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THE THEORY OF RULES AND THE MORAL PROVISION OF PUBLIC GOODS

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Abstract

Contributing to collective actions or public goods for moral reasons is an important fact of society and its economy. These reasons have often been elaborated by moral philosophers such as Kant and Rousseau, or can be "lateral reciprocity" or moral matching, or Rawls's notion of stability. Their modeling leads to distinguish rules of comparative fairness or sharing from their application. Applying rules makes unanimity and Pareto efficiency imply each other for "consistent" direct or "deviation" rules and for "moral teams". With even low degrees of morality, large numbers of participants are often favourable to non-free-riding.

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1. Rules and the public good

When Adam Smith left moral sentiments for wealth-creating self-interested exchange, he founded economics but left economists with major problems in the related fields of public goods and distribution which require moral sentiments about issues such as fairness, impartiality and the civic virtue of cooperation. Explaining free contributions and finding best policies for collective actions and public goods certainly constitute a basic challenge for economics. It is also the essence of the central concern of Adam Smith's colleagues, the Enlightenment philosophers of the late eighteenth century. Nothing is gained by continuing to keep these two disciplines apart: the precise deductions of one and the conceptual depth and

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psychological subtlety of the other can associate on their common grounds of methodological and moral individualism and normative concerns, and this may help solving the severe difficulties that both face (the differences in style of these two ways of understanding society should be overcome and should rather make them complementary). These philosophies include the social approach of David Hume's "conventions" (with a clear view of the technical aspect of the public good problem); Jean-Jacques Rousseau's political conception – presented as a public good problem – of a hypothetical "social contract" transmuting separate individuals into citizens unanimously implementing the "general will"; and Immanuel Kant's deontic categorical (i.e. unconditional) imperative to "follow the rule that one could want to be universally obeyed".[1] These moral philosophies relate to the theory of constitutions (such as Buchanan's in economics), to Rawls's late-enlightenment concept of a stable rule of fairness, inspired by Kant, and to "communicative ethics" (e.g. Habermas and Apel). They also relate to the economic analyses of the "lateral reciprocity" or moral matching of freely contributing one's fair share when other people contribute theirs and to the philosophical notion (Rawls) of reciprocally accepted terms of cooperation. It is important that these theories are but elaborations of common actual social conducts and reasonings against free riding, referring to norms, implicit agreements, fair contributions, matching reciprocities, good citizenship, dialog, and "folk-Kantianism" revealed by reasons such as "I contribute because what if nobody contributed?" (the main reason people give when they are asked why they often abstain from polluting public places or vote in large elections).

Practically, when the set of individuals' actions, notably self-interested ones, leads society into an unwanted situation – for instance one of the general type in which other possible situations are preferred by everyone (a failure of Pareto efficiency) –, society tends to produce rules of individual conducts such that the unwanted result does not occur or is attenuated if everybody abides by them. These rules are followed because of personal morality (or other people's judgments), public coercion, or both. The modes of realization and motives are often more or less mixed in various ways. Rules that secure Pareto efficiency may be unanimously preferred to their absence. Public constraints may be democratically or even unanimously chosen. Some people may follow some rules freely "if other people obey them too" and coercion may only serve to guarantee each agent's action for the others. Classical economic rules for securing Pareto efficiency are Pigou taxes for externalities or, for public

[1] See Rousseau (1762), Kant (1785, 1907), Nagel (1991) and Birnbaum, Lively and Parry (1978).

goods, Lindahl proportional pricing or the more general non-proportional compared contributions (but, possibly, only simple linear ones) that permit to reach any Pareto-efficient state.[2]

The study of rules, notably for the provision of public goods, has both a normative and a descriptive-explanatory objective: choosing policy and understanding social norms and the behaviour they induce. Judging them refers basically to the two complementary criteria of Pareto efficiency of the result of their application and of fairness – plus, perhaps, less basic properties such as simplicity. Classically, such normative judgments can either be incorporated in the structure of a maximized social welfare function – with its limitation to the comparison of individuals' welfare and classical difficulties – or result from evaluations of fairness in the direct consideration or comparison of people's situations. The realization of rules has two stages, their choice and their application. This is a necessary structure of ruled or principled choice because, by definition, a rule has to have several possible applications (at least notional ones) – such a choice is not ad hoc. This provides notably the possibility to deal with the fairness issue at the level of the choice of the rule and, consequently, to have an application unanimously desired given that the rule applies. The method of analysis has to associate properties revealed by economic modeling and classical philosophical thought.

This thought is, first, briefly recalled, with the inspiration it may provide, the logical problems it meets, and the very few economic studies that it inspired. The various interesting properties of rules are noted. The basic general structure is the following. For a given allocation problem, a *general rule* is a rule that first, is followed by all individuals jointly (possibly according to the specific characteristics of each), and second, is defined as being applied in several specific overall social situations (such a situation implies a situation for each individual).[3] Such a rule is (socially) *consistent* when all these individuals prefer the same of its possible overall applications (this implies given that it applies to all). Hence, their own allocations, contributions or acts that all individuals prefer or choose given that a general rule applies constitute an application of this rule if and only if this rule is consistent. *Deviation rules* are general rules that define deviations from any given state; unanimous preference for an absence of deviation from a state according to a deviation rule defines the consistency between this rule and this state. A rule of fairness can be either (directly) comparative by

[2] Kolm (1970a, 1970b), Bilodeau and Gravel (2004).
[3] Rules are modeled in section 3.2.

comparing individuals' situations (e.g. contributions or allocations) or individualized (for instance indicating each individual's share of each level of a cost or of a benefit). Consistent rules are a subset of the general rules.[4] Deviation rules can a priori have any structure, but they can describe a more restricted conception of fairness only, comparing states and local.

For public goods or collective action, and for this structure of social interaction only, consistent and deviation rules make unanimity and Pareto efficiency imply one another: a set of individual contributions is Pareto efficient if and only if it is the unanimously preferred application of a consistent rule, and if and only if it is unanimously preferred to all deviations from it according to any given deviation rule. With well-behaved preferences and costs, the possibility to reach all Pareto-efficient outcomes even holds when restricting to rules of the simplest forms: two-parameter linear rules for consistent rules (each individual contribution is a linear – affine – function of each other or of the total cost of the good), and proportional deviations for deviation rules (deviations of individual contributions are proportional between them or to the variation of the total cost). Particular structures of utilities and linear rules provide notable applications (including cases of Pareto-efficiency for socially inconsistent rules).

An alternative to these solutions by comparative or sharing rules and their application is an a priori moralization of individuals' preferences into the same interest-respecting social maximand, which makes participants constitute a "moral team" (Radner calls "team" a set of individuals who seek to maximize the same thing). We finally show how the presence of some moral conduct often makes the large number of participants favourable to the absence of free riding.

2. Applications

This logic of rules can apply to several types of moral conducts, reasons or policies.

2.1 Rousseau's problem

[4] They will be referred to as "direct consistent rules" if there is a risk of confusion with deviation rules.

For Rousseau (1762), the very essence of a society is a public good issue: each citizen gives up her contribution for the benefit of all but beneficiates from the joint contributions of all. This is realized in a structured way, however. Rousseau's social contract divides into two conceptually successive parts: a social compact choosing the basic rules of society and a social treaty applying them. He sees this second level as the unanimous choice of citizens abiding by the first choice that provides the means to characterize this "general will" (lack of unanimity just reveals citizens' imperfect information about the general will, which can justify majority voting – Condorcet's (1785) famous study of voting is an attempt to solve this Rousseauan question).

This social moral theory is, therefore, a conceptual construction of the life of society as a two-stage process where the first chooses rules for solving conflicts and the second applies them with unanimity. It can be made operational and precise in various ways. The compact may describe the general will as a social maximand, for instance a classical social welfare function, that citizens seek to maximize at the second stage. This view of society as a "moral team" amounts to Condorcet's interpretation of Rousseau. Yet, the fact that concepts of fairness are not always based on comparisons of preferences or utility and the difficulties of the aggregation of transitivities render problematic the very representation of the social choice by a social maximand.

