ECONOMIC INEQUALITY

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1. INTRODUCTION

People die in revolutions fighting inequalities. "Men are free and equal in rights" is the political basis of democratic societies, and these rights may more or less include rights to consumption or income. Inequality, notably of incomes, is related to practically all important problems of society, as cause or as effect. But one generally has to compare unequal situations: When is one less unequal than the other ? Does the transfer of one dollar from a richer to a poorer diminish inequality, given that it certainly augments inequality between the relatively poor receiver and the still poorer and equally poor, and between the relatively rich giver and the still richer and equally rich ? The standard pattern of democratic economic growth is that differences in income increase whereas relative differences decrease : does it augment or diminish inequality ? If the two-person income distribution (2,4) is as unequal as the distribution (1,2) because the ratios of incomes are the same, is inequality also unchanged when the distribution (0.01,1) is transformed into the distribution (0.1,10) by the multiplication of all incomes by the same number 10? In the tranfer case, whatever its effect on inequality, the transfer from richer to poorer may be good because it alleviates one person's poverty, or it may be just.

Hence, three facts characterize the issue of social, economic, and simply income inequalities. First, this question is very important for judging society and because people use it for judging society. It also is important for sociological and economic description and explanation. Second, this issue is amenable to mathematical formulation and deductions. Third, the actual meaning of social inequalities is delicate, subtle, sometimes elusive, and always polysemic, even when consideration is restricted to as relatively simplistic terms as incomes or consumption. Indeed, it refers to logic, to social ethics through distributive justice and through judgments about social structures and social sentiments, to the sociology of classes, hierarchies, concentration, spreading, stratification, isolation and polarization, and to sociopsychology as regards the sense of justice, norms, envy, sentiments of inferiority or of

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superiority, preferences for distinction or for conformity, and so on. This threefold characteristic of social importance, mathematical formalization, and subtelty in meaning, gives this field a place in the social sciences which is both central and particular.

2. THE CHALLENGE

Inequality increases or decreases, or is higher in this place than in this other, are expressions that one often heard or read, from people who seem to attach a great importance to this issue and who speak as journalists (or the like), or – alas – as scholars. Such views are highly problematic. Indeed, given any two distributions (of anything – say of income for instance), I can practically always prove that one is more unequal than the other, or the converse, with reasons that will all be convincing in themselves. This means that such comparisons are absurd as long as one has not sufficiently specified which kind or properties of inequality one is talking about. For example, a classical question is whether economic growth goes with higher or lower inequality (say in income distribution). We will shortly see that for a given measure of inequality, one often has to consider an "absolute form" and a "relative form", the latter being the former divided by the mean. Now, economic growth in a democratic country, with some redistribution, standardly entails that absolute inequality increases and relative inequality decreases.[1]

Hence, one can speak of higher or lower inequality only in specifying what kind of inequality one is considering, and the relevant kind of inequality depends on what one wants to do with this concept. A concept of inequality depends on two items, its substance (i.e., inequality of what?) and its structure.

3. INEQUALITY AS DISPERSION OR AS INJUSTICE

There are two types of reasons to be concerned with inequality: a descriptive or positive reason, and a normative reason concerned about justice or fairness. In fact, one never speaks

[1] For example, in a country where half the population earns 100 per person and the other half earns 200 per person, a balanced growth at the rate of 6% makes these incomes become 106 and 212 respectively, but a transfer of 2 from the rich to the poor makes these numbers become 108 and 210. The difference in incomes has increased from 100 to 102, an increase of 2%. Yet, the difference relative to the mean has passed from 100/150 to 102/159, which is a decrease of 4%.

of inequality, but, rather, of dispersion or of injustice. Each of these two types of reasons has two subcases.

Positive views may be purely descriptive, or they may relate inequality to other factors as a consequence or a cause. When inequality is taken as a cause, the relation determines the relevant type of inequality. For example, if we suggest that growth may be influenced by inequality, the reasons, theory and model of this influence determines the adequate substance and structure of the inequality in question (this may concern income and savings, the structure of incentives, education, and so on).

Inequality matters for ethics either because we are concerned with poverty and assume that lower inequality reduces it (for instance through transfers with limited disincentive effects), or because inequality per se is deemed unfair (cf. Aristotle: "justice is equality, as everyone thinks it is, quite apart from other considerations"). The latter reason is a requirement of rationality: if person x receives y because of her characteristics z, then, if person x' also has characteristics z, she should also receive y, prima facie. Yet, this equality may be impossible, or it may interfere with another relevant moral value – for instance, an unequal distribution can make everybody happier, or the ideal equality of something else also matters. The relevant substance of this inequality depends on the underlying social ethical theory and may be, notably, income, wealth, consumption, welfare, the bundle of consumption goods, specific goods (health, housing, education, a basic staple, etc.), basic rights, freedom of choice, and so on.

