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Economic inequality

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Introduction

When some people are treated more or less favourably than others without a seemingly valid reason, this inequality arouses a judgment of injustice which is conveyed in the term “inequality”. Such inequalities are a major issue for judging societies or policies and are often compared across time or societies, in particular by the media and politicians. Such comparisons are a priori highly problematic, however, since, given any two unequal distributions of some item, one can most of the time show that anyone is more unequal than the other and the converse, with reasons, comparisons and measures which, a priori, may all seem convincing.

Does, for instance, growth tend to augment or diminish inequality? Balanced growth followed by a fiscal partial redistribution of the gains diminishes inequality measured by ratios but may augment inequality measured by differences. Inequality based on ratios is not changed when the pair 0.01 and 1 becomes the pair 0.1 and 10. Inequality based on differences is not changed when the pair 1 and 2 becomes the pair 11 and 12. Does a transfer from a richer to a poorer diminish inequality? It augments the pairwise inequalities between the richer and the still richer or equally rich and between the poorer and the still poorer and equally poor. One can pass from the income distribution of Australia to that of France (adjusted for population) by a sequence of such transfers, and yet Australia, with its large homogeneous middle class, seems a more egalitarian society.

During the first two thirds of the 20th century, scholars developed measures of inequality (Gini, Theil), and comparisons of distributions (Lorenz), and proposed reflections about the effects of transfers (Pigou, Dalton) and of variations in the same proportion (Dalton, Taussig), by the same amount (Dalton, Cannan, Loria), or in population size (Dalton).

After such considerations about dispersion with a vague feeling of injustice, a revolution in the rational ethical analysis of unjust inequality came with the last third of the 20th century. It was opened by two remarks. First, *the effects of inequality and notably injustice are among the reasons for the overall ethical evaluation of a distribution, and they can be measured by the cost of inequality implied by this evaluation.* Second, *a number of important basic properties of the comparison of inequalities happen to be logically equivalent*, thus providing the basis of the modern ethico-logical analysis of economic inequalities.

The measures of economic inequality derived from overall ethical evaluations[1]

Overall ethical evaluation and inequality

The cost of inequality, notably of its injustice, is implicit in any overall ethical evaluation, and therefore its measure can be derived from this evaluation. However, the converse view is also relevant. An overall ethical evaluation is the synthesis of moral judgements about the various relevant aspects and properties of a situation. One or some of these aspects or properties can concern inequality and in particular distributive injustice. Then, direct moral judgements about inequality matter. The overall judgement aggregates the various particular ones in a way that has to respect properties of consistency.

Consider the simplest and important case of the distribution of incomes – or any other desired quantity (other cases will be noted shortly). There are n individuals indexed by i=1,... n. Denote as x_i the income of individual i. A distribution is a set of n x_i, one for each individual i. Such a distribution is *equal* when all the x_i are equal. The sum $X=\Sigma x_i$ is the *total* or *social income*. The *average income* is $\bar{x}=X/n$. For an equal distribution, $x_i=\bar{x}$ for all i.

The overall ethical evaluation needs only be by judgements of better or worse. It is described by an ethical evaluation function $W(x_1,\ldots x_n)$ which takes a higher value when the distribution is considered to be better. The *nature* of this function is not further specified, and, hence, this function can be replaced by any increasing function of itself; that is, it is ordinal, and any increasing function of it is one of its specifications.

[1] The rest of this presentation of the basic properties of economic inequality consists of a simplified version of Kolm (1966, sections 6 and 7).

Moreover, we assume that the situation improves if one income increases while no other decreases, a property called the *benevolence* of the overall judgement. This translates as function W being an increasing function of the x_i.

Finally, the present concern about the ethics of inequality leads us to assume that all judgements relevant here about how to share a given total income X can be expressed through judgements about the inequality of the distribution.

The equal equivalent income

For the overall evaluation, a distribution can be replaced by any other that gives the same level to function W. In particular, it can be replaced by one such distribution which is also equal. The individual income of this latter distribution is called the *equal equivalent income* of the initial distribution, and it is classically denoted as $\bar{\bar{x}}$. It is therefore defined by the equality

$$W(x_1,\dots x_n)= W(\bar{\bar{x}}, \dots \bar{\bar{x}}). \tag{1}$$

This level $\bar{\bar{x}}$ is uniquely defined because function W is increasing (benevolence). Hence, *the equal equivalent income of a distribution is the individual income of the equivalent equal distribution*. It is the individual income such that, if all individuals had it, the resulting equal distribution would be as good as the distribution in question.

