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EQUAL-EQUIVALENTS FOR INEQUALITY, WELFARE AND LIBERTY: CONCEPTS AND POLICY

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Abstract

The concepts of the "equal-equivalents" permit the definition of one-dimensional and multidimensional inequalities, of individual "welfare" (the same function for all individuals) and, as a result, of classical inequality properties and of the optimal allocation in "macrojustice" (optimum income taxation and transfers, amounting in particular to equal liberty of choice in different domains).

Summary

If an equality is the object of a moral judgment – for instance, it could be an injustice –, comparisons and measures of this inequality may be derived from an overall evaluation of the social situation. Two characteristics of this judgment are relevant. First, the judgment often takes the form of an ordering, for instance with a maximand function. Then, indexes of inequality can be derived from comparisons between averages and "equal-equivalents" (Kolm, 1966b), i.e. individual allocations such that, if every "individual" had the same, the overall allocation would be as good as the one under consideration. For multidimensional inequalities in bundles of quantities of several goods, the equal-equivalent allocations are those of the "equal-equivalent manifold". The second aspect refers to the "substance" which motivates the judgment. This substance often refers to concepts of welfare or of freedom. "Welfare", a descent from classical utilitarianism, was a common reference in economics and in political philosophy. In 1971, Rawls argued both that this criterion is never used in actual social choices of "social justice" ("macrojustice") and that it should not be used there. However, a closer analysis shows that what is convincingly objected to is not the reference to individuals' happiness but only inter-individual *differences* in hedonic capacities (capacities to enjoy) and in tastes. Actually, the concept of individual *welfare* is commonly used in distributive judgments, with the implicit (or explicit) assumption that an individual's welfare

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is a concave function of this individual consumption, the same function for all. The classical technical concept is the individual's "utility function". The individual utility functions can be cleaned of inter-individual differences in hedonic capacities and tastes in order to provide the relevant individual welfare function thanks to the basic concept of the "equal-equivalent utility function". This permits both to make sense of classical properties of inequality analysis, and to determine the optimum macrojustice allocation, and income transfers, taxes and subsidies. The result (ELIE for Equal-Labor Income Equalization) has a number of ethically and logically meaningful definitions and properties, including equal liberty of choice (with different domains). This outcome divides individual hedonic and productive or earning capacities into two parts, one which is self-owned and the other the benefits from which in welfare or income are equally distributed.

I – Situation

If some inequality is an injustice, comparisons and measures of this inequality can be derived from an overall social ethical evaluation. From social metaethics, this has two consequences:

1. A standard such evaluation, particularly in economics, is *classical welfarism*: max $W\{u_i(x_i)\}$. This implies two structures:

a) It is an *ordering*, with a maximand $U\{x_i\}=U(X)$. $U[\{x_i\}]$ written as $U\{x_i\}$.

b) It uses "*welfare*", u_i : what does this mean?

The validity of this principle raises the central modern debate. John Rawls writes in 1971 that it is never used in actual choices and that it should not be used.

If not, which value should we use? Is it freedom?

2. Social ethics demands that its principles be judged according to *all their aspects*: all their properties, axioms and consequences (see, for instance, Plato's "dialectics" in *Republic* or Rawls' "reflective equilibrium" for applications).

II - Notations

There are n comparable "individuals" (more generally "justiciables") indexed by i. An "allocation" for i is denoted as $x_i \in D$ $\quad \{x_i\} = X \in D^n$

Particular cases $D=\Re^m$.

case m=1 $x_i=y_i$ income

m>1 multidimensional inequality

m=2 important for "optimum income tax":

$x_i=y_i$, ℓ_i or $\lambda_i=1-\ell_i$

disposable income labour leisure

Domains of justice

1) Levels of justice

macrojustice: overall income distribution from main social resources; main ones: human (capacities) for production or earning and for consumption or enjoyment. Rawls calls his topic: "social justice" which is macrojustice (he says "macro" and "not micro").

≠*microjustice*: multifarious situations, specific for circumstances, goods, people.

mesojustice is about important goods that concern everybody.

Ex. health, education, are mostly "being goods" or "ontological goods" which contrast with "having goods" (and "doing goods" which are means, freedoms).

2) "Spheres of justice": Max Weber (1962) says that people want equality for each specific type of good. This induces Michael Waltzer's (1982) concept of "spheres of justice". He forgets that one sphere is particularly large: income in a market setting. "Spheres" are separated by lack of substituability or compensatibility: what is the income value of living one year longer? Health and education are largely "sphered goods".

