\Re^\nu \to \Re$ function which takes higher values when the y_i are in some sense "more equal". For instance, it can be a quasi-concave or a Schur-concave function. Income y^e is an equal-equivalent of the incomes y. It is well-defined. Denoting $F(e\eta)=f(\eta)$ it writes $y^e = f^{-1} \circ F(y)$. It is a specification of function F which need only be ordinal. In particular, it is an increasing function of each y_i. A particular structure of function F is additivity, $F = \Sigma\varphi(y_i)$, with an increasing strictly concave function φ, and y^e is the corresponding "generalized mean" $y^e = \varphi^{-1}[\nu^{-1}\Sigma\varphi(y_i)]$. This permits to define corresponding functions F for groups of incomes of different sizes ν. When function F or φ is "more concave" in some sense, the replacement of these y_i by y^e in an overall evaluation describes a higher inequality-aversion about their distribution. This aversion and function F may depend on the specific reason for this preference for lower inequality.

When all reasons are taken into account as shortly noted, the result will be an evaluation function (or policy maximand) of all the y_i which is an increasing function of each y_i and is higher when the y_i the inequality between which is unfair for some reason become more equal. The increasingness guarantees non-waste (Pareto efficiency if this translates to utilities).

There is a number of criteria. Each of them partitions the set N of the individuals i into subsets in which the y_i should be equal from this point of view (for instance some family structure, health status, age, etc. on the side of needs, effort and desert, merit, seniority in occupation, any entitlement on the side of supply, nations if these are the accepted entities of solidaristic distribution, and so on). Individuals who are identical according to all criteria should ideally have the same income, and then y_i is replaced by an egalitarian equal-equivalent of them. However, we specify one criterion and choose a function F which corresponds to the estimated "badness" of the corresponding inequality (it may or may not depend on the situation of the other criteria). Then the inequalities stigmatized for this reason (criterion) have been taken care of. We are left with the other criteria and the corresponding coarser partition of the individuals. We again replace the incomes in each of these new subsets by an egalitarian equal-equivalent of them, but do it with a function F which corresponds to the inequality aversion for another specific reason (criterion). The process continues till the exhaustion of criteria. At the last stage, there is no partition any longer and only aggregation with some inequality-aversion. This last unique egalitarian equivalent – or simply the function F – is the evaluation function or social ethical maximand. The result depends on the order chosen for the criteria.[51]

For instance in the simple but important application of income produced by labour plus the effects of policies, there is an a priori larger or smaller inequality aversion about the consequences expressed by an evaluation function $U(y)$ for $y=\{y_i\}$, plus a value of the desert principle "to each according to her work or effort". If individual i produces effort x, y_i is replaced, in $U(y)$, by an egalitarian equal-equivalent of the y_i of the individuals who provide the same effort x, denoted as y^x, and the evaluation or maximand becomes

$$U(\{e_x y^x\})$$

[51] Except if all the equalizing averages are generalized means with the same function φ. See Kolm (2001b). A literature considers one criterion only, for instance need (such as family size) in Atkinson and Bourguignon's (1987) sequential analysis, or effort in Roemer (1998). Roemer takes min for function F and a final non-egalitarian sum of mins (hence this is not an increasing function of all items, and augmenting the lowest – in each category – is generally particularly costly because of decreasing returns of policy although there are several of them). This should then be submitted to sequential analysis across efforts (a marginal dollar should be valued more not only when given to a poorer for the same effort but also, for the same income, when given to someone who works harder: the multidimensional dominance then applies).

where e_x is a vector of n_x ones where n_x is the number of these persons.

However, the equalizing policies, for individuals different in earning capacities and preferences, tend to induce Pareto inefficiency (notably if people are free to choose their labour, which provides the responsibility reason for desert in addition to effort). Taking utilities as end-values may not be thought relevant and their ordinality does not permit much in specification of inequality. Efficiency is a priori respected by people being entitled to the full product of their labour, due to both their effort and the capacities it uses, that is, according to *merit* rather than to desert. However, the consequential inequality in the y_i may then be large and desert is discarded. A solution is to resort to *partial or relative merit* in the sense that an individual *i* keeps her earnings above some benchmark $\tilde{y}_i < y_i$ given to her. The policy is then concerned with the $\tilde{y}_i$ only. The $\tilde{y}_i$ may then replace the y_i in maximand *U*, and they may be fully equalized as is the case with an ELIE policy where they are $k\overline{w}$ which is also the same income for the same labour *k*, that is according to desert.