The equal equivalent income $\bar{\bar{x}}$ is a function of the distribution $(x_1,\dots x_n)$ and of the function W (it is a 'functional' of function W). The expression $W= W(\bar{\bar{x}}, \dots \bar{\bar{x}})$ shows that it is an increasing function of the value or level W. Hence, it is a particular specification of this ordinal evaluation function. Moreover, it has the nature of an individual income.

If the initial distribution $(x_1,\dots x_n)$ is equal, $x_i= \bar{x}$ for all i, and hence, from equation (1), $\bar{\bar{x}} = \bar{x}$. If the evaluation function W has a specification of the form Σx_i, equation (1) writes $\Sigma x_i =n\bar{\bar{x}}$, and hence $\bar{\bar{x}} = \bar{x}$ again. This form of W implies that the ethical evaluation resents no injustice in any inequality resulting from the distribution of a total income $X=\Sigma x_i=n\bar{x}$.

The basic ethically derived indexes

If the inequality in the distribution (x_1,... x_n) is morally bad, in particular unjust, this implies that the equal sharing of the total income $X=\sum x_i$, the equal distribution $(\bar{x},... \bar{x})$, is better, that is

$$W(x_1,... x_n) < W(\bar{x},... \bar{x}). \quad (2)$$

A discrepancy between these two values of function W measures a moral cost of inequality. Note that $\bar{x}$ is the equal equivalent income of the equal distribution $(\bar{x},... \bar{x})$. Inequality (2) also writes, given definition (1),

$$W(\bar{\bar{x}},... \bar{\bar{x}}) < W(\bar{x},... \bar{x}),$$

which implies $\bar{\bar{x}} < \bar{x}$. A cost is a difference between two values. Since function W is ordinal, a difference (or a ratio) in values of W is a priori not meaningful with respect to this property. However, the operation of difference (and ratio) is meaningful between quantities. It is, therefore, for the specification of W that is the equal equivalent $\bar{\bar{x}}$. Hence, the difference $\bar{x} - \bar{\bar{x}}$ is a cost in income term of the inequality of distribution (x_1,... x_n). However, the cost can also be expressed in relative terms, by ratios, or for the whole population, as expressed by the following six classical meaningful indexes:

$I^a = \bar{x} - \bar{\bar{x}}$: *absolute* (per person) *inequality*;

$I^t = nI^a = X - n\bar{\bar{x}}$: *total inequality*;

$I^r = I^a / \bar{x} = I^t / X = 1 - (\bar{\bar{x}} / \bar{x})$: *income relative inequality*;

$I^e = I^a / \bar{\bar{x}} = I^t / n\bar{\bar{x}} = (\bar{x} / \bar{\bar{x}}) - 1$: *equal equivalent income relative inequality*;

$\eta = \bar{\bar{x}} / \bar{x} = 1 - I^r$: the *equal equivalent yield* of the distribution;

$\gamma = \bar{x} / \bar{\bar{x}} = 1 + I^e$: the *unit cost of the equal equivalent income*.

Each of these six indexes is a priori meaningful and it turns out to be the relevant one for specific questions met in the theoretical and applied analyses of inequality.

If the distribution is equal, or if Σx_i is a specification of the evaluation function W, $\bar{\bar{x}} = \bar{x}$, $I^a = I^t = I^r = I^e = 0$, and $\eta=\gamma=1$. With an unequal distribution and a cost of inequality, $\bar{\bar{x}} < \bar{x}$, $I^a>0$, $I^t>0$, $I^r>0$, $I^e>0$, $\eta<1$, $\gamma>1$. With an extreme inequality-aversion, the smallest of the x_i, $\min_i x_i$, is a specification of function W, then $\bar{\bar{x}} = \min_i x_i$, and the six indexes have the corresponding values.[2] For the general function *W*, each index is in between these two limiting values.[3,4]

Elementary properties

When the evaluation function *W* has a certain structure shortly noted – which is in particular satisfied if it has specifications of the form $\Sigma f(x_i)$ where function *f* is increasing and concave (it increases less and less when x_i increases by successive equal amounts) –, the foregoing ethical evaluation-consistent measures of inequality classify distributions according to a comparison which has a number of other remarkable properties, such as: a transfer from a richer person to a poorer one of less than half the difference in their incomes diminishes inequality, the Lorenz curve of a distribution of a given total income is above that of another, and a number of other meaningful ways to compare inequalities. Before showing these properties, let us note a few more elementary properties that will be used.