Notation (continuation)

e = n-duplication: $e\alpha = \alpha,\alpha,\alpha... \alpha$ n times.

$X = e\,x$: equal allocations ($x_i=x$, $\forall i$)

$x_i \in P_i \;=\; P \subset D$

$X = \{x_i\} \in Q \subset \Pi P_i = P^n \subset D^n$.

$q=\{x: e\,x \in Q\}$ possible equalities.

Multi-meaning properties (continuation).

most famous example: the 20 or so properties mathematically equivalent one of which is the "transfer principle" (T.P.), for m=1 (y).

n=2: y' is "inclusion more equal" than y when $[y_1', y_2'] \subset [y_1, y_2]$

$\Rightarrow$ progressive transfers (P.T.) inclusion-reduce inequality.

n>2: small P.T. inclusion-increase pairwise inequalities between the lower, receiving income and other incomes equal and lower, and between the higher, yielding income and other incomes as high and higher: The overall effect on inequality is ambiguous. There are 2 solutions.

1. Consider other properties for $n \geq 2$ that become the T.P. for n=2

Notably:

1) *Concentration* (linear uniform concentration towards the mean):

$y_i' = \alpha y_i + (1-\alpha)\ \bar{y}$, $\quad \alpha \in [0,1[$

2) (Balanced bi-) *Truncations* $y'_i = a$ if $y_i \leq a$, b if $y_i \geq b$, y_i if $a<y_i<b$, with $a \leq b$ and $\Sigma y'_i = \Sigma y_i$.

2. Consider other properties equivalent to a sequence of P.T.

in particular: welfare:

"compare $\Sigma u(y_i)$ for all concave u". (1)

Meaning?

$\Sigma u_i(y_i)$= utilitarianism. But what is u without i?

There are two classical answers, both based on uncertainty and VNM theory, but mistakenly applied.

1) Lerner (repeated by Leontief, Samuelson, and Sen at the Biarritz conference): u_i uncertain, stochastic $\tilde{u}_i$ i.i.d. $\mathrm{E}\,\tilde{u}_i = u$.

but VNM theory says: max $\mathrm{E}\,F(\Sigma \tilde{u}_i)$, for some function F which has no reason to be an affine function.

2) Allocation Original Position. Any individual with a concave $u(y)$ receives any of the actual y_i with equal probabilities. $Eu(y) = (1/n)\Sigma u(y_i)$

But α) VNM theory says that if u=welfare, the risk-relevant utility is not u but $v=f[u(y)]$ for some f : $f' > 0$.

$v'=f' u'$.

$v''=f'u''+f''u'$: u concave does not imply v concave.

\+ - + +

or

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β) Theories of Original Position take individual evaluation in uncertainty as social ethical value. But the social ethical value refers to social ethics, society, others (not egoism in uncertainty).

Condition (1) Assumes: ∃ meaningful function $u(y)$ increasing concave, same for all individuals. This would denote "individual welfare". meaning? relation with the u_i?

III – Ordering, equal-equivalents

$U(X)$ maximand

Denote $U(e\,x)=V(x)$

$\bar{x}=(1/n)\Sigma x_i$. if $D=\Re^m$ *average*

define $\bar{\bar{x}}$: $U(X)=V(\bar{\bar{x}})=U(e\,\bar{\bar{x}})$ (2) *equal-equivalent*

Case $D=\Re$, y_i income. U increasing: $\bar{\bar{y}}$ is well-defined (figure 1).

Figure 1

Inequality indexes I from $\bar{y}$ and $\bar{\bar{y}}$. *6* are useful for specific questions:

$I^a=\bar{y}-\bar{\bar{y}}$, $I^r=I^a/\bar{y}$, $I^t=nI^a$, $\eta=\bar{y}/\bar{\bar{y}}=1-I^r$, $\gamma=\bar{y}/\bar{\bar{y}}=1/\eta$

absolute relative total yield unit cost

$I^a/\bar{\bar{y}}=\gamma-1$ excess unit cost, unit loss.

Case $D=\Re^m$ $m>1$ multidimensional. x_i^j = dimension (e.g. quantity) j for individual i.

$x_i=\{x_i^j\}_i$ (a m-vector).

(2): $\bar{\bar{x}}\in E$ *equal-equivalent manifold* ($(m-1)$ – manifold, hypersurface).