6. Conclusion. Taking ethics seriously: a needed but challenging program for inequality studies

By relating explicitly to social ethical evaluation, the field of economic inequalities entered an era of explicit meaningfulness naturally matched by richness and generality in formal structure. A most useful part of the present program of the field – it is suggested – would be to systematically build on the whole of social ethical thought. This includes both scholarly social ethics and justice theory and the perceptive and integrated analysis of common opinions. The main types of issues raised by this program are readily seen. Philosophy usually discusses types of equality, whereas the actual situation is inequality and the relevant judgment is in terms of comparing inequalities – which substantially enriches the issue. Unveiling the relations between the various principles and criteria, already a rich tradition in normative economics,[52] will be embedded in a broader and deeper meaning and directed by it. A basic problem is to translate – in Bernard Williams terms – *thick principles* which are richly suggestive but imprecise into the relevant *thin principles* formalizable but the social importance of which is not clear at first sight. Important problems are raised by the issue of the meaningful properties of the relevant material of inequality: a quantity (income), multidimensional, various types of liberty (of choice, basic rights, non-domination,

[52] See a survey in Thomson 2008.

autonomies), ranking and ordinal, co-ordinal, cardinal and co-cardinal, and the vast issue of selecting the appropriate representations and proxies. The various spheres of justice raise different issues, notably concerning the distinction macro, meso, micro. Inquiries about common opinions are often essential. A recurring theoretical question in various forms is the consistency between concepts of equality and Pareto efficiency. Multicriterial justice is the common situation for broad issues (e.g. income distribution). The relations of inequality to social sentiments and their consequences is a whole field in itself, not only relative deprivation and sense of fairness, but senses of community, solidarity and reciprocity, altruism, and envy and sentiments of inferiority and superiority. Last but not least, the final issue is not to know the world but to improve it, and theoretical and empirical analysis should be related to the corresponding social choice and policy.

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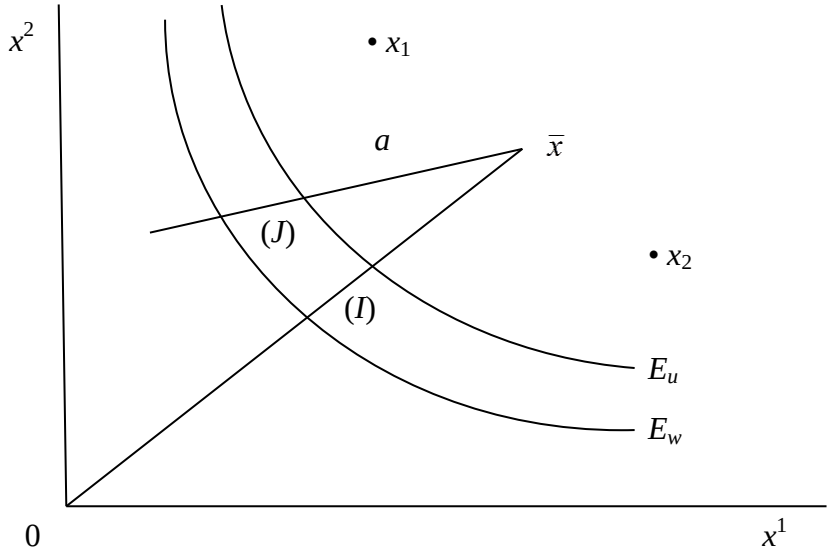