These obstacles suggest, as alternative representation, the more modest and straightforward one of considering – according to Rousseau's concepts – the general will as the will of all when they obey some general rules established at the first stage. This stage can thus focus on its core issue: the choice of rules of fairness that provide an answer to the conflicts between the interests or desires of the various individuals. At the second stage, the actions are performed under the constraint that the chosen rule applies to everybody, but, given this condition, with people's standard preferences. Rousseau's idea is that this second-stage choice should be unanimous. This implies that the "compact" chooses a consistent rule. It may also choose a deviation rule and the will of all chooses a state consistent with it. However, the principle of unanimity is also violated, in a sense, if the final outcome fails to be Pareto efficient (since, then, there exist other states that are unanimously preferred – with the possible indifference of some people). The maximization of a standard preference-respecting social welfare function realizes Pareto efficiency, of course. But it turns out that, for the public-good structure of social interaction that Rousseau sees in society, and for this structure only, the rules of all the noted types following a concept of consistency – direct or deviational, comparative or individualized – entail Pareto efficiency, whereas any Pareto-efficient set of individual contributions can be the outcome of rules of each of these types.

2.2 Constitutions

Relatedly, the standard structure of choices in a society actually consists of two successive levels. First a constitution, which is a set of rules, is chosen. Second, the other choices are constrained by everybody abiding by these rules. In economics, J. Buchanan (1975, 1991) has emphasized this structure of the social choice. A good constitution should provide all the rules for solving possible conflicts between citizens, all oppositions between their desires or interests. This may include rules for the provision of public goods. However, the unanimity in the application of the constitution usually includes the type of unanimity constituted by free agreements (notably for markets).

2.3 Kant's problem and the nascent Kantian economics

Laffont (1975) proposed a "Kantian" solution to the public good problem by assuming identical individuals (same utility function and income) who contribute to a public good assuming that each other contributes the same amount (duplication) and maximizing their (standard) utility function. The result is consistent (all prefer the same amount of public good), Pareto-efficient, and, indeed, a Lindahl solution. This model faces several problems, however. Kant demands that an individual first chooses a rule, and then applies it. When applied, a rule indicates each individual's contribution. Then, the rule preferred by an individual maximizing her utility function is one in which all the others contribute all they have to the public good and she contributes only little (not at all for a relatively "small" individual). This individual thus chooses extreme free riding. So does any other who reasons similarly, contrary to the rule preferred by the initial individual (who can know or foresee this behaviour). This individually chosen rule is a "self-centered rule". The solution to this problem consists in restricting the rules chosen to the set of what can be called *impartial rules*, that is, rules that are not so that a reason for their choice is that they favour more the person who chooses them or people she likes. Self-centered rules are not "universalizable". Moreover, people have in general different utility functions and incomes.

Even restricting to impartial rules, “Kantian” individuals choosing a rule the application of which maximizes their different (standard) utility functions will a priori choose different rules. Then each chooses assuming that the others follow a rule different from the one they follow if they are also “Kantians”, and this individual can know this. One side aspect is that the Kantian meta-rule or principle does not apply to itself: an individual cannot assume that every other follows the rule that this other prefers and the rule that she herself prefers since they are a priori different and lead to different choices. On the whole, the result is inconsistent and generally not Pareto efficient.

This may not happen if the rule is defined broadly with few alternatives – perhaps two, “do or do not do this” – and all individuals prefer the same alternative as universal rule. Kant has this in mind, indeed, as shown by his examples such as the choice between “lie or do not lie” without consideration of specific circumstances. This also applies to “vote or do not vote”, a very important issue since the main reason people give to explain why they vote in large elections in which their own vote makes no difference is the “folk-Kantian” question: “what if nobody voted?”[5] However, the inconsistency remains with more specific questions, such as the specific level of contribution to a public good.

However, Kant says that you “could want the result” of the rule you choose for hypothetical application by all (universal). He also emphasizes that rational moral conduct is not derived from “inclinations”, that is, tastes or desires, precisely what standard preferences or utility functions mean. Therefore, if the rules are chosen by maximization, it should be that of some ethical functions. In a pioneering elaborate study, Bordignon (1993) describes “Kantian” contributions to a public good by individuals each of whom chooses her contribution by assuming that each other pays the same amount, and by maximizing a sum of utilities each of which is her own utility function applied to the situation of each individual of the society (the form could be other than a sum, for instance a maximin). The result is not Pareto efficient, and Bordignon compares it with a political choice which is also inefficient. By contrast, people may agree or be persuaded to maximize the same classical social welfare function increasing function of individuals’ utilities.[6] They constitute a moral team and the

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[5] If the rule includes the nature of the vote, then the Kantian conduct consists in ruling out strategic voting.

[6] Kolm (2008a). A theory of the formation of such a unanimously maximized social welfare function as the result of balanced mutual influence in a process of fair social dialogue is presented in Kolm (2000, 2004 chap. 20). Habermas’s (1981, 1983) “communicative ethics” and its dialogical

outcome is Pareto efficient. All these solutions make some use of individuals' utility functions. It is, indeed, necessary that these functions appear somewhere if the outcome is to be Pareto efficient. However, these solutions also introduce other structures of a moral/social nature. Moreover, Kant, in examples, also considers self-interest for choosing the rules: one should help the needy because one may need help sometimes. He even writes: "Interest is the occasion for reason to become practical, that is, to become a cause that determines will". Therefore, "reason" is the rule (general and in particular universal), and its specific application occurs when interest, constrained by reason, chooses an action.

Then, the choices of individuals remain described by their standard utility functions and one focuses on the choice of rules to secure the desired properties and notably Pareto efficiency. This is the case of the Pareto-efficient rules of payments for public goods that permit reaching any Pareto-efficient state with unanimity given the rule, possibly restricted to linear rules (Kolm 1970a, 1970b – Lindahl prices are a particular solution). It is also an application to public goods of Bilodeau's and Gravel's (2004) concept of "Kantian rules". The rule can also be for deviations from the chosen state that everybody finds worse than this state, which also can yield any Pareto-efficient solution ("deviational Kantianism").[7]

2.4 Matching and lateral reciprocity

A common reason and conduct is described by expressions such as: "Given that other people contribute, I contribute too", "I take my fair share of the burden given that the other people take theirs", etc., without this constituting a binding exchange or agreement. This is fair matching or lateral reciprocity – i.e. reciprocity with co-contributors whereas simple reciprocity refers to ordinary gifts and means providing a return gift. This raises two issues, the definition of the matching and implementation. The stability of the outcome requires, first of all, that all participants share the same conception of the fair sharing of the burden between them, a property of consistency. This identity can itself result from reciprocity in acceptance of the same rule of fairness. In philosophy, this is the basic property of Rawls's concept of a "well-ordered society". Indeed, Rawls (1980) emphasizes "a conception of the *fair terms of social cooperation*, that is, terms each participant may reasonably be expected to accept,

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convergence towards a shared moral view in the "ideal speech" situation provides philosophical underpinnings of such a model.

[7] Kolm (2008a).

provided that everyone else likewise accepts them. Fair terms of cooperation articulate an idea of reciprocity and mutuality: all who cooperate must benefit, or share in common burdens, in some appropriate fashion as judged by a suitable benchmark of comparison". Or again (1982): "in a 'well-ordered society' each citizen accepts the principles and knows that everyone accepts them as well… Moral persons are said to have both the capacity and the desire to cooperate on fair terms with others for reciprocal advantage".