Figure 2

Measures of multidimensional inequality

$U(X)=V[(1-I)\ \bar{x}\ ]$ overall relative inequality.

$U(X)=V(\bar{x}-a\,J_a)$ absolute. Case $a=\bar{x}$: $J_{\bar{x}}=I$.

$a\in\Re^m$ case $a\in\Re$ numéraire

choice of vector a: importance of goods; and social ethical choice, dialogue, etc.

as for coefficients of a price index.

IV – Welfare Case $U(X)=W[\{u_i(x_i)\}]$

IV-1 The ethical value of "welfarism".

Our moral intuition about the relevance of the criterion of "welfare" can be tested by a few questions.

1) *Macrojustice, income tax*. Should you pay a higher income tax than someone else because she likes dollars more than you do, notably those taken away and one is utilitarian, or less than you do, notably the remaining ones and one is egalitarian (in utility)? Should you pay more or less than the other because the other (or you) has a cheerful character (which may lead one to enjoy a euro more or to regret its absence less – opposite effects again)? Or are these psychological characteristics or differences thought to be irrelevant for this issue, that is, people would be deemed entitled to their benefits and accountable for their shortcomings (as enforcing money transfers or modulating the income tax to compensate for differences in physical beauty is neither practiced nor – it seems – advocated)?

2) *Tastes*. Should you finance someone else's beverage because she only likes expensive wines? This classical "expensive tastes" argument extends in two ways. The other person may have to compensate you for your inability to experience such delicate gastronomic pleasures. And utilitarians meet "cheap tastes": should you finance the other's beverage because she likes cheap beer, and hence generates low-cost utility? Differences in tastes raising "different kinds of problems" is precisely the reason Mirrlees gives in 1971 for taking identical individual utility functions for determining the income tax.

3) *Desires*. Rawls (1982) takes up still another possible meaning of "utility" when he notes that, for "social justice" (the present macrojustice), "Desires and wants, however intense, are not by themselves reasons in matters of justice. The fact that we have a compelling desire does not argue for its satisfaction any more than the strength of a conviction argues for its truth".

4) *Liberal, earning*. Finally – since income is mostly earned – should I take the 10 dollars you just earned because I like them more than you do (or more than you dislike the labour with which you earned them)?

IV-2 From reflexions of the type of the first ones, Rawls concludes that individual utility functions $u_i(\ )$ are irrelevant for macrojustice ("social justice"). However, what these questions actually show is that what is irrelevant for macrojustice are the *differences* in the individual functions $u_i(\ )$ rather than these functions themselves.

Therefore, if a social ethic and in particular a conception of distributive justice wants to be "welfarist", it has to consider a concept of inter-individually comparable individual "welfare". This has to be defined by derivation from individual "utility functions" by erasing

differences in these functions. These differences are then due to other causes of individual "utility" or preferences, in particular the differences in capacities to enjoy and in tastes that the above questions have suggested to be morally irrelevant for the evaluation of the distributions under consideration. The theory of the "equal-equivalent utility function" provides the technical solution to this problem and, hence, the required definition of the "individual welfare".

IV-3 The equal-equivalent utility function

Consider an equal allocation $e\,x$.

Define function $u(x)$ by: $W\{u_i(x)\} = W[e\,u(x)] = w[u(x)]$ (3)

def of w

W increasing $\Rightarrow u(x)$ is well-defined.

u is the *equal-equivalent utility function*.

u is utility minus specific | hedonic capacities $u_i(\ )$ | which are averaged away, thanks
| and tastes | to W as averaging function.

It is *individual "welfare"*, the *same function for all*.

$(3) \Rightarrow u(x) = w^{-1} \circ W\{u_i(x)\}$. (3')

For balance, W has to be symmetrical. This requires that the u_i are comparable by > or = (They are "fundamental utility", at least co-ordinal).

This u, which gives individuals' $u(x_i)$ which can be compared and evaluated by the social ethical function W and are "in between" utilities u_i and goods or income x_i , also seem to be a proper definition of the *midfare* that G. Cohen (1989) seeks as the ethically relevant "currency of justice".

IV-4 Two equal-equivalent allocations

→There thus are 2 possible definitions of *equal-equivalent allocations*, with -utilities;
or
-welfares.

def $\bar{\bar{x}}$ def u

$W\{u_i(x_i)\} = W\{u_i(\bar{\bar{x}}_u)\} = W[eu(\bar{\bar{x}}_u)]$ $\bar{\bar{x}}_u \in E_u$ "utility".