Figure 1. The equal-equivalent manifolds

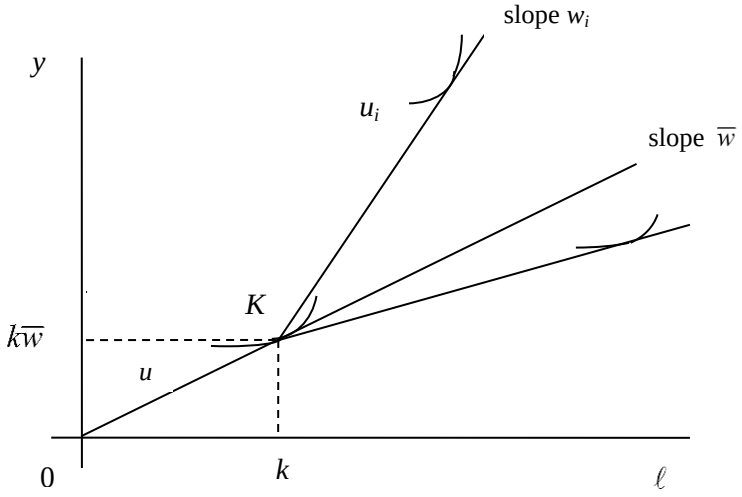

Figure 2. The two-part income: Equality and liberty

