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# Social Ethics and Rationality, new directions for the Optimum Production of Social Justice: Meaningful welfare, equal liberties, social solidarity

***Serge-Christophe Kolm***[1]

**Content**



[1] School for Advanced Studies in Social Science, Paris.

**Summary**

After brief remarks about the history of rational inequality measurement and the basic conceptual challenges it faces, this paper continues with three parts. In part I, in order to eliminate a basic logical flaw the relevant individual welfare is defined. This leads to the distinction of the social ethical end-values of the various theories, and in the end to the specific structure of income distribution that also achieves equal basic and real freedoms. This structure and its properties are presented in part II (including the actual tax system it inspired which proves its possibilities). Part III analyzes the possibilities and properties of solidaristic inequality reduction, notably by the formal analysis of Rawls's social stability theory of justice and of Kant's categorical imperative corrected for its basic flaw.

## F**oreword: a few remarks on the study of unjust inequalities**

### **1. History (recent): 3 books (5 volumes): 1966, 1968, 1969. Simultaneous writing.**

A – In **1966**, two things relevant for our purpose occurred.

1) Publication of the book **Monetary and Financial Choices (Modern Theory and Techniques)**, Paris: Dunod, 1966 (in French).

Much of it is on choice in uncertainty. It includes in particular:

- What was later called *Second Order Stochastic Dominance* (and Third Order and variants too), with all the various *equivalent properties*.
- The "linear uniform *Concentration* to and *Expansions* from the mean", i.e. what Rothschild and Stiglitz later called "decreasing risk" and "increasing risk".
- Various uses of the *certainty equivalent*.
- *Measures* of risk, notably all those using the certainty equivalent.

And so on.[2]

2) The 1966 **Biarritz International Economic Association Conference on Public Economics**, with the paper **The Optimal Production of Social Justice** (jointly in English and French).

[2] A number of concepts and results presented in this volume and interesting for income (or other) distribution have not yet been applied to this topic.

The book and the paper were **written simultaneously** (1964, 1965; at Harvard and MIT; lectures and papers).[3]

B – 1966: **Collected papers and Proceedings of the Conference**,by H. Guitton and J. Margolis. They were later published in *book form*:
1968 in French: Economie Publique, Paris, CNRS.
1969 in English: Public Economics, London, Macmillan.

**2. Remarks on formal landmarks**

On formal grounds, a few facts can be noted from the beginning of these studies, *in the Biarritz paper*. The first ones concern inequality in the distribution of a quantity such as income.

(1) The comparisons between the **equal equivalent income** $\bar{\bar{y}}$ and the mean $\bar{y}$ are crucial.

$$W(\{y_i\}) = W(e\bar{\bar{y}}), \quad e\text{: } n\text{-vector of ones.}$$

In particular, case of W additive, symmetrical: **all the general means** are considered (including $f(y)=y^{\alpha}$, Log, $e^{\beta y}$).

*All the 6 comparisons between the equal equivalent* $\bar{\bar{y}}$ *and the mean* $\bar{y}$ *using ratios and/or differences have a specific important meaning*: relative $(\bar{y}-\bar{\bar{y}})/\bar{y} = 1-(\bar{\bar{y}}/\bar{y})$), absolute $\bar{y}-\bar{\bar{y}}$, total $n\cdot(\bar{y}-\bar{\bar{y}})$, yield $\bar{\bar{y}}/\bar{y}$, unit cost $\bar{y}/\bar{\bar{y}}$, excess unit cost $(\bar{y}/\bar{\bar{y}})-1=(\bar{y}-\bar{\bar{y}})/\bar{\bar{y}}$ (they also measure the social waste of inequality).

Further concepts for non-symmetrical *W* are presented.

(2) The basic comparisons are with **different total amounts and means**. The case of same total income or mean is a particular case (the "constant-sum case"). The sum of the *m* lowest as a function of *m* or of *m/n* was called the **concentration curve** in statistics textbooks. The corresponding **concentration-curve dominance** is considered. Lorenz-curve dominance is too.

[3] Modigliani seminar, Solow, etc. Stanford (Arrow). The reception at the Biarritz conference by the various participants was also interesting (Erik Lundberg said it is "mathematical theology", Sen, Musgrave, Malinvaud, Samuelson, Dorfman, Chenery, etc.).

(3) Both measures that are **intensive** (i.e. invariant to scale)[4] and **equal-invariant** (i.e. invariant under equal additions) are considered, with the corresponding specific measures (**including the intensive**).

(4) For each measure $I$, its **absolute form** $I^a$ and its **relative form** $I^r = I^a / \bar{y}$ (sometimes to the equal equivalent $I^a / \bar{\bar{y}}$) are *a priori always* **considered jointly**.

• Hence the interest of *synthetic measures* which have an intensive relative form and an equal-invariant absolute form.