The matching contribution to public goods is sometimes mentioned and is analyzed by Sugden (1984) and Kolm (1984). Sugden's elaborate and important proposal considers a specific rule by which each individual contributes a meaningful function of the contributions of all the others whatever they are. The outcome is not non-fortuitously Pareto efficient. This means that this rule is a kind of rule of fairness in contributions which does not imply a rule of fair sharing of the total surplus. By contrast, other types of rules of fairness are consistent with Pareto efficiency or even induce it. Notably, consistent binary comparative rules (or even linear ones only), lead to Pareto-efficient sets of contributions and can induce all such sets.

Moreover, people who contribute for a motive of reciprocity should be sufficiently sure that others contribute their fair amount. The surest way is that others are forced to contribute. Then, these contributions are no longer free. However, if all the people in question have this motive, everybody is forced to contribute, and yet she also wants to provide this contribution because the others provide theirs. This constraint is reached but not binding, although it plays a useful or necessary role. These contributions are actually both forced and free. This is for instance the case of fair taxes to which everyone freely consents. Other possibilities are provided in sequential situations with some people who may contribute without full contributions by all others, possibly unconditionally for some, and with a population heterogeneous in this respect.[8]

2.5 Rawls's stability and dialogical unanimity

Rawls (1980) calls a rule of fairness stable (or inherently stable) when it generates its own support. That is, if people live with such a rule, then they support it. This opinion is a publicly shared point of view, arising in a "shared political culture" with deliberation and democracy,

[8] See, notably, Fershtan and Nitzan (1991), Admati and Perry (1991), Marx and Matthews (2000), and Masclet, Willinger and Figuières (2007).

and such a society is a “well-ordered society”. There is a “congruence of the right and of the good”, that is, of the rule of fairness and of people’s desires. A number of other theories consider similar rules of fairness or justice unanimously wanted by the people to whom it applies as the result of a process of social dialogue by which people’s moral opinions mutually influence one another (this convergence is the “ideal speech” of Habermas’s (1981, 1983) “communicative ethics”, and models of such convergence have been analyzed[9]). These notions deserve three remarks. First, the Rawlsian stability of a general rule of fairness implies, in particular, that people want the same specific application of it. This latter condition constitutes a minimal stability useful when the general rule is chosen otherwise, such as by an agreement of the exchange type, possibly a hypothetical one (i.e. a social contract), a political choice, tradition, etc. Second, the desire of people implies that of all people, i.e. unanimity. Then, minimal stability characterizes consistent general rules, applied directly or as non-deviation. Such rules are the conceptions of the right that make individuals’ conceptions of the good coincide. Third, if the outcome is not Pareto efficient, people will probably agree to shift to one of the states that they unanimously prefer (with the possible indifference of some of them). This destroys stability. However, if the rule concerns the contributions to a public good to be chosen, whether it is direct or deviational and comparative or sharing, it turns out that the outcome is Pareto efficient – and the public-good structure of interaction alone has this property – , and all Pareto-efficient sets of contributions can be obtained by such rules (even usually by simple linear direct rules or proportional deviation rules only).

2.6 Tax, tariff, price

A rule for the financing of a public good can be a tax, a tariff or a price. This may be chosen by a collective decision between the people concerned, however, for instance by a political decision for a tax.

3. Rules

We first note a number of properties of rules that one “could want”, and then present the general logic of rules.

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[9] Such models for individuals’ social welfare functions (Pareto’s “utilities” as opposed to “ophelimities”, or Bergson’s functions) and for distributive justice are presented in Kolm 2000 and 2004, chap. 20.

3.1 Properties

(1) *Pareto efficiency*. Pareto efficiency of the outcome is valuable for its classical meaning of the absence of waste of individuals' welfare, and because its absence is a kind of amputation of the freedom of society and a failure of democracy.

(2) *Universal power*. A type of rules has universal power if all Pareto-efficient states can be obtained by a rule of this type.

(3) *Fairness*. Since Pareto-efficient states differ from one another by the distribution of society's possibilities between individuals, the complementary property is the fairness of this distribution implied by the rule. The material the distribution of which induces the basic judgment of this fairness can be various items (not necessarily welfare).

(4) *Impartiality*, defined above, is an aspect of fairness.

(5) *Form*. The rules can have various forms. An individual's fair situation (allocation, contribution, etc.) may be compared with that of every other's (as equality is), with those of all others jointly, or with some global situation (for instance the amount of a public good). The rule may be direct or deviational. It may consist of some function to maximize.

(6) *Unanimity*. A rule has the property of unanimity if all individuals prefer the same of the possible states in which the rule applies for everybody. This property gives a particular interest to the division of the social choice into two stages, the choice of such a rule and its application. The second stage, a social choice given that the rule applies, is unanimous, without conflict, oppositions and the usual problems of social choice. The first stage, therefore, concentrates these problems and can focus on their basic issue: the definition of fairness. Examples of unanimity-inducing rules are rules for fair lateral reciprocity or Rawlsian stability, Rousseau's compact defining a general will unanimously applied, the solution of the Kantian problem of choosing consistent universal rules (Bilodeau's and Gravel's (2004) "Kantian rules"), Lindahl's (1924) pricing for public goods, or the more general rules of payments for public goods that can reach all Pareto-efficient states and can be restricted to linear forms (Kolm 1970a, 1970b).

(7) *Social consistency*. If each individual's allocation or contribution is chosen as her preferred one assuming that a general rule is followed by all, and if these choices, with the same rule, correspond to one another by constituting a specific application of this rule, then the rule is socially consistent. This amounts formally to the property of unanimity.

(8) *Stability*. A general rule is stable if all individuals prefer either both this rule and a specific application of it (Rawls) or such an application given the rule (minimal stability, amounting to the properties of unanimity and consistency).

(9) *Logical consistency* of binary comparative rules. If z_i denotes an item relative to individual *i* (allocation, contribution, etc.) and *i*, *j*, *k*, … denote individuals who can have different relevant characteristics, a binary rule of comparative fairness defines "z_i for *i* is as fair as z_j for *j*", and this rule is logically consistent if this binary relation between individuals is reflexive, symmetrical and transitive. Fairness is indeed often such a binary concept (as is the related concept of equality).

(10) *Simplicity*. The simplicity of rules is an interesting property. Rules with the smallest number of parameters are, in particular, noteworthy. For instance, individual contributions to a public good in relation to one another or to the total cost of the good may be linear (affine) or proportional.

(11) *Specific cases*. What are these rules in specific cases such as similar individual preferences or incomes or particular forms of preferences?

If, when everybody follows the same general rule, people prefer unanimously (properties (5) and (6)) a Pareto-efficient state (property (1)) and since Pareto-efficient states differ from one another by the distribution (of welfare and hence of its causes), this implies that the principle of distribution is fully incorporated in the rule.

3.2 The logic of rules

Very generally, denote as $z_i \in \tilde{Z}$ something which is chosen to apply to individual *i*, $C_i \in \tilde{C}$ a sufficient set of characteristics of individual *i*, for each of *n* individuals. For simplicity (sufficient here) the functions introduced are considered to be one-to-one. A *point-rule* for choosing the *n* z_i is a function *r* giving $z_i = r(C_i)$ for each *i*. A (binary) *comparative rule* says that for a given z_i, there is a corresponding $z_j = R(z_i, C_i, C_j) = \rho^i_j(z_i)$. For instance, if z_i holds, then it is fair that z_j has this value, for some comparison described by this rule of comparative fairness.[10] Such a comparative rule defined for all pairs *i*, *j*, is *logically consistent* if it is

[10] Such a comparative rule commonly results from some kind of egalitarian comparison between some function of the pairs (z_i, C_i) and (z_j, C_j) ("justice is equality", and the concept of equality refers to a comparison between two individuals).