A distribution to two persons (n=2) (x_1', x_2') is *inclusion more equal* (more equal by inclusion) than another (x_1, x_2) if x_1' and x_2' are in between x_1 and x_2, with the possibility that x_1' or x_2' is equal to x_1 or x_2 if the other is not also equal to the other x_1 or x_2 (a strict inclusion of the segments between the two incomes).

Comparisons are *constant-sum* when they compare distributions with the same total *X* or average $\bar{x}$ (for a given number *n*).

If, when only a subset of the x_i changes, a comparison of the distributions does not depend on the levels of the other, unchanging x_i, this comparison for n>2 is said to be *independent* (or separable). Independence for the overall evaluation occurs if and only if a specification of the ordinal function *W* has the additive form $\Sigma f_i(x_i)$.[5]

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[2] This particular *W* is no longer strictly increasing in all its arguments.
[3] Further concepts have been defined when the overall evaluation is such that, for some distributions, $\bar{\bar{x}} > \bar{x}$.
[4] In a didactic and influential article, Atkinson (1970) also considered the equal equivalent income $\bar{\bar{x}}$ (the 'equally distributed equivalent income') and the relative measure I^r .
[5] It suffices that the independence property holds when only a properly chosen set of subsets of the x_i changes, which can be reduced to n!1 subsets, or to all pairs of x_i, or to n!1 chosen pairs.

If the incomes x_i are the only characteristics that relevantly differentiate the individuals for the problem at hand, the comparisons or measures are unchanged if the x_i are permuted (*invariance under permutations*). The corresponding functions – such as W – are *symmetrical* (by definition of the term). Note that this implies in particular that peoples' different specific tastes, needs, utilities, other possibilities, etc. are found not to be relevant. In particular, such a W cannot be a classical social welfare function depending on individuals' utilities since individuals' utility functions are a priori different.[6] If it means 'welfare', this is welfare evaluated otherwise, by a judgement not following the individuals' evaluations of their own welfare, and the meaning of this concept has to be explained (which has not been done yet). However, we will consider a property that holds for all such judgements having some general properties. This symmetry is assumed in this simple presentation, but the cases in which it is not relevant have been studied. Symmetry plus independence of the function W hold if and only if it has a specification of the form $\Sigma f(x_i)$.

The core moral logic of economic inequalities

The basic ethical comparisons of economic inequalities

The transfer principle

A *progressive transfer* is a transfer from a higher income to a lower one of less than the difference (or not higher than half the difference). The *transfer principle* proposes that a progressive transfer diminishes inequality.

The transfer principle can be justified by the assumption that the unchanged incomes are irrelevant for the comparison and, given that it maintains the total sum constant, either the fact that it inclusion-reduces the inequality between the changing incomes, or the assumption that the increase in the poorer person's 'welfare' overcompensates the decrease in the richer's 'welfare', for amounts which are equal (concavity of the functions f in an additive evaluation $\Sigma f(x_i)$).

[6] Justifying the symmetry from such a function by a lack of information about individual utilities is possible but analytically delicate (cf. Kolm in Silber 1999).

'Social welfare'

If the overall evaluation of the distribution is both separable-independent and symmetrical, the ordinal function W has specifications of the form $\Sigma f(x_i)$. This cannot describe classical utilitarianism $\Sigma u_i(x_i)$ because the same function f applies to all x_i.[7] If this refers to 'welfare', this is a concept different from the individuals' evaluations of their own welfare. This raises two questions: what can this evaluation mean, and what can it be? The second question is in part eschewed by the consideration of comparisons that holds for *all* functions f that are increasing (benevolence) and concave. This latter property means that an extra euro increases evaluation or 'welfare' more the lower the income to which it is added. It also is a property of 'satiation' in the evaluation or 'welfare' effect of individual income.