4. A FRAMEWORK

For presenting the most basic issues concerning the structure of inequality, let us focus on the case of one quantity to each social entity, described – for the sake of concreteness and because it is an important case –, as the statistical distribution of income to individuals. The basic issue is: how does this inequality vary when this distribution varies? There are two categories of relevant variations: transfers among individuals and cases when all incomes very in the same direction (this can be the effect of growth, of the distribution of a bounty, or of taxation).

There will be n individuals indexed by i, x_i denotes the "income" of individual i, $x=\{x_i\}$ denotes the set or bundle or vector of the x_i, $\bar{x}=(1/n)\Sigma x_i$ is the average income, and $I(x)$ denotes an index of inequality: it is a number that increases or decreases as inequality does. The logical issue is the structure of the function $I(x)$. A basic point is that each inequality index has two forms, an absolute form $I^a(x)$ and a relative form $I^r(x)=I^a(x)/\bar{x}$ (total equality $I^t(x)=n\,I^a(x)$ is also sometimes relevant). The distinction is irrelevant for transfers that leave the average $\bar{x}$ unchanged, but it is crucial in the other cases. Absolute and relative inequality also represent inequality per person and per dollar, respectively.

We will now point out the very most basic properties of the comparisons and measures of inequality.

5. EVALUATION-CONSISTENT INEQUALITIES, EQUAL-EQUIVALENT

A sentiment of badness or injustice in the distribution is a priori derived from an overall evaluation of the distribution x. It can notably derive from the inequality. Inequality is absent when all individuals have the same income. The income level $\bar{\bar{x}}$ which, attributed to all individuals, gives a situation as good or as bad as allocation x is the equal-equivalent income of this distribution.[2] If all x_i are equal, $\bar{\bar{x}}=\bar{x}$. If inequality is bad, $\bar{\bar{x}}<\bar{x}$. The difference $\bar{x}-\bar{\bar{x}}$ measures the badness of inequality in income units. It is a measure of the injustice of the inequality in income units. It can be taken as a measure of this unjust inequality. It is an absolute (per person) form of the measure, $I^a(x)=\bar{x}-\bar{\bar{x}}$, and the relative (per dollar) form is $I^r(x)=(\bar{x}-\bar{\bar{x}})/\bar{x}=1-\bar{\bar{x}}/\bar{x}$. From this relative injustice, there results that $\bar{\bar{x}}/\bar{x}$ is a measure of relative justice. Specific cases are derived from other properties shortly presented.

6. PROGRESSIVE TRANSFERS, RECTIFIANCE

A transfer from a richer to a poorer of less than the difference in their income is a progressive transfer.[3] Such a transfer doubtlessly diminishes the inequality between these two incomes. However, it need not diminish the overall inequality if there are other individuals. For

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[2] Kolm 1966.

[3] Numerous historical economists have considered such obvious transfers (Taussig, Loria, Pigou, Marshall, Divisia, Dalton, etc).

instance, transferring 1 from a person who has 4 to a person who has 1 diminishes the inequality between them. But if another person has 4 and another has 2, this transfer between the two former ones breaks down two strict equalities of 4 and 2 respectively. The effect on the overall inequality is a priori ambiguous.

An inequality index that nevertheless decreases under progressive transfers is called rectifiant (it "satisfies the transfer principle"). This term also applies to an overall evaluation of the distribution that likes progressive transfers (its evaluation-consistent measures of inequality are rectifiant). Rectifiance has a number of possible reasons. One is evaluation independence (or irrelevance of fixed incomes): favouring variations in a subset of incomes does not depend on the levels of the other, unchanging incomes. Another reason refers to comparing the effects on the welfare of the two individuals.[4]

7. GENERAL COVARIATIONS. INTENSIVENESS AND EQUAL-INVARIANCE

The other family of transformations are those where all incomes vary in the same direction. Does inequality depend on differences or on ratios? Is it maintained when differences or when ratios remain the same? When all incomes are added the same amount or are multiplied by the same number? That is, is inequality equal-invariant (invariant under addition of the same amount to all incomes) or intensive (invariant under multiplication of all by the same number – the classical term used in the sciences)?

The answer can be: both. Indeed, for a family of inequality indexes called synthetic inequalities, the absolute form is equal-invariant and the relative form is intensive. Since, by definition, $I^r = I^a / \bar{x}$, there results that the absolute form is also multiplied by a number when all incomes are multiplied by this number: as the sciences say, it is extensive.