reflexive, symmetrical and transitive: for all *i*, *j*, *k*, $z_i = \rho_i^i(z_i)$, $z_i = \rho_j^i(z_j)$ implies $z_j = \rho_i^j(z_i)$, and $\rho_j^i \circ \rho_k^j = \rho_k^i$. Then, for each z_i, there is a set $z=\{z_j\}$ of n z_j corresponding to it by this rule. This set is equivalently parametrized by any of its z_j, or by the $\zeta \in \tilde{Z}$ of an added hypothetical individual of characteristics $C \in \tilde{C}$. In all cases, since $\zeta = z_j$ and $C = C_j$ for any *j* is possible, one can write $z_i = R(\zeta, C, C_i) = \rho_i(\zeta)$ for all *i*. Denote as $\rho = \{\rho_i\}$ the set of the *n* functions ρ_i. For any given ζ, the point-rule $z=\rho(\zeta)$ is a *specification* of rule ρ. If individual *i* has a preference over the set *z* described for instance by a utility function $U^i(z)$, then the specification of the rule ρ that maximizes U^i, defined by $\zeta_i = \arg\max_\zeta U^i[\rho(\zeta)]$, is individual *i*'s *preferred specification of the rule*. ζ_i will be assumed to be unique (the cases in which it would not be are fortuitous). A rule is *socially consistent* if all the individuals have the same preferred specification, $\zeta_i = \zeta^*$ for all *i*. Such a rule, both logically and socially consistent, is simply *consistent*.

A particular type of logically consistent rules consists in *deviation rules* that define a state $z' \in \tilde{Z}^n$ as a deviation from an a priori given other state $z \in \tilde{Z}^n$ and holds when $\tilde{Z}$ is in a vector space (representing quantities of goods), $\tilde{Z} \subseteq \Re^m$. A deviation rule $\tilde{\rho}$ for $z' \in \tilde{Z}^n$ from a given state $z \in \tilde{Z}^n$ and with a deviation parameter $\tilde{\zeta} \in \Re^m$ is $\tilde{\rho}(z, \tilde{\zeta})$ defined as $z' = z + \tilde{\rho}(z, \tilde{\zeta})$ with $\tilde{\rho}(z, 0) = 0$. The *non-deviation* with this rule is socially consistent if all individuals prefer $\tilde{\zeta} = 0$ and hence $\tilde{\rho} = 0$ and $z' = z$ ($\arg\max_{\tilde{\zeta}} U^i[z + \tilde{\rho}(z, \tilde{\zeta})] = 0$ for all *i*). Then, such a state *z* and the deviation rule $\tilde{\rho}(\tilde{z}, \tilde{\zeta})$ defined for the generic $\tilde{z} \in \tilde{Z}^n$ are said to be consistent with respect to one another. When this deviation rule is given, the choice is that of a state *z* consistent with it.[11]

3.3 Applications

This structure applies in particular to the noted cases. (1) The z_i can be allocations, taxes, etc., imposed to individuals *i* according to some rule (for instance of fairness). (2) In lateral reciprocity, if each individual *i* chooses the $z_i = r(C_i)$ of a given point-rule or her preferred

[11] Examples and specific applications of these concepts are provided shortly.

$z_i=\rho_i(\zeta^*)$ of a given consistent comparative rule if all others do the same, these point-rules may hold (given the noted particularities in implementation). (3) With Kantian individuals who choose by maximizing their utilities, inconsistency and inefficiency remain a priori if each chooses a point-rule or a general rule, but consistency is secured if they choose their preferred specification of any given consistent rule or the z_i that constitute a z consistent with a given deviation rule; this turns out to achieve Pareto efficiency also for contributions to a public good (and with this structure of interaction only), and this Kantian ethic has universal power (it can implement all Pareto-efficient states). (4) A model of Rousseau's social compact chooses a consistent rule ρ, which secures the unanimous choice of ζ^* constituting the general will; or the compact chooses a deviation rule $\tilde{\rho}(z, \tilde{\zeta})$ and the general will chooses a corresponding consistent z; these outcomes turn out to be Pareto efficient for contributing to a collective action or public good (and with this structure of interaction only); and this Rousseauan conception has universal power. (5) The choice of a consistent rule or of a z consistent with a deviation rule is a possible realization of Rawls's notion of moral stability, reinforced by the noted Pareto efficiency for application to public goods.

4. Rules for public goods

4.1 Comparative and sharing rules

Applying these concepts to the financing of a public good, $z_i \in \Re_+$ is individual i's free contribution or tax (depending on application), and $Z=\Sigma z_i$ is the cost of the public good, taken as the public good itself (*w.l.g.*). Consider a logically consistent rule $z=\rho(\zeta)$. The $\zeta \in \Re$ can be in particular any of the z_i (then ρ_i for this i is the identity function). Functions ρ^i_j are normally increasing, which we assume. Then, functions $\rho_i(\zeta)$ can be taken as increasing and parameter ζ can be replaced by any increasing function of itself with a corresponding contravariant change of functions $\rho_i(\zeta)$.[12] We have $Z=\Sigma z_i=\Sigma \rho_i(\zeta)$. Inversing gives the increasing function $\zeta(Z)$. Then, Z can be taken as a particular ζ, and functions $s_i(Z)=\rho_i[\zeta(Z)]$ are *sharing functions* that give each individual i's contribution to a given level of the public good Z if the comparative rule ρ is followed. The comparative rule between contributions z_i is thus

[12] In particular applications it will turn out to be convenient to consider decreasing functions $\rho_i(\zeta)$ for all i, which does not change the theory.

translated into the corresponding *sharing rule* of the cost Z of the public good for any Z. With $\bar{z}=Z/n$ denoting the average contribution, functions $\sigma_i(\bar{z})=s_i(n\bar{z})$ use homogeneous variables and provide the *deviations from average* of each contribution for each average, $\delta_i(\bar{z})=\sigma_i(\bar{z})-\bar{z}$ with $\Sigma\delta_i(\bar{z})=0$. Functions ρ_i will be taken as differentiable for this presentation. We have $Z=\Sigma s_i(Z)$ and hence $\Sigma s_i'=1$.

4.2 Pareto efficiencies

Consider increasing differentiable strictly quasi-concave utility functions of individuals i, $u^i(x_i, Z)$, with given initial incomes y_i, $x_i=y_i-z_i$, $Z=\Sigma z_i$, and a logically consistent rule $z=\rho(\zeta)$.[13] Denote $v^i(x_i, Z)=u_2^i/u_1^i$, $V^i(\zeta)=v^i[y_i-\rho_i(\zeta), \Sigma\rho_j(\zeta)]$, and ζ_i the ζ preferred by individual i, which satisfies, for an interior solution,[14]

$$-u_1^i\rho_i'(\zeta_i)+u_2^i\Sigma_j\rho_j'(\zeta_i)=0$$

or

$$V^i(\zeta_i)=\rho_i'(\zeta_i)/\Sigma_j\rho_j'(\zeta_i). \qquad (1)$$

$Z_i=\Sigma_j\rho_j(\zeta_i)$ is the level of Z preferred by individual i for the rule $\rho(\zeta)$ (Z can also be taken as parameter ζ, with $\rho_j(\zeta)=s_j(Z)$).

The condition for Pareto efficiency for interior solutions is

$$\Sigma v^i(y_i-z_i, \Sigma z_j)=1 \qquad (2)$$

for the actual choices of the z_i. It is in particular satisfied by the maximization of all u^i under the rule $\rho(\zeta)$ in the two following cases.

1) *The rule is consistent* (Bilodeau and Gravel, 2004, for “Kantian rules”).

Then, indeed, $\zeta_i=\zeta^*$, the same for all i, $z_i=\rho_i(\zeta^*)$ for all i, $v^i=V^i(\zeta^*)$, and, adding conditions (1) for all i, condition (2). Therefore, if all individuals agree on (or are imposed) a rule of fairness such that, when this rule is followed, their self-interest leads them to prefer the same

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[13] If g denotes the quantity of the public good of cost $Z(g)$ and $\tilde{u}_i(x_i, g)$ individual i's utility function as function of x_i and g, with $u_i(x_i, Z)=\tilde{u}_i(x, g)$, function u_i is quasi-concave if function $\tilde{u}_i$ is quasi-concave and g is produced with non-increasing returns to scale, i.e. function $Z(g)$ is convex, that is, in the classical case.