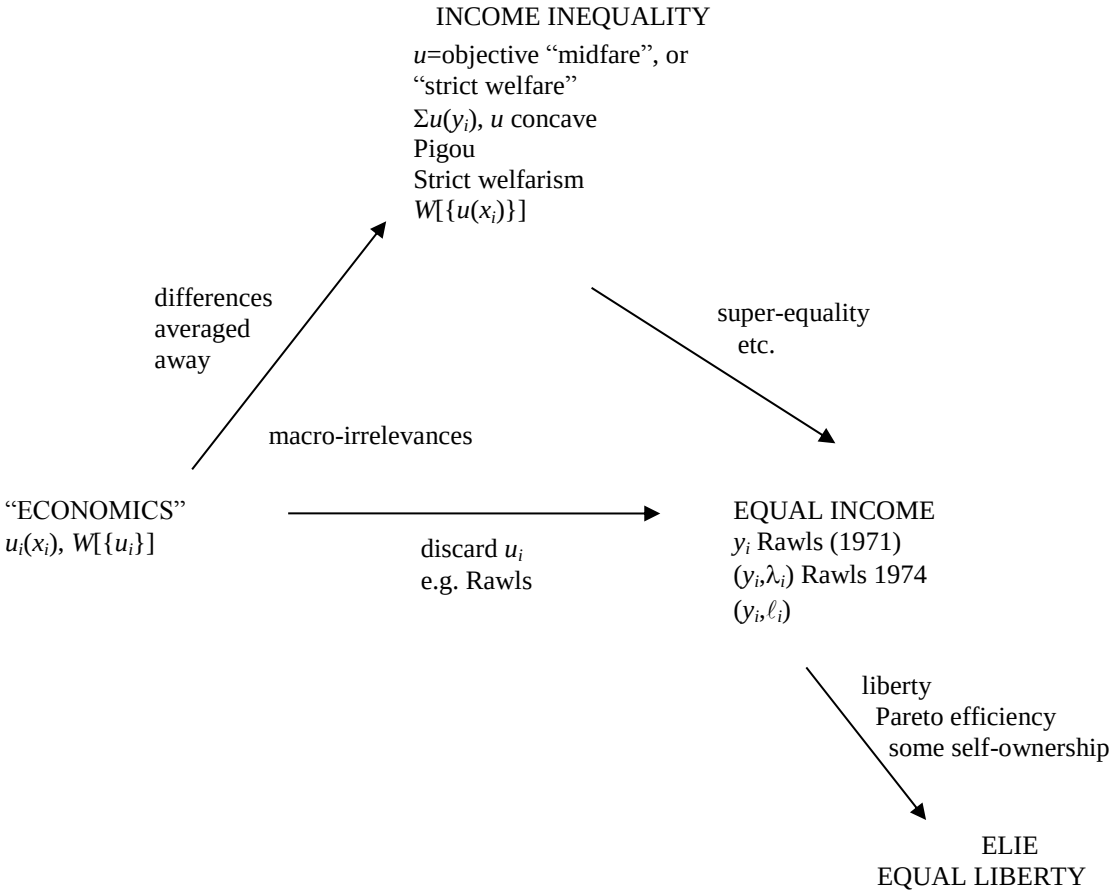

Figure 3. Four theories

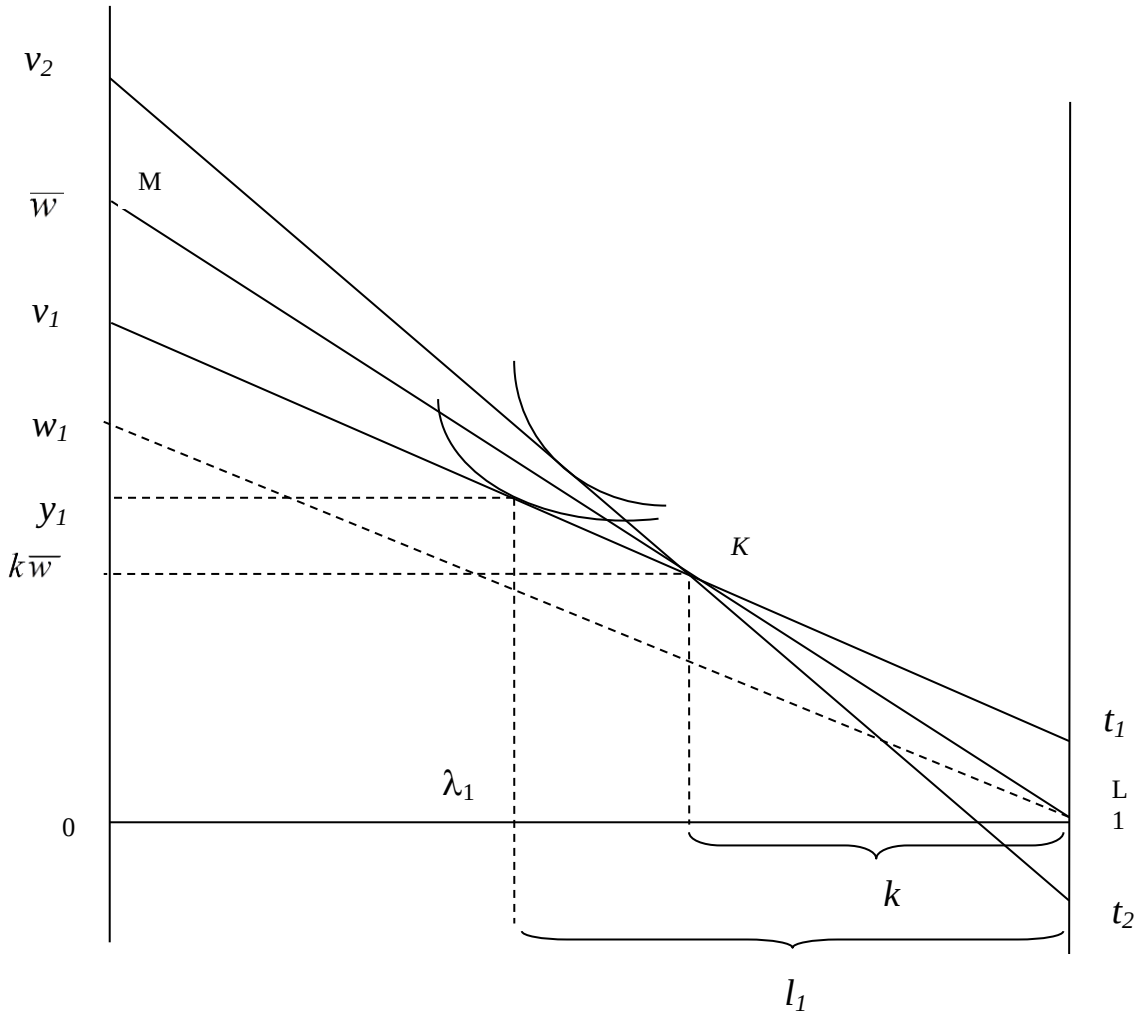

Figure 4. Equal-freedom budget sets