Concentration curve and Lorenz curve dominances

Denote as y_m the sum of the m lowest x_i. That is, if the numbering i of the x_i are rearranged in such a way that the new x_i are in a non-decreasing order ($x_1 \leq x_2 \leq \ldots \leq x_n$, that is, $i>j$ implies $x_i \geq x_j$), y_m is $y_m = \sum_{i=1}^{m} x_i$. Then, $y_n = \Sigma x_i = X$, the total amount.

Elementary textbooks of statistics call the curve of the y_m as function of m (or of m/n) the *concentration curve* of the distribution of the x_i.

The Lorenz curve of this distribution is y_m/X as a function of m/n.

When the x_i are all equal, these two curves are straight lines with these x_i and 1 as respective slopes.

A curve is said to be above another when it is somewhere above and nowhere below.

A distribution *concentration-dominates* another when its concentration curve is above that of the other, that is, for distributions $(x_1, \ldots x_n)$ and $(x'_1, \ldots x'_n)$, $y_m \geq y'_m$ for all m and $y_m > y'_m$ for at

[7] However, this additive form is the case where the remark of note 6 applies.

least one *m*. *Lorenz-domination* is similarly defined for Lorenz curves. Both comparisons coincide when comparing distributions with the same total $X = X'$, that is, in 'constant-sum comparisons'. Then, a preference for a higher concentration or Lorenz curve is called isophily (*isophilia* is the Greek term for inequality-aversion).

Averaging

A distribution is in a sense less dispersed than another if all its items are averages of those of the other. Distribution ($x'_1,\ldots\ x'_n$) is a (linear convex) average of distribution ($x_1,\ldots\ x_n$) when $x'_i=\Sigma_j\, a_{ij}\, x_j$ with $a_{ij}\geq 0$ for all i and j and $\Sigma_j\, a_{ij}=1$ for all i. If the total sums are equal $X = X'$, notably for a redistribution, this implies the last equality of

$$\Sigma\, x'_i=\Sigma_{i,j}\, a_{ij}\, x_j=\Sigma_j(\Sigma_i\, a_{ij})\, x_j=\Sigma_j\, x_j,$$

and therefore

$$\Sigma_j\, (1-\Sigma_i\, a_{ij})\, x_j=0.$$

We consider such transformations that are independent of the initial distribution (the a_{ij} do not depend on the x_k), and applicable to all distributions. The foregoing identity then implies $\Sigma_i\, a_{ij}=1$ for all j. Such a_{ij} constitute a *bistochastic matrix*, i.e., a non-negative matrix whose sums of the elements in each row and in each column amount to 1. This transformation of x into x' is an *averaging*.

If $a_{ii}=1$ for all i (hence $a_{ij}=0$ if $i\neq j$), $x'_i=x_i$ for all i, nothing is changed, the transformation is an identity. If all a_{ij} are only zero or one, the transformation is a permutation of the x_i. If $a_{ij}=1/n$ for all i, j, $x'_i=\bar{x}$ for all i (a 'complete averaging'). If, for $0\leq\alpha\leq 1$, $a_{ij}=\alpha/n$ for all i, j with $i\neq j$ and $a_{ii}=1-\alpha+\alpha/n$ for all i, $x'_i=(1-\alpha)x_i+\alpha\,\bar{x}=x_i+\alpha\cdot(\bar{x}-x_i)$ for all i. This is a *concentration* of the x_i (a uniform linear concentration towards the mean): each x'_i is an average between x_i and the mean $\bar{x}$, it goes the same fraction α of the way towards the mean; the concentration amounts to an equal redistribution of the same fraction α of the x_i; it amounts to a decrease of all incomes in the same proportion followed by an increase of the same amount (which restores the total amount). A progressive transfer is a particular averaging: if $x_i>x_j$, $0<t<1$, $a_{ii}=a_{jj}=1-t$, $a_{ij}=a_{ji}=t$, $a_{kk}=1$ for all $k\neq i, j$, and $a_{kl}=0$ for the other entries, $x'_i=x_i-t\cdot(x_i-x_j)$, $x'_j=x_j+$

$t\cdot(x_i-x_j)$, and $x'_k = x_k$ for all $k \neq i, j$. Of course if all the x_i are equal, all the x'_i are also equal to them. Moreover, an averaging of an averaging is an averaging.