[14] Only interior solutions are considered explicitly in order to avoid technicalities and focus on meaning.

outcome, or to choose individual actions that correspond to one another according to this rule, the result is Pareto efficient.

2) *The linear case*.

That is, the two following properties hold:

(1) The rule is linear,

$$\rho_i = a_i\zeta + b_i \tag{3}$$

with constant $a_i>0$ and b_i for all i (this includes, in particular, the *proportional* rule where $b_i=0$ for all i, that is, the z_i are in given proportions – for example the z_i are proportional to some income of the individuals i, the y_i or others –, and in particular *equality* or *duplication* in which all the z_i are equal).

(2) All utility functions are quasi-linear in the public good which is produced at constant price taken as 1, i.e. have a specification of the form $u^i = \varphi_i(x_i)+Z$.

Then, indeed,

$$v^i = 1/\varphi'_i(x_i) = 1/\varphi'_i(y_i - a_i\zeta - b_i), \tag{4}$$

condition (1) writes

$$V^i(\zeta_i) = 1/\varphi'_i(y_i - a_i\zeta_i - b_i) = a_i/\Sigma a_j, \tag{5}$$

and, adding conditions (5) for all i,

$$\Sigma V^i(\zeta_i) = 1. \tag{6}$$

However, with this structure of functions u^i,

$$V^i(\zeta_i) = v^i[x_i, \Sigma(a_j\zeta_i + b_j)] = v^i[x_i, \Sigma a_j\zeta_j + b_j)] = v^i(x_i, Z)$$

and therefore relation (6) is the condition of Pareto efficiency of the state $x_i=y_i-z_i$ and $z_i=a_i\zeta_i+b_i$ for all i. Note that, a priori, these ζ_i are different for the different i and these z_i do not correspond to one another according to the rule (the result is not a specification of the rule).

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The consistent rules ρ for given functions u^i have to satisfy the n equations

$$V^i(\zeta^*) = \rho'_i(\zeta^*)/\Sigma\rho'_j(\zeta^*). \tag{7}$$

These n equations can in general determine (uniquely or not) n real number parameters. The two ways in which these parameters intervene naturally are found. In one case, these parameters constitute a particular set of contributions z, as in the theory of deviational rules (section 4.7). In the other case, by symmetry each of the n parameters is a parameter $\gamma_i \in \Re$ of one function $\rho_i(\zeta)$. Then, the consistent rules ρ are of the form $\rho_i = f(\gamma_i, \zeta)$ for a two-variable function f having, as function of ζ, the properties of $\rho_i(\zeta)$ and for the rest arbitrarily chosen in the relevant range, and the n functions (7) determine the γ_i.

Any Pareto-efficient solution can be obtained by consistent rules. If z is this state, such a rule satisfies, in particular, $\rho(\zeta^*)=z$ and the $\rho_i'(\zeta^*)$ are proportional to the $V^i(z)$.

4.3 Consistent linear rules

The simplest rules are linear,

$$\rho_i = a_i\zeta + b_i \qquad (3)$$

for all i with constant $a_i>0$ and b_i. Then, equations (7) become

$$v^i(y_i - a_i\zeta^* - b_i,\ \zeta^*\Sigma a_j + \Sigma b_j) = a_i/\Sigma a_j. \qquad (8)$$

This implies

$$a_i\zeta^* = v^i \Sigma a_j \zeta^* \qquad (9)$$

which means that the sharing of the amount $\zeta^*\Sigma a_j$ follows the Lindahl rule. The total amount $Z^* = \zeta^*\Sigma a_i + \Sigma b_i$ is divided into two parts, one divided into arbitrary fixed amounts b_i (but a priori $b_i \gtrless 0$), and the other allocated according to the Lindahl rule. However, there are two typical cases in which, respectively, the a_i or the b_i are given, and the n others are determined by the n equations (8). Denote $a=\{a_i\}$ and $b=\{b_i\}$ the vectors of the a_i and b_i respectively. These two cases are denoted by functions $a(b)$ and $b(a)$ in which, respectively, b or a are given and a or b is a function of it given by equations (8). A Lindahl solution is $a(0)$, with given $b=0$, hence a *proportional rule* $z=a\zeta$ in which the $z_i=a_i\zeta$ are in the same proportions a_i determined by these equations; then $a_i=v^i\Sigma a_j$ and $Z=\Sigma z_j=\zeta\Sigma a_j$ imply the classical $z_i= v^iZ$.

The choice of the given b or a depends on the specific problem.[15] Given a_i or b_i can be related to characteristics of the individuals i, or be equal. For instance, the given a_i may be some income of the individuals, y_i possibly augmented or diminished by some other payment,

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[15] The level of ζ^* can a priori be chosen arbitrarily, with the contravariant changes that, for solutions $a(b)$, the a_i vary inversely proportionally to ζ^*, and, for solutions $b(a)$, if ζ^* is augmented by any number h each b_i is diminished by ha_i.

and the Lindahl parts of the contributions are proportional to it. They may be equal and provide an equal Lindahl part of the payments. The given b_i may be equal (of any sign), which gives the "equally augmented Lindahl solutions". For n=2, any linear rule with unequal a_i is identical to a rule of this category.

One can also consider linear sharing rules

$$z_i = \rho_i = s_i(Z) = \alpha_i Z + \beta_i \tag{10}$$

with given α_i (a priori >0) and β_i, and, since $\Sigma z_i = Z$, $\Sigma\alpha_i = 1$ and $\Sigma\beta_i = 0$. For a consistent such rule all individuals i prefer $Z = Z^*$ such that

$$v^i(y_i - \alpha_i Z^* - \beta_i, Z^*) = \alpha_i, \tag{11}$$

and hence $\Sigma v_i = 1$. The financial transfers consist of two parts. First, there is a balanced redistribution of income in which each individual i yields or receives the amount $\beta_i \gtrless 0$ (yields $\beta_i > 0$ or receives $-\beta_i$ if $\beta_i < 0$) with $\Sigma\beta_i = 0$. Second, each individual i contributes to the financing of Z^* with $\alpha_i Z^* = v^i Z^*$ i.e. according to the Lindhal rule. Equations (11) can determine Z^* and, for instance, the α_i for given β_i or the β_i for given α_i.[16] Lindahl solutions correspond to given $\beta_i = 0$ for all i. A priori equal α_i give the β_i that permit equal Lindahl payments. Given α_i may be proportional to some characteristics of individuals i.

Consistent linear rules are the consistent rules with the smallest number of parameters ($2n$ in general) that permit reaching any Pareto-efficient solution with quasi-concave preferences. For such a state z, it suffices to choose a consistent linear rule with $\rho(\zeta^*) = z$ and a_i proportional to the $V^i(\zeta^*)$. In particular, these rules can be sharing rules (with $2n$-2 independent parameters and $\alpha_i = V^i(\zeta^*)$ for all i, which determines the β_i).

Since Lindhal solutions are often considered, it is possible to present consistent rules as a generalization of them. Lindahl solutions correspond to proportional consistent rules: $\rho_i = a_i\zeta$ or $\alpha_i Z$ for all i. There are in fact successive levels of generalization: consistent linear sharing rules with possibly non-zero initial redistribution β_i, a priori given or more generally;

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[16] The n equations (11) and either $\Sigma\alpha_i = 1$ or $\Sigma\beta_i = 0$ make n+1 a priori independent equations for determining Z^* and either the α_i or the β_i.

consistent linear rules with $a(b)$ which provide, in a sense, a Lindahl solution in addition to any given set of contributions b rather than from the particular b=0 only; consistent linear rules in general (notably with $a(b)$ or $b(a)$); consistent sharing rules or consistent rules in general. All these cases lead to Pareto-efficient unanimous choices. The drawback of Lindahl solutions is that they impose a particular distribution (or particular distributions). By contrast, as we have noted, any Pareto-efficient solution can be reached by consistent rules or sharing rules, and by linear such rules with quasi-concave utilities.