$W\{u(x_i)\} = W[eu(\bar{\bar{x}}_w)] = W\{u_i(\bar{\bar{x}}_w)\}$ $\bar{\bar{x}}_w \in E_w$ "welfare".

Cases $D=\Re^m$.

Case m=1: $\bar{\bar{y}}_u$ and $\bar{\bar{y}}_w \Rightarrow$ 6 more indices of inequality I_w ...

Case m>1 (figure 3).

Figure 3

E_u and E_w are both iso-u loci $\Rightarrow$ hence they do not intersect. For all defined I, they provide the same ranking.

Comparison: $W\{u_i(x_i)\} > W\{u(x_i)\}$ if there is a positive correlation between allocations and tastes-preferences.

There are two limiting forms

equity-"no-envy"	$u_i(x_i) \geq u_i(x_j), \forall i,j$	| Plus 1 strict >
"adequacy"	$u_i(x_i) \geq u_j(x_i), \forall i,j$	|

Case m=1, $\bar{\bar{y}}_w < \bar{\bar{y}}_u$. $I_w > I_u$ for I^a, I^r, I^t, η.

From (3): u_i increasing in $x^k \ \forall i \Rightarrow$ idem u.

The equal-equivalent utility function will be put here to two uses: define the welfarist "equivalent property", and, more importantly, determine the optimum distribution in macrojustice.

IV-5 The welfarist "equivalent property"

Consider the case $W=\Sigma$ (utilitarianism),

$W\{u_i(x)\} = nu(x)$, $u=(1/n)\Sigma u_i$

$W\{u(x_i)\} = \Sigma u(x_i) = (1/n)\ \Sigma_{i,j}\ u_i(x_j)$

For the case $x_i=y_i$, $D=\Re$ (m=1)

u_i concave $\forall i \Rightarrow u$ concave. Information u_i concave $\forall i$, hence u concave.

For$\forall$ u concave: this is the property.

V – Macrojustice

A case m=2: $x_i=y_i$, ℓ_i (labour) or $\lambda_i=1-\ell_i$ (leisure). One good only is transferable, y.

Classical "optimum income tax" theory uses max $W\{u(x_i)\}$ with concave functions.

(Mirrlees: "u because differences in tastes are irrelevant for this issue".)

This amounts to minimize a bidimensional inequality.

$$W\{u(x_i)\} < W[en^{-1}\Sigma u(x_i)]$$

$$W\{u(x_i)\} \leq W[en^{-1}\Sigma u(x_i)] < W[en^{-1}u(\bar{x})]\,.$$

= for case Σ

(utilitarianism)

The ethical end-value is the equalizand (that which should ideally be equalized). Hence this is x_i rather than $u(x_i)$ (equal $x_i \Rightarrow$ equal $u(x_i)$, not converse).

Hence the "first-best" is equal x_i.

Rawls's ideal is equality in "primary goods"

His economic primary goods are in 1971: y_i

in 1974 (post Musgrave(1974)): 2: y_i and λ_i.

Consider equal x_i. But which equal allocation?

$x_i = x,\ \forall i$

$W\{u_i(x)\} = w \circ u(x)\,.$ Max $W \Leftrightarrow$ Max u.

Note y_i =y, $\ell_i = \ell$, $\forall i$. max $u(y, \ell)$ |

constraint $\Sigma y_i = \Sigma \ell_i w_i \Rightarrow y = \ell\ \bar{w}$ | $\ell = k$ ⎫

| ⇒ ⎬ K (see figure 4)

| $y = k\bar{w}$ ⎭

Figure 4

This best egalitarian solution has *3 vices*:

there is no | freedom: each *i* prefers other (y_i, ℓ_i) to K

| Pareto-efficiency: idem.

| Partial self-ownership (see question 4 of IV-1).

Remedy to all these three issues: free (action, exchange, labour) from K (figure 4).

i chooses ℓ_i, earns extra income $(\ell_i - k)w_i$ (if $\ell_i > k$, see below), untaxed.

This result has the following meanings, structures and properties:

1 *The two-part income*

$$y_i = \quad k\bar{w} \quad + \quad (\ell_i - k)w_i \qquad (4)$$

egalitarian *liberal*

(equal income for equal labour)

according to: *desert* *merit* (labour and capacity w_i).