• Population effects.

And in other works:

-*Multidimensional* inequalities.[5]

-The principle of *diminishing transfers* (third order stochastic dominance and variants).[6]

-*Intermediate measures* with "augmented incomes" which have to have the form presented because of the joint relevance of the relative and the absolute forms of the measure.[7]

And later:

-Application of the multidimensional case to the least unequal Pareto-efficient allocation and consequences. Relation with the theory of equity-non-envy.

-Inequalities in *liberty*, with several concepts.

-Applications to optimum distribution.

**3. Philosophy: Economics meets social ethics**

Equality is a mathematical concept that induced the guillotine (notwithstanding liberty and fraternity). When inequality means injustice rather than difference or dispersion only, the indignation it can arouse can go so far as to induce revolutions and lead people to choose to

[4] Intensive is the sciences' term for homogeneity of degree zero for a proper reason – i.e. not for unit invariance, which is dealt with by contravariant transformations of the functions.
[5] 1973 (1975, 1977).
[6] 1972 (1976b).
[7] The intensive and equal-invariant measures have to be a relative and an absolute form, respectively. If the equal-invariant measure were a relative form, the corresponding absolute inequality would increase under an equal addition to all incomes. This was bypassed by Bossert and Pfingsten's erroneous objection to the intermediate measures. Their alternative proposal also presents the contradiction that it cannot be derived from an additive social evaluation whereas this is the main justification of the transfer principle which they think is indispensable (since it amounts to "irrelevance of constant incomes") – see chapter 1 in the *Handbook of Income Inequality Measurement*. Similarly, if the intensive measure were an absolute form, the corresponding relative inequality would decrease under an equiproportional increase of all incomes.

kill and be killed. It is the vice that violates the "first virtue of society" as Aristotle and Rawls call justice (which Aristotle says "is equality as everybody knows"). The corresponding inequalities are, therefore, an essential feature of societies. The study of their socially relevant properties is a field of social ethics and normative economics. Some time ago, economics started to apply the standards of social ethics to this topic, while keeping its own characteristics of a largely mathematized social science and of its specific field. These standards include:

***1) External consistency*** by relating measures of inequality to the overall evaluation of society (hence the crucial role of "equal equivalents").

***2) Complete evaluation*** which requires appraising a property from all its angles and implications – e.g. Plato's "dialectics" or Rawls's "reflective equilibrium" –, as splendidly exemplified by the twenty or so properties different in intuitive meaning but mathematically equivalent that include the transfer principles, concentration-curve dominances and the Schur-Ostrovski-Birkhoff-von Neumann structures (rectifiance, isophily, averages and mixtures preferences, etc.).[8]

***3) Associating rationality with moral opinion, logical "intuition" and semantic understanding*** in an integrated analysis, for instance to evaluate the effect of some transformations of a distribution on inequality, rather than forgetting some relevant modes of knowledge, or selecting some property almost by chance, without sufficient consideration of all effects on meanings.

***4) Tests of internal consistency including for meaning***, such as checking the transfer principle against the different egalitarian property of clusters or the possible meanings of the highly problematic concept of "welfare".

## 4. Suggestions for research

### *1) Rational foundations*

Unfortunately, this set of major advances may be invalid. It is based on a central contradiction, an inconsistency between the relation with the overall social evaluation on the one hand and the set of basic equivalent properties on the other hand. Utilitarianism writes $\Sigma u_i(x_i)$, not $\Sigma u(x_i)$ ($x_i$=$y_i$, or $(y_i, \ell_i)$ ($\ell_i$=labour), etc.). What does this *u* without an *i*

[8] Some of the relevant formal relations and properties were known to mathematicians but many were not; some mathematicians were also influenced by discussions with economists about distributions and their transformations (Claude Berge, Paul Lévy, André Lichnerowicz, Benoît Mandelbrot, etc.).

mean? (Not even a "fundamental utility" $u_i(x_i) = u(a_i, x_i)$ – where $a_i$ is an individual parameter of any nature – occasionally meaningful).

The first research needed may be to secure foundations, so as to build on rational grounds and to think with actually meaningful concepts.

The present paper proposes to solve this specific problem to begin with (section I). However, when one pulls this thread, what comes out is a full questioning of the concept of optimum distribution and taxation, with a solution that amounts to equality in liberty with different domains of choice (section II).

This has, in particular, consequences for inequality measurement. It suggests focusing directly also on *redistribution* from the market outcome rather than on the resulting incomes only. In particular, this shows the meaningfulness, as a measure, of the *equivalent equalization duration*, that is, the fraction of time such that, if there were full income equalization during this time, the decrease in a measure of inequality would be the same as in the actual redistribution. This is the degree of an equivalent *concentration* of total incomes. It is equal to the relative decrease of any synthetic inequality index. This measure is, for instance, from 1 to 2 days a week in national redistributions (from the USA to Scandinavian social-democrat national communities).