4.5 Particular structures

When the a priori given structure of the rule ρ has less than n independent parameters to be determined, the rule cannot be consistent in general. This includes, for instance, equal $z_i=\zeta$, equal final private income obtained with rule $z_i=y_i-\zeta$, contributions proportional to given incomes $z_i=y_i\zeta$ or to any other characteristics. This is a drawback since such rules are common.

However, similarities between the individuals open possibilities. If the n equations (7) or (8) are the same, they can determine the value ζ^* of a parameter ζ. This can happen, notably, in cases in which the functions u^i are ordinally the same. Then, denoting as u a common specification,

$$u^i=u(x_i, Z)=u(y_i-z_i, Z). \tag{12}$$

The only remaining difference between the individuals and between their respective equations is that of their income y_i. This difference may be eliminated in three ways, two of which with more specific given structures. Denote $v=v^i$ for all i.

1) The chosen contributions equalize the remaining incomes, $z_i=\rho_i=y_i-\zeta$ for all i, and

$$v(\zeta^*, Y-n\zeta^*)=1/n \tag{13}$$

where $Y=\Sigma y_i$ is total income.

2) If the y_i happen to be the same, $y_i=\eta$, equal $z_i=\zeta$ provide the solution, with

$$v(\eta-\zeta^*, n\zeta^*)=1/n \tag{14}$$

(Laffont, 1975).

3) If function u is quasilinear, $u=x_i+w(Z)$, $v=w'(Z)$ and an additive rule $z_i=b_i+\zeta$ gives

$$w'(\Sigma b_i+n\zeta^*)=1/n. \tag{15}$$

With a strictly quasi-concave increasing function u, equations (13), (14) and (15) have a unique solution ζ^* (equations (13) and (14) amount to maximizing this function under the linear constraint $nx_i+Z=Y$).

4.6 Applications and the public-good specific unanimity-efficiency implication

If the z_i are taxes or tariffs for financing the public good, presenting the rule in the form of the sharing function, $z_i=s_i(Z)$, shows that consistent rules are the payments that lead all individuals to prefer the same level of the public good. Given such a tax function or tariff, they unanimously choose this level by political action (e.g. votes). Consistent rules constitute the general form of the spirit of Lindahl pricing (which is the proportional consistent rule). They permit to reach all the Pareto-efficient solutions (whereas the Lindahl rule determines the distribution(s)). Linear rules are sufficient for this and constitute the set of rules that permit it with the smallest number of parameters. For lateral reciprocity (matching), Kantian conducts or Rawlsian stability, the rule may be a social moral norm, a part of the civic culture, or a convention. For a Rousseauan compact, consistent rules induce unanimity defining the general will and implying Pareto efficiency.

Consistent rules for contributing to a public good are the social mechanisms that associate Pareto-efficiency and unanimity about the choice of the good: with such a rule unanimity entails Pareto efficiency and, conversely, any Pareto-efficient outcome can result from unanimity under such rules. Consistent rules do this by endorsing the distributive question which makes individuals disagree about the choice of a Pareto-efficient solution, by the comparative fairness they imply.

Moreover, consistent rules are the mechanisms associating Pareto-efficiency and unanimity in this way that is *specific to the public-good structure*. Indeed, assume that each individual utility depends on the z_j not necessarily through their sum $Z=\Sigma z_j$ but possibly more generally, as $U^i(x_i, z)$ with $x_i=y_i-z_i$ and $z=\{z_j\}$. Denote $U^i_{x_i} = \partial U^i / \partial x_i$, $U^i_j = \partial U^i / \partial z_j$ and $V^i_j = U^i_j / U^i_{x_i}$. Let $\rho(\zeta)$ be a consistent rule with a unique unanimously preferred $\zeta=\zeta^*$. Then, with $z=\rho(\zeta)$, for each i and interior solutions,

$$\Sigma_j V^i_j \, \rho'_j(\zeta^*) = \rho'_i(\zeta^*). \qquad (16)$$

However, each z_j is a public good and the corresponding condition for Pareto efficiency is

$$\Sigma_i V^i_j = 1. \qquad (17)$$

Conditions (16) imply non-fortuitously conditions (17) only when, for each i, the V^i_j are the same for all j, $V^i_j = V^i$. Then, indeed, conditions (16) imply $\Sigma V^i = 1$, that is, conditions (17) for all j. This implies that, at least marginally, the U^i depend on the z_j by their sum $Z = \Sigma z_j$, that is, this externality has the structure of a public good.

4.7 Ruled deviations

A basic property is: *if a state z of individual contributions z_i to a public good is unanimously (weakly) preferred to all ruled deviations from it for a given deviation rule, it is Pareto efficient*. This holds whatever this given deviation rule is.

Indeed, let z and $\tilde{\rho}(z, \tilde{\zeta})$ denote this state and the given deviation rule. Each function $\rho_i(z, \tilde{\zeta})$ is smooth and increasing in $\tilde{\zeta}$ and denote $\tilde{\rho}'_i(z, \tilde{\zeta}) = \partial \tilde{\rho}_i(z, \tilde{\zeta}) / \partial \tilde{\zeta} > 0$. For each i,

$$u^i = u^i[y_i - z_i - \tilde{\rho}_i(z, \tilde{\zeta}), \Sigma z_j + \Sigma \tilde{\rho}_j(z, \tilde{\zeta})] \qquad (18)$$

is maximum at $\tilde{\zeta} = 0$ if, for an interior solution,

$$-u^i_1 \cdot \tilde{\rho}'_i(z, 0) + u^i_2 \cdot \Sigma \tilde{\rho}'_j(z, 0) = 0$$

or

$$v^i(x_i, Z) = \tilde{\rho}'_i(z, 0) / \Sigma \tilde{\rho}'_j(z, 0) \qquad (19)$$

for $x_i = y_i - z_i$ and $Z = \Sigma z_j$. Summing up for i gives

$$\Sigma v^i(x_i, Z) = 1, \qquad (20)$$

the condition for Pareto efficiency.

The concept of ruled deviation, and this result, applies to the fields of Kantianism, lateral reciprocity, general will and stability. It demands one to choose a state such that nobody prefers its variations according to some given rule. Then each individual i provides her z_i of this set z. The n equations (19) a priori determine the n z_i (uniquely or not).

The rule $\tilde{\rho}\,(z,\tilde{\zeta}\,)$ can be, for instance, proportional $\tilde{\rho}_i = a_i\tilde{\zeta}$ with n numbers a_i and $v^i = a_i/\Sigma a_j$. These a_i may be some income of individuals i, for instance y_i. They may be equal, corresponding to equal deviations ρ_i, with $v^i = 1/n$. They may also be z_i, corresponding to proportional deviations $\rho_i = \tilde{\zeta}\, z_i$, and to $z_i = v^i\Sigma z_j$ which is the Lindahl rule.

A deviation rule can a priori be any function of the parameter $\tilde{\zeta}$ with the noted properties. It need not depend on individual preferences. This is a major difference with the direct consistent rules.