2 $y_i = w_i\ell_i \quad + \quad k \cdot (\bar{w} - w_i)$ (4').

earned *transfers*

$= t_i = -T_i$

3 t_i = *ELIE*: equal-labour income equalization.

4 Policy not based on ℓ_i: *no disincentive effect*. Pareto-efficient.

5 = each yields to each other the product of the same labour $(k/n)w_i$:

general equal labour reciprocity.

Promotes reciprocitarian sentiments (voluntary).

6 *productivity-progressive transfers*: each *i* yields to each *j* less productive ($w_j<w_i$) the same fraction of the difference in productivities,

$(k/n)(w_i-w_j)$.

7 = *equal universal basic income* ($k\overline{w}$) *financed by an equal labour* (k) *of each.*

8 Total income: earned plus leisure.

value of leisure: $w_i\lambda_i$. From (4') (with $\ell_i+\lambda_i=1$),

$$Y_i = y_i + w_i\lambda_i = (1-k)w_i + k\overline{w} = P_i = P(1,w_i) \qquad (5)$$

def

(equal uniform) *concentration* (to the mean) of total productivities.

(generalizable transfer principle).

9 (5): $P_i = P(1,w_i)$ = price index of prices of income/goods (1) and labour/leisure (w_i) with coefficients $k\overline{w}$ and $(1-k)$.

From (5), Y_i/P_i is the same for all *i*.

This means *equal real income, purchasing power, freedom of choice*.

10 also *equal free exchange* (labour) *from an equal allocation* (K).

Note: $k\leq\ell_i$, $\forall i$.

1) De facto: actual redistributions produce the same decrease in inequality as ELIE with k from 1 (US) to 2 (Scandinavia) days/week.

2) Ethics: if $k>\ell_i$, $k-\ell_i$ is a part of leisure

-taxed at value of labour if $w_i>\overline{w}$: is rejected;

-receives wage complement $w_i-\overline{w}$ for hours that produce no wage if $w_i<\overline{w}$: is absurd.

If involuntary unemployment: there is an extension of the theory (Kolm, 2004).

11 There is a minimum income $k\overline{w}$ (from (4)).

Implementation

1) *Incentive compatible*

i chooses type and amount of labour ℓ_i. Capacity w_i = most remunerative possible labour.

max $u_i(y_i, \ell_i)$ on (4) $\Rightarrow$ chooses the highest w_i for $\ell_i - k > 0$.

This reveals productive capacities.

2) wage labour = 9/10 of labour in modern economies.

Firms cannot hide $\{w_i\}$.

3) Tax w_i. For instance: *exempt overtime income* over rather low benchmark.

$$T_i = k \cdot (w_i - \overline{w}) = (k / \ell^o) w_i \ell^o - k\overline{w}$$

Applied in France since 2007. ℓ^o = 35 h/week or idem in days/year for executives. And a uniform tax rebate. (and for partial labour ℓ_i: exempt "supplementary hours" from the labour contract).

Actually, an individual productivity is due in part to her natural capacities and to education which comes from her own effort, her family effort and influence and public education. In the detailed analysis (Kolm, 2004), the individual learning effort are a part of her labour, public education financed by a tax on earned income is distributionally neutral, and the given resources w_i reduce to natural talents and family influence and support.

VI – Conclusion

1. The 4 nodes of income justice

Figure 5 summarizes the main stages and reasons of the foregoing discussion.

Figure 5

2. Distribution of benefits from capacities: the entitlement-accountability cut

The main resources of a society are the human resources or *capacities* of its members. They are of two kinds and each piece or the benefits from it can be allocated according to two alternative principles. There are the *productive* or *earning capacities*, of value w_i for individual *i*, and which produce income (and consumption). And there are the *hedonic capacities*, individuals' capacities to enjoy, represented by the utility functions $u_i(\ \ )$ for each individual *i*. Since a capacity is held by an individual, its holder, a piece of it or the benefit from this piece can be a priori allocated to its holder who then is *entitled* to this piece or benefit, and also *ipso facto* is *accountable* for any corresponding shortcoming of this capacity. This is *self-ownership*. The other *prima facie* alternative is *equal sharing* of this

benefit (and of this accountability) between the members of the society, or some criterion referring to some tangible equality.