Share reshuffling

Divide each individual income into a series of shares, each share being the same fraction of the income for all incomes. Then, reshuffle the shares corresponding to the same proportion among the individuals, that is, perform a permutation of these shares among them. The permutations of the shares for the various fractions are unrelated. Formally, consider numbers $\lambda_k>0$ with $\Sigma\lambda_k=1$, and permute the shares of each k, $\lambda_k x_i$, among the individuals i, with independent permutations.

Mixtures

Denote as $x=\{x_i\}$ the vector of the incomes x_i. A permuted vector of x is x^π obtained by permuting the x_i of x by the n-permutation π (i.e., $x_i^\pi=x_{\pi(i)}$ for all i). The absence of relevant individual characteristics other than their incomes x_i implies that the x^π are equivalent. Then, a *mixture* of a distribution x is an average (a linear convex combination) of the x^π, $x'=\Sigma\lambda_\pi x^\pi$ with $\lambda_\pi\geq0$ for all π and $\Sigma\lambda_\pi=1$.

Since, in share reshuffling, if one writes π_k the permutation corresponding to share k, the result is $x'=\Sigma\lambda_k\, x^{\pi_k}$, mixtures and share reshuffling are clearly equivalent (each instance of one in an instance of the other). These transformations are not permutations when $\lambda_\pi\neq1$ for all π for a mixture, and, for a share reshuffling, $\lambda_k\neq1$ for all k (hence, there are at least two shares) and the permutations of the shares are not all identical.

A transformation that is not, in fact, a permutation is called *strict*.

The fundamental equivalences of ethical inequality comparisons

Each of these properties has a flavour of comparing more or less unequal distributions. Their meaning in this respect is very strongly reinforced by the fact that they are mathematically equivalent.

Indeed, when comparing 2 distributions $x=(x_1,\dots x_n)$ and $x'=(x'_1,\dots x'_n)$ with the same amount $X = X'$, the following properties are equivalent.

1) x' can be obtained from x by a sequence of progressive transfers.

2) The concentration or Lorenz curve of x' is above that of x.

3) $\Sigma f(x')>\Sigma f(x)$ for all increasing and strictly concave functions f.

4) x' is a strict averaging of x.

5) x' results from a strict share reshuffling of x.

6) x' is a strict mixture of x.

Moreover, if the distributions can have different amounts, say $X'\geq X$, the following properties are equivalent.

1) X' can be obtained from X by a sequence of progressive transfers or increases in incomes.

2) $\Sigma f(x'_i)>\Sigma f(x_i)$ for all increasing strictly concave functions f.

3) The concentration curve of distribution x' is above that of distribution x.

Clearly, these relations cannot be both ways between two distributions; if they hold from x to x' and from x' to x'', they hold from x to x'' (transitivity). They thus constitute an ordering of the distributions. For distributions with the same total amount, this is an important sense of comparisons by more or less unequal. Yet, they do not compare all distributions: they do not compare them when their concentration curves intersect. Other criteria can then be added.

An evaluation function $W(x)$, increasing, symmetrical and such that $W(x')>W(x)$ when x' relates to x as in the preceding relations, and the corresponding ethical evaluation-consistent inequality indexes, are called *rectifiant*, or, respectively, Schur-concave and Schur-convex[8] (the functions $\Sigma f(x_i)$ with increasing and concave f constitute a sub-class of such functions).

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[8] After I. Schur whose articles of 1922, 1923 and 1936 first considered the effects of the transfer principle and averaging on such functions (rectifiance means, more generally, the satisfaction of the transfer principle whether the functions are symmetrical or not).

Finally, there are types of redistributions or transformations of distributions that are more inequality-reducing structures than the others. The two polar cases of the particularly inequality-reducing transformations are the *concentrations* in which all incomes diminish their distance to the mean in the same proportion, and *truncations* where all incomes above a level are reduced to this level and all below a lower level are augmented to this level. Both have important applications in normative economics – this is notably the case for concentrations in the theory of optimum distribution, taxation and aid.