This takes account of all the ethical conceptions deemed relevant for overall distribution (both egalitarianism possibly derived from the proper welfarism and classical liberalism). For this reason, important policy applications came fast,[9] whereas other scholarly proposals based on ethical judgments not accepted by the public have not yet seen the beginning of an application after nearly forty years of existence.

***2) Policy, politics, social justice***

In the presence of unjust inequalities, indeed, "the objective is not to study the world but to transform it". That is, when one shows unjust inequalities, one probably has the duty to provide, with this study, policy advice showing how they can actually be reduced. Hence the noted emphasis on redistribution policy.

Providing such advice that is efficient is not easy because it implies understanding the conception and implementation of social justice in the society in question. Basic issues may be enlightened by models such as those proposed in section III of this paper, which show the properties and possibilities, in this respect, of theories such as Rawls's stability conception of

[9] See section II-2.

justice and a Kantian categorical imperative corrected for its basic mistake (that is, different "Kantian" agents want different "maxims" to be universalized as soon as the problem is sufficiently specific).

***3) Why equality?***

"Inequality is higher, has decreased, etc." is common language. However, given any two distributions, it is probably almost always possible to prove that any one is more unequal than the other, with measures or rankings that seem to have sound, normal and rational properties only. It is easy to see which structural parameters should be emphasized to obtain this result.[10] To compare or measure inequality as for anything else, we can do something well only by reference to why we do it. If we compare or measure some inequality or dispersion because of its influence on something else (growth, social unrest, etc.), then the theory of this effect gives the proper criterion of comparison or measurement. But what about the ethical judgment about inequality? If we are concerned about inequality because of its effects on poverty, relative deprivation, envy, jealousy, sentiments of inferiority or superiority, exclusion, isolation, elitism, polarization, clustering, spreading, stratification, hierarchization, and so on, we have elaborate measures of all these concepts and an efficient study considers them directly. Material inequality then may simply be an indication that some other distribution might solve the problem if issues of incentives and political and social forces permit it.

We may also observe that people have opinions of logico-ethical nature about inequality and start from this, say in an extended Amiel-Cowell approach. There can be both ethical values and structural views about the formal conception of inequality. This has both a moral and a factual dimension. Factually, people's opinion may influence or impose policy when it is that of leaders or of voters. Morally, abiding by public opinion may be commended. However, people's social ethical views are not like consumers' preferences and tastes that may have to be respected. They imply external effects by nature. They are supposed to constitute objective judgments, and nevertheless they are importantly formed by particular influences and experiences which may a priori have no moral value. Their judgments about inequality may be biased by various effects of perception which do not necessarily have

[10] For instance: emphasize particular segments of the distribution; choose an intensive, an equal-invariant, or some intermediate measure; consider various possible emphases on clusters with various returns of distance; take the absolute or the relative form of measures; consider the many possibilities of choosing the variables (which income, relative to what, etc.); appropriately divide into subgroups and then aggregate; and so on. The comparisons immune to such arrangements would only be very particular artificial constructs.

ethical significance such as salience or framing effects of various types (an analog to Kahneman-Tversky's prospect theory for uncertainty is revealing). Moreover, more or less equal or unequal is only a part (sometimes a small part) of people's social ethics (issues of earned income and labour, of needs of various types, of the nature of the community the distribution within which is considered, etc.). Kant, for one, ridicules folk ethics and wants the requirements of reason alone. But his view is a program, not an achievement.

There is, however, an intrinsic and direct reason for equality, which simply is that it is a requirement of reason – hence not even a moral reason per se. This is that equality is a requirement of rationality taken in its most common sense of accepting to give a reason, to justify. A complete reason for treating a person in a particular way is a function of a set of characteristics of this person that is deemed relevant. If another person has an identical set of these characteristics, this function indicates the same treatment. Treating them unequally is unjustified, arbitrary, irrational in this sense. This is, of course, *prima facie*, in the absence of an overpowering reason such as impossibility or the interference of some other value deemed relevant. Then, what is wrong with inequality is that it is arbitrary, unjustified.[11] This suggests that the proper criterion for ranking or measuring inequalities is their arbitrariness. This raises issues of both logic and ethics concerning the moral relevance of causes (for instance, section II below concerns the degree of relevance or irrelevance of given capacities for disposable income, as conceived in actual, socially implementable ethical views, depending on the community in question).

People are indeed sometimes upset by treatments because they are arbitrary, unjustified (at least not justified by a good and sufficient reason). This applies to the resulting inequalities. But is this property, in itself, always objectively sufficiently repulsive to attach so large an importance to inequality? It depends on inequality of what. In particular, a certain type of inequality has another intrinsic vice, of a different nature. It is common to suggest that equality and liberty oppose one