The great classical ethical theories of distributive justice define themselves along these lines (figure 6). *Classical liberalism* is *full self-ownership* of all capacities, productive and hedonic. Classical welfarism takes as end-values individuals' happiness, say the u_i , by maximizing some aggregate "social welfare function" $W(\{u_i\})$. This can be some ideal highest equality in the u_i, perhaps maximin or leximin in the u_i, or the effect of a symmetrical strictly quasi-concave (or concave) function W. This objective is the aim of the policy allocating the allocations x_i. Rawls (1971), Dworkin (1981) and others, deny relevance of the utility functions $u_i(x_i)$ in this process. Rawls' s ideal is a basic equality in "primary goods", with, in 1971, income as the economic primary good and a second-best egalitarian solution as $\text{maxmin}_i y_i$ (the "difference principle"). This implies self-ownership for hedonic capacities.

Figure 6

The obtained ELIE solution cuts across both types of capacities for defining the domains of self-ownership and of equal allocation. These two cuts correspond to one another. Labour k is a fraction of productive or earning capacities if total time is measured as 1. The corresponding products are equally shared by the ELIE policy. The complement $1–k$ is self-owned, but individuals i who freely choose to work ℓ_i do not put all this resource to earning use if $\ell_i<1$ ($\lambda_i>0$); the rest is leisure λ_i. As for the hedonic part, welfare u only is the same function for all, and the individuals are entitled to or accountable for its difference with their own utility functions u_i. These two capacity "cuts" are related since coefficient or equalization labour k has been obtained from the maximization of $u(\ell\overline{w}, \ell)$ for ℓ. Since function $W(\{u_i\})$ determines the equal-equivalent utility function u, it determines the value of k. Classical liberalism is ELIE with k=0. However, this simple and richly meaningful index k of the policy is bound to be a direct object of the social ethical and political debate and choice.

In addition to this sharing of the various capacities, the policy requires a principle concerning the comparison and aggregation of the chosen individual end-values (for instance the u_i or the y_i). Equality in these variables is a possibility, with the highest possible equal value. When equality is not possible or is dominated by unequal sets of these variables which give more to each i, the second best egalitarian solution can be maximin or leximin (for instance Rawls's "difference principle" for incomes, or "practical justice" as leximin in

comparable u_i – "fundamental utility" – in *Justice and Equity* (Kolm, 1971)). The solution can be a maximand $W(\{u_i\})$ or $M(\{y_i\})$, with an egalitarian preference for symmetrical concave functions. The case opposite to equality is the highest sum, Σu_i (utilitarianism) or Σy_i (highest social income). Then, equality refers to comparisons of variations of the variables. See figure 6.

This result concerns macrojustice. In addition, there can be policies – including solidaristic public transfers, social insurances and free or controlled ordinary insurances – for issues of mesojustice (notably health care and education) and microjustice for exceptional needs, situations or accidents.

Finally, the concept of equal-equivalence, applied to any allocation, notably one-dimensional or multidimensional, or to utility functions, permits one to derive, from the overall ethical evaluation, comparisons and measures of the ethical wrongness of inequality – notably of its injustice –, the morally relevant interpersonally comparable concept of "welfare", and the general meaning-rich structure of optimum income distribution, transfers and taxation.

The obtained rational ethical equalities are bound to be morally accepted or required as just by all citizens, thus permitting a society of peace, good social relations and efficient cooperation.

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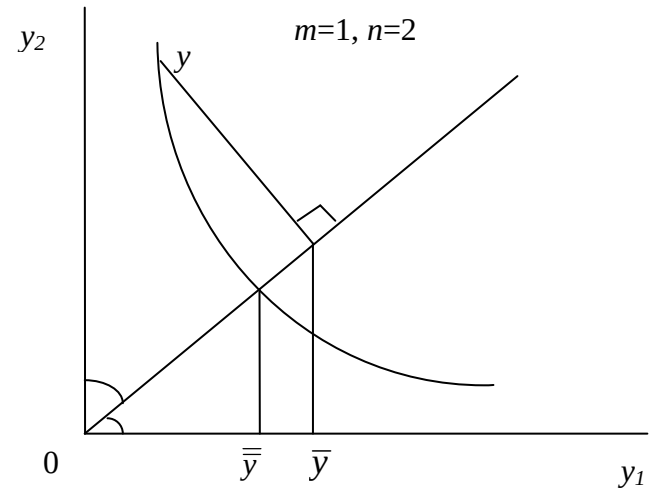