The converse of the above property also holds: any Pareto-efficient set of individual contributions to a public good is consistent with some deviation rules. With quasi-concave utility functions, for contributions z_i and interior solutions it suffices to take the proportional deviation rule $\tilde{\rho}_i(z,\tilde{\zeta}) = \tilde{\zeta}\cdot v^i(y_i - z_i, \Sigma z_j)$ for all i with $\tilde{\zeta} \in \Re$. Indeed, $\tilde{\rho}'_i(z,0) = v^i(x_i, Z)$ and the condition for Pareto efficiency (20) implies the condition (19) for z to maximize u^i under the rule. Therefore, *a set of individual contributions is Pareto efficient if and only if it is consistent with respect to a deviation rule.*

This also shows that, with quasi-concave functions u^i, all Pareto-efficient sets of contributions are consistent with respect to deviation rules of the simplest form, with only one parameter: the proportional deviation rules $\tilde{\rho}_i = a_i\tilde{\zeta}$ for all i, by taking $a_i = v^i(x_i - z_i, \Sigma z_i)$ for the set of contributions z.

A deviation rule can also be presented as a sharing rule since $\Sigma\tilde{\rho}_i(z,\tilde{\zeta}) = Z' - Z = \Delta Z$ where $Z' = \Sigma z'_i$ and $Z = \Sigma z_i$, and hence, since the $\tilde{\rho}_i$ are increasing functions of $\tilde{\zeta}$, $\tilde{\zeta} = \varphi(\Delta Z, z)$ and $\tilde{\rho}_i(z,\tilde{\zeta}) = \tilde{\rho}_i[z, \varphi(\Delta Z, z)] = \tilde{s}_i(z, \Delta Z)$ which, since $\Sigma\tilde{s}_i = \Delta Z$, denote individual i's share of each variation ΔZ of the public good from state z. Functions s_i also write $\tilde{s}_i = \tilde{\sigma}_i(z, \Delta Z/n)$ which gives individual deviations from the mean deviation $\tilde{\delta}_i(z, \Delta Z/n) = \sigma_i - (\Delta Z/n)$.

5. Other issues: teams, number and “warm-glow”

5.1 Moral teams

The various philosophies, conducts and policies in question can also be attached to the very different method of maximizing a classical preference-respecting social welfare function, the increasing function $U(\{u^i\})$. Individuals i may choose or be imposed their respective variables z_i such that $z=\{z_i\}$ maximizes U. For a policy this is standard welfare economics, but such a choice may also constitute a rule of moral conduct. When individuals choose their z_i freely with this objective, they constitute a moral team. They may do it given that the others have the same objective in lateral reciprocity or matching. A Rousseauan social compact may define in such a way the "general will" that citizens implement by this maximization. The highest U may also describe moral/social preferences of Kantian agents, indicating what they "could want" as moral duty-bound entities (rather than utility maximizers following their "inclinations" or tastes), and they act assuming that all the others behave similarly with the same objective.

When the z_i chosen by individuals i are independent variables (as their contributions to a public good are), the solution $z^*=\{z_i^*\}$ that maximizes U is a Cournot-Nash equilibrium of these individual choices of the z_i. This can be used for reaching this optimum by a processus without explicit cooperation.

The outcome is both Pareto efficient (in the u^i) and unanimously wanted in the appropriate cases. The problem is, of course, the choice of function $U(\{u^i\})$.[17] Normatively, this choice amounts to evaluating the distribution between individuals by comparing their welfares (pleasures, happiness). Not all ethically relevant choices can be made in this manner (direct comparisons of allocations or issues of freedom may be relevant – even if Pareto efficiency is required, for choosing between Pareto-efficient states). By contrast, the choice of comparative rules of fairness can be more general, flexible and direct for the consideration of this ethical question. However, the choice of a direct consistent rule $\rho(\zeta)$ requires the consideration of individuals' utilities (in particular, the $\rho_i'(\zeta^*)$ satisfy conditions (7)). Yet this is not the case of the choice of deviation rules $\tilde{\rho}(z, \tilde{\zeta})$.

[17] Such a commonly agreed upon social welfare function can be a moral-social norm, it can result from an agreement possibly implicit (a social contract), from a process of a priori mutual influence in a social dialogue about the public good and justice – as previously noted – or from a political process.

5.2 Moral contribution to public goods or collective action and number

When what moral behaviour should be is determined – for instance by some Kantian-like reasoning, an implicit agreement (i.e. a social contract), reciprocity or moral matching, or a simple social or moral norm –, realizing it is a virtue, perhaps a duty, it may arouse moral satisfaction, perhaps pride or praise, and behaving differently may elicit guilt, shame or remorse, and blame, contempt or scorn from others or towards oneself. When this behaviour is not forced and remains free, realizing it depends on the agent's evaluation of these sentiments and judgments, by comparison with other possible relevant effects of these acts, notably various possible other advantages or disadvantages for the actor or for other people when she cares about them. An important determinant of this choice concerns whether the moral judgment due to the nature of the act in itself depends on these other advantages or disadvantages or not, and how it depends on them when this is the case. One of the reasons of this importance is that, when the number of beneficiaries from and contributors to a public good is large, the common case turns out to be that the advantage (and cost to others) of an individual's free riding vanishes. Then, if there is some moral "cost" in free riding by itself, a low degree of morality suffices to check it if the number of people is sufficiently large. If the moral cost vanishes when the advantage of free riding (and its cost to other people) vanishes, then whether it checks free riding or not depends on the speed of this variation. The conclusion is that, as a result of this moral motive, there is a large scope of cases in which the large number of participants is favourable to non-free-riding (contrary to other classical effects).

Whether or not there can be moral badness independently of advantages or disadvantages of the action is an important moral issue. Kant tends to think that the two should be independent. They are not commensurable because they do not belong to the same realm: one has a price and the other has a dignity. To lie is bad in all cases. The issue is duty rather than (other) moral satisfaction. For other views, guilt depends on the effects. This may be jesuistic casuistry or simple forgivingness for faults with unimportant consequences.

The effect of number on free riding is easily seen by considering identical individuals. The conclusions are valid for almost all individuals in the other cases. Hence, consider n identical agents producing cooperatively an amount Z of a public good produced at constant price taken as 1, by providing an impartial equal contribution $c=Z/n\geq 0$ each. Number n is large ($n \to \infty$) and is taken as a continuous variable. The optimum and efficient levels are $c(n)$ and $Z(n)$. For monotonic $c(n)$, since c is bounded (would it only be by individuals' incomes), $c(\infty)<\infty$ and, therefore, $c'(\infty)=0$. Denote $c(\infty)=a\geq 0$. If $a>0$, $Z=an$ for large n. Assume that the individuals have a quasi-linear utility function $u=v(Z)-c$ with an increasing strictly concave and twice differenciable function v. For given n, the optimum, Pareto-efficient Z and c satisfy $v'=1/n$. If $Z=an$ with $a>0$, integrating this v' gives $v=a\text{Log } Z+\gamma$ with a constant γ. This is a two-parameter family of functions v, among all functions v satisfying the assumed general properties. Therefore, except if there is a particular reason for functions v to have a logarithmic form, a priori almost always $a=c(\infty)=0$. This implies $c'<0$ for large n.

If such an individual decides to free ride, she gains her contribution c and is affected by possible effects of her action on the amount of the public good Z. These effects can a priori be of several possible types. This "small" agent certainly does not consider that her withdrawal from cooperation makes the other people cease to cooperate between them, or retaliate by producing little of the good in order to punish her (against their interest in a one-shot situation). Denote as φ the gain from free riding.[18] If the amount Z does not change, $\varphi=c$. This implies that the others augment their contribution. If the others leave their contributions unchanged, Z becomes $Z-c$, and

$$\varphi=v(Z-c)-v(Z)+c=c-c\,v'=c\cdot[1-(1/n)]\simeq c.$$

In a third case the others reorganize their cooperation between them and produce their optimum $Z(n-1)$. Then,

$$\varphi=v[Z(n-1)]-v[Z(n)]+c=c-v'Z'=c-(c/n)-c'\simeq c-c'$$

for large n, since $v'=1/n$ and $Z'=c+nc'$. Since in all cases $c'(\infty)=0$ because $c(\infty)<\infty$, in all cases $\varphi(\infty)=c(\infty)=a$, that is 0 in the general case $a=0$ and $a>0$ in the borderline case of functions v logarithmic for large n.