Inequality under co-variations of incomes

The foregoing mainly emphasizes the effects of transfers or redistributions on inequality, hence comparisons of the inequality of distributions with the same total amount. However, cases in which all incomes vary in the same direction are also important. Does general growth, or an equal distribution of a benefit or a charge, augment or diminish inequality? This depends on the relevant concept of inequality.

The two polar cases are those in which inequality does not change when all incomes vary in the same proportion and by the same amount, respectively. In the former case, inequality is what the sciences call an *intensive* property. In the latter case, inequality is said to be *equal-invariant*.

Measures of inequality derived from a separable evaluation that are *intensive* are the *relative* inequality with a *power* or a *logarithmic* individual welfare function ($f(x_i) = x_i^{\alpha}$ with $\alpha>0$, or $\log x_i$), and those that are *equal-invariant* are the *absolute* inequality with an *exponential* individual welfare function ($f(x_i)=1-e^{-\beta x_i}$, $\beta>0$). One consequence is that one cannot derive both an intensive and an equal-invariant measure of inequality from the same separable ethical evaluation.

Nevertheless, there is another class of measures of inequality, the *synthetic* measures, with an absolute form $I^a(x)$ and a relative form $I^r(x) = I^a(x)/\bar{x}$, such that the relative form is intensive and the absolute form is equal-invariant. One consequence is that the absolute form is also ‘extensive’, that is, multiplied by a scalar when all incomes are. These absolute forms are the linearly homogeneous functions of the differences ($x_i - \bar{x}$) or ($x_i - x_j$). They include some of the most common measures of inequality such as the Gini index $\Sigma|x_i - x_j|$, $\Sigma|x_i - \bar{x}|$, or the standard deviation.

Moreover, one can derive, from a separable ethical evaluation, measures of inequality that are intermediate between the intensive and the equal-invariant measures. The simplest case is the ‘income-augmented’ intensive measures, which apply the intensive measures to new variables that are the incomes plus a non-negative constant. The measures are intensive when the constant is zero and equal-invariant when it tends to infinity.

For intensive or equal-invariant measures, one can reduce the comparison of the inequality of two distributions to constant-sum comparisons by respectively multiplying or increasing all the incomes of one of the distributions by the same number.

Conclusion

The foregoing properties constitute only the basics of the standard economic theory of unjust inequality. Many other properties are added. In particular, they describe the effects, on this inequality, of: transfers depending on the levels of or differences in incomes; the addition of several types of incomes to the same people; the aggregation of populations with intra-group and inter-group inequalities; growth; the income tax; characteristics which may relevantly differentiate the persons such as needs, size and type of family, labour provided, merit or desert, or various rights; judgements that violate the transfer principle, for instance because they attach importance to clusters of incomes (size of income classes); and so on.

The theory then considers the inequalities in other items than income or a single quantity, notably the multidimensional inequalities in a bundle of goods (to begin with in both income and labour or leisure, or in income, health, education and housing); inequalities in various types of freedom, power or opportunities; inequalities in ranks or status; etc.

The nature of the items often implies particular properties of the comparison and measures of inequality. This happens even with the simplest case of quantities. For instance, if health is measured by the duration of life, it may be, on average, better to die at 35 rather than at 34 than to die not only at 95 rather than at 94 (concavity of the function f), but also at 5 rather than at 4 (non-concavity of f).

In other cases, the basic reference is not equality but some other particular distribution. For instance, it might be the outcome of markets, which has a possible moral justification from freedom of exchange (or self-ownership). In this case, the relevant concept is the degree of equalization achieved by redistributions from this state. For example, present-day redistributions at national levels are equivalent, in this respect, to fully equalizing the incomes from the labour of one to two days per week. Such durations turn out to be richly meaningful measures of the degree of equalization or solidarity in the community.

Finally, the issue of inequality is very closely related, both in fact and in analyses, to other very important economic and social phenomena such as poverty, polarization, segmentation, clusters, class or cast structure, exclusion, isolation, eliticism, envy, status, etc.

The literature on economic inequality is very large and cannot be presented here. For interested readers, Silber (1999) offers an excellent bibliography source.

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