Figure 1. One-dimensional inequality

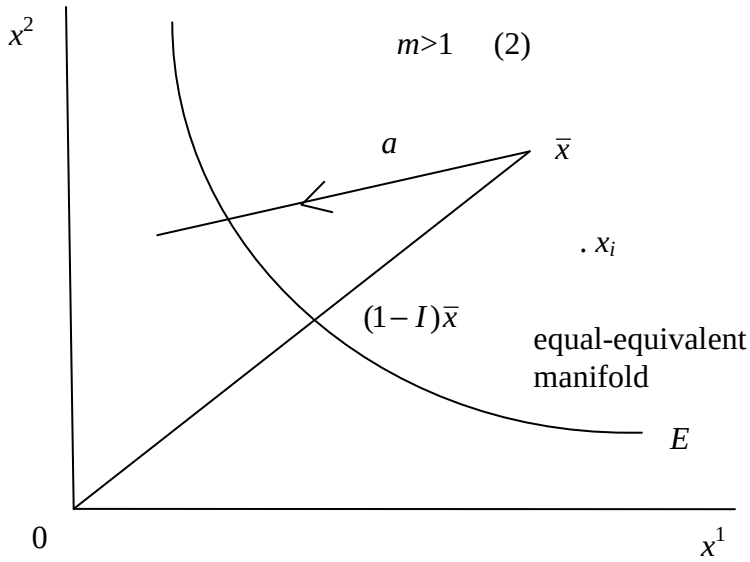

Figure 2. Multidimensional inequality

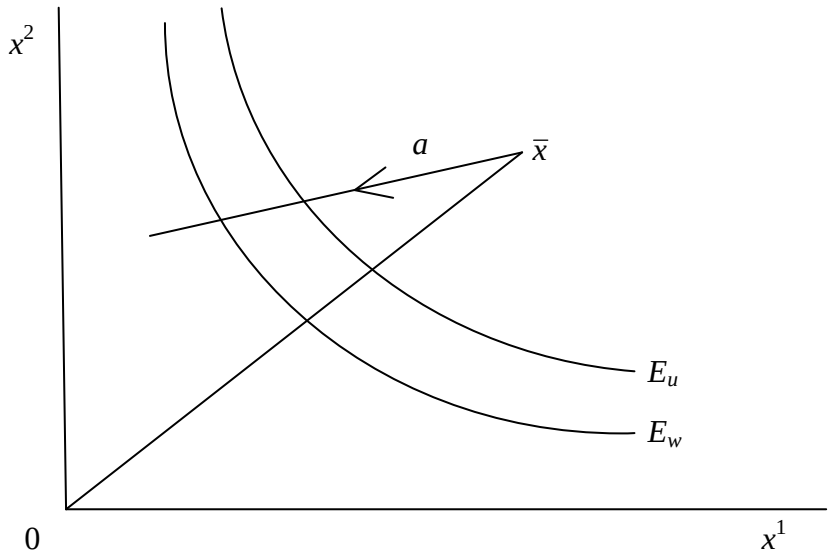

Figure 3. The two equal-equivalent manifolds