Therefore, if free riding has a moral cost in itself (shirking an implicit agreement, non-reciprocating, violating a Kant-like reason, not obeying a norm, etc.) then in the general case in which $\varphi(\infty)=0$ this moral cost suffices to check free riding for sufficiently large n, whatever its level and in particular no matter how small it is. If the moral cost is the higher the higher

[18] The free rider may produce some of the good in a non-cooperative interaction with the others, but for a "small" agent the consequence is that she does not produce any amount (from the theory of the core for non-excludable public goods – Kolm 1987).

the benefit, say $\mu(\varphi)$ with $\mu'>0$, then the same result holds if $\mu(0)>0$ or, if $\mu(0)=0$ if $\mu'>1$for sufficiently small φ.[19, 20]

5.3 "Warm glows"

Another possible explanation of individual contributions to non-excludable public goods can be direct preferences for one's contribution for reasons other than the appreciation of the amount of the good it creates, for instance in order to be praised or to feel praiseworthy (the latter is Andreoni's "warm glow"). The virtue may simply be to "be a good co-operator". However, what may seem particularly virtuous is to contribute to "moral public goods", contributions to which have an intrinsic moral value. The paragon of these goods is helping the poor – a public good for altruists –, but culture, the environment, research, or the defence of values can provide other cases. However, such motives, in themselves, do not solve the problem of the lack of cooperation and, hence, do not a priori induce Pareto efficiency. Yet, if this efficiency is provided by taxation and public provision, with standard individual concerns for private and public goods only, this should crowd out almost all non-cooperative free private contributions. Then, concern about contributions in themselves may explain why such contributions often remain. However, this explanation meets a number of problems.[21] The main one is that if one $z_i>0$ is explained in this way, then with a large number of beneficiaries

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[19] Exception to this conclusion has to find some reason for the logarithmic structure of utility. Could one find it in some kind of Weber-Fechner law ("sensation varies as the logarithm of excitation")? Moreover, taxes in large national societies hardly vanish. Yet they cover a fair number of public goods jointly. In addition, a part of them aims at redistribution which is a public good solely insofar as it is the implementation of people's altruism (compassion) or sense of justice, which is not all the explanation (there is also politically induced redistribution). Some moral motives for paying one's taxes exist (they seem to be quite different according to national cultures). Moreover, the situation may be different if the moral sentiment attaches to contributions to each public good separately. This is a reason for favouring a presentation to the taxpayer of the various uses of the budget and of the contribution of her taxes to each of it.

[20] $nv'=1$ and $v''<0$ imply $Z'>0$ or $-c'/c<1/n$ or $E^c=nc'/c>-1$. Also $c=Zv'(Z)$. Differentiating $nv'=1$ gives $(E^c+1)E^{v'}+1=0$ where $E^{v'}=Zv''/v'$. If $\alpha=\lim_{n\to\infty} E^c$ and $\beta=\lim_{n\to\infty} E^{v'}$, then $1+(1+\alpha)\beta=0$ with $\alpha\geq-1$ and $\beta\leq0$. For large n, functions $v(Z)$ and $c(n)$ behave, respectively, as

$$v=bZ^{1+\beta}+\delta \text{ with } -1<\beta<0$$

or

$$v=a\text{Log } Z+\gamma \text{ if } \beta=-1,$$

and

$$c=an^{\alpha}$$

with constant $b>0$, $a>0$, δ and γ. With function v of logarithmic form, $\beta=-1$, $\alpha=0$, $c=a$. In all the other cases $-1<\beta<0$, $\alpha<0$ (excluding $c(\infty)=\infty$), $c(\infty)=0$, $c'/c=\alpha/n$, hence $\varphi\simeq c$ with all forms of free riding.

[21] See Kolm (2008a, 2008b).

almost all of them should be satiated with the public good (for instance, almost all non-poor think that the poor have enough). One may also note that one cannot be praiseworthy or praised as a moral person if seeking this judgment is the motive for the contribution (since this is not a moral motive). A lower type of appreciation of the person (by herself or others) may be attached to the fact that some of her wealth contributes to the production of the public good. However, this sacrifice includes both the free contribution and the distributive tax, and it turns out that the individual's preference about it cannot explain a positive free contribution. Nevertheless, an individual's free contribution can paradoxically be explained by the fact that other people prefer this contribution plus the tax this person pays to be lower (for reasons of comparative sentiments such as envy, sentiments of inferiority or superiority, equality, conforming, etc.). In the end, the difficulties of "warm-glow" explanations lead to the consideration of the moral motives analyzed here.

6. Conclusion

Voluntary contributions to collective action and non-excludable public goods provide a main challenge to the explanation and organization of societies. Explanations by sequential actions and warm-glow alone meet important difficulties and limitations.[22] Correlated action without enforced agreements rest on moral/social sentiments and reasons that are present and the theory of which has often been elaborated by classical moral philosophers. This includes, notably, folk- and full Kantian ethics, implicit or putative agreements hypostasiatied into social contracts among which Rousseau's is both an explicit public good issue and probably the most elaborate on moral grounds, as well as lateral reciprocity or moral matching, civic virtue and Rawls's "stability". The core concept that comes out is a theory of rules that divides the problem into two stages: the choice of a rule that fully encompasses the issue of fairness, and its application in the unanimous choice of a specific outcome. The main type of such rules is rules of fairness that relate individuals' contributions with one another or with the output. There are two types of these rules: direct consistent rules in which everybody prefers the same application, and the unanimous opposition to ruled deviation. For both, the outcome is Pareto efficient, and all Pareto-efficient states can be obtained in this way. These

[22] Purely self-interested sequential provision is powerless beyond a small number of participants. Providing less as retaliation to punish some other who contributes too little (a free rider) also punishes all other beneficiaries and is not even felt for a small agent in a large number. The very logic of sequential punishing is problematic. Standard analysis (such as those of "folk theorems") rest on some kind of implicit agreement to begin with, and this has to be explained.

properties characterize the public good structure of the interaction. The direct consistency principle limits the conceptions of fairness it can implement, whereas all such conceptions are possible for the non-ruled-deviation principle which, however, concerns the fairness of variations only. The rule may result from a moral reflexion about which principle of fairness is relevant, from social dialogue and deliberation, from any type of social contract or from a political process, or it may be a convention, a norm, an element of a shared political culture, a tradition or a habit. In another type of solution, the individuals unanimously choose a Pareto-efficient outcome by being converted to maximizing the same social welfare function as a moral team. However, this restricts the rules to comparisons of welfare and to this form which is unusual in actual life. Finally, non-free-riding for a moral reason requires in general degrees of morality that are lower the larger the number of numerous participants; hence this large number is often favourable to voluntary contribution.

Some of the sequels to this study are clear. The question of information cannot be studied before one has determined what agents need be informed about. Its method consists in considering explicitly agents' uncertainty about the various items, including preferences.[23] The basic issue of moral motives is a field in which there is probably more to be found. The basis of this investigation is the analysis of people's explanations and discourses about their moral judgments and the reasons for their own actions.[24] This requires psychological enquiries and philosophical considerations to determine the situations, relations, scope, logic and structure of these motives. Empirical investigations and experiments can then bring important confirmations and precisions. The incorporation of the outcome into models of conducts shows the various properties, explains behaviour and permits particular applications. These applications include public policy about rules, taxes, and, importantly, moral education and the promotion of civic behaviour.[25]

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[23] As with the determination of optimum tariffs, taxes or rules in Kolm (1970a, 1970b).

[24] See Brekke, Kverndokk and Nyborg (2003).

[25] Remember that Rousseau considered his work on political philosophy, *The Social Contract*, as an appendix to his work on moral education, *Emile*, published simultaneously.

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