Synthetic absolute forms can be, for instance, proportional to $\Sigma|x_i - \bar{x}|$, $\Sigma|x_i - x_j|$ (Gini), $[\Sigma(x_i - \bar{x})^2]^{1/2}$ (standard deviation).

However, synthetic measures cannot possess other properties, such as being evaluation-consistent measures defined above. Then, indexes of inequality can have one of

[4] Kolm 1966 and, for an exhaustive analysis of the various possible reasons for rectifiance, 1999.

the two properties without their other form having the other, or they can be intermediate in being "subequal" and "superintensive" – i.e., an equal addition reduces inequality and an equiproportional increase augments it (the democratic growth suggested above can maintain such an inequality).

8. EVALUATION INDEPENDENCE

Rectifiance ("the transfer principle") for an evaluation of the distribution occurs in particular when this evaluation can be represented as favouring higher values of a function of the form $\Sigma f(x_i)$ with an increasing and concave function f. This is sometimes seen as "social welfare" as sum of individual welfares ($f(x_i)$ for individual i), but such an interpretation requires a very careful explanation.[5] Then, the evaluation-consistent inequality measure has an intensive relative form if function f is a power function and an equal-invariant absolute form if function f is exponential or logarithmic. An adequate intermediate structure is with a function f which is a power of variables x_i+c where c is a positive constant ("income augmented measures"): the former case occurs when $c=0$ and the latter one when c tends to infinity.[6]

9. INEQUALITY UNAMBIGUOUS COMPARISONS

We have seen that a progressive transfer reduces inequality in the pair it affects but not necessarily in the overall distribution (for $n>2$). The transformations that extend the unambiguous reduction of inequality of a progressive transfer if $n=2$ to the case of any n are not keeping the transfer between two incomes only. They are other transformations defined for any n and which boil down to progressive transfers when $n=2$. These general unambiguous inequality-reducing transformations are of two kinds in simple forms (plus more complex intermediate cases).[7]

In truncations of a distribution, all incomes higher than a given level b are brought down to level b (an "upper truncation"), and all incomes lower than a level $a\#b$ are augmented to a (a "lower truncation"). The "floor" a and the "ceiling" b can be such that the sum Σx_i (and hence the average $\bar{x}$) are maintained. The result then is a "balanced bitruncation".

[5] Kolm 1996, pages 59 to 61.
[6] Kolm 1976.
[7] Kolm 1999.

Truncations are the only transformations such that the transformed incomes of each pair of incomes are in the initial interval of the pair (each pair becomes "inclusion more equal" or does not change).

In a concentration of a distribution, the same fraction of all incomes is equally redistributed. Each income becomes the same weighted mean between its former value and the average $\bar{x}$. This amounts to two transformations: an equiproportional reduction of all incomes – which maintains intensive inequality –, and an equal increase of all incomes – which maintains equal-invariant inequalities. The weight, that is, the fraction in which each income goes towards the mean, is the degree of the concentration.

For any redistribution of incomes, one can define the degree of equalization it achieves as the degree of the concentration which leads to the same level of some inequality index in the end (the concentration "equivalent" to this redistribution as concerns inequality). It is easily seen that if this index is a synthetic measure of inequality, the degree of equalization is the relative decrease of its absolute form.

Similarly, there is a balanced bitruncation inequality – equivalent to any redistribution for a given inequality index (balancedness, the preservation of the sum, implies that the ceiling b is a function of the floor a, and the equality in inequality determines the level a). The case $a=b=\bar{x}$ is a full equalization. The differences, either absolute $\bar{x}-a$, or relative $1-a/\bar{x}$ measure a degree of imperfection of the equalization.

These measures are often particularly meaningful. For instance, the degree of equalization based on the equivalent concentration leads to the measure of present national redistributions as amounting to an equal redistribution of the incomes earned during one to two days per week (from the United States to Scandinavia).

10. CONCLUSION

Few concepts are as meaninglessly used as that of inequality. This led to the birth, in the early sixties, of a thriving branch of economics devoted to the analysis of the logic of inequalities and its applications. These applications cover normative economics and distributive justice; multidimensional inequalities in various basic goods; inequalities in freedom and opportunity;

the relations between inequalities and growth and politics; sociological considerations such as isolation, group size, clustering, and polarization; the relation between inequality and various social sentiments such as envy and altruism, and consequences such as charity and social unrest; and so on. These applications extend now from economics to various other fields such as ethics, sociology, politics, and social psychology. The meeting of new problems stimulate in turn the analysis of meaningful logical and formal properties of the concept.

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