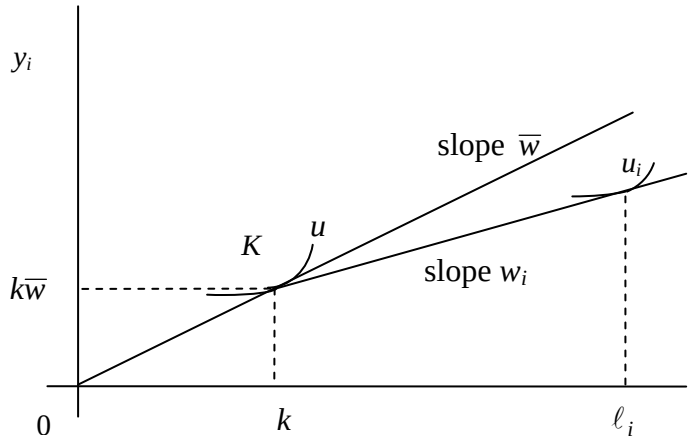

Figure 4. Equal-Labour Income Equalization

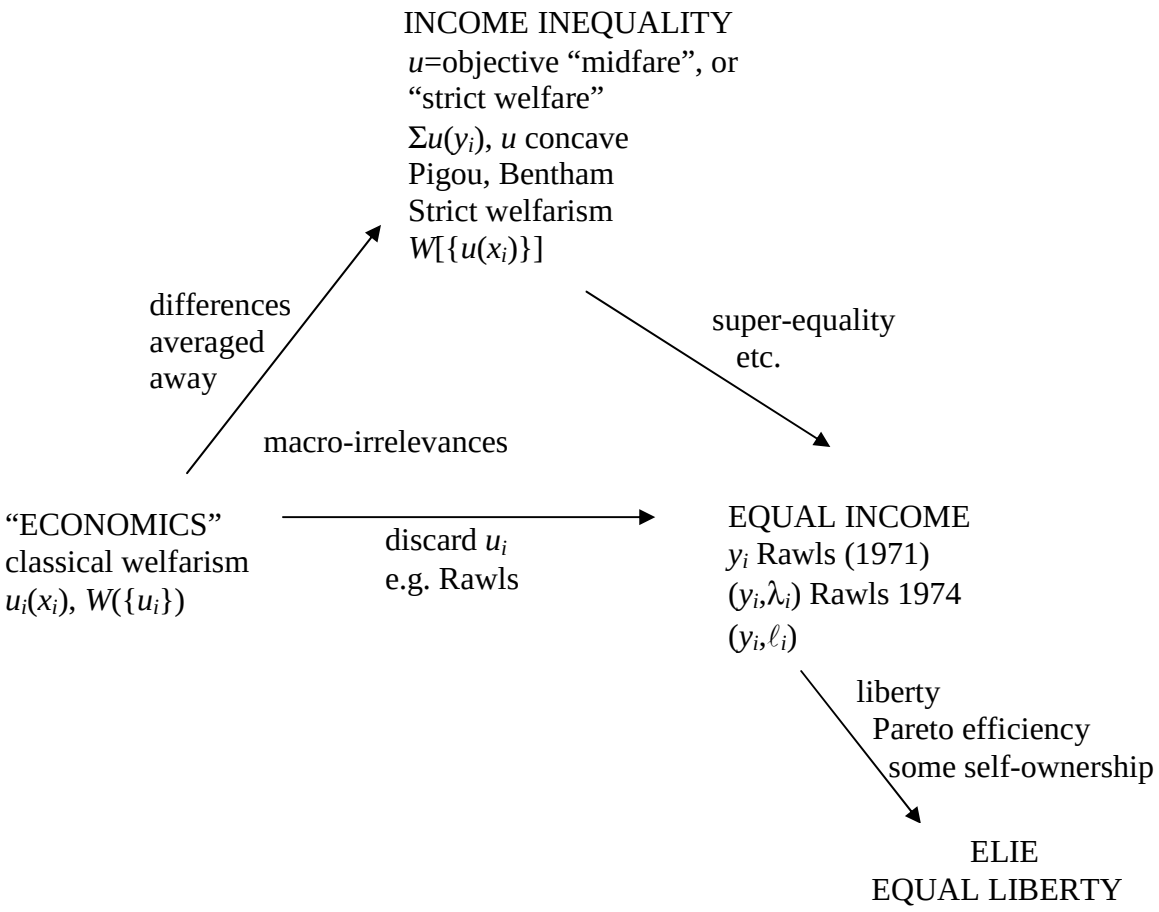

Figure 5. The 4 nodes of income justice

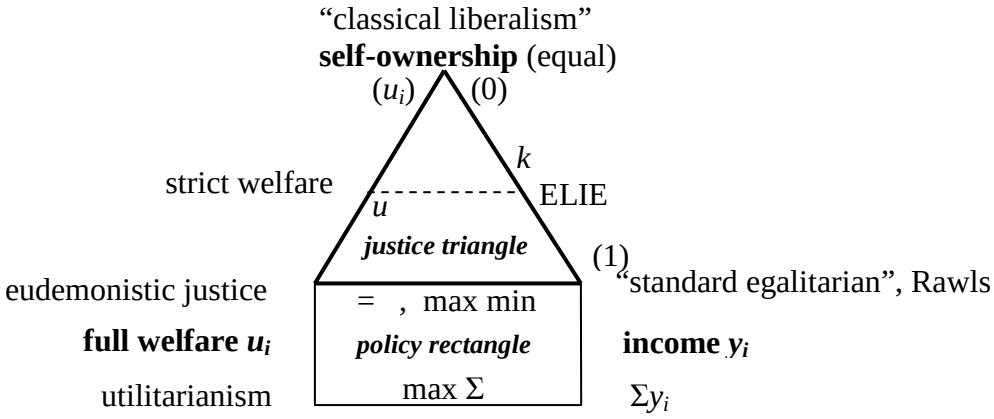

Figure 6. The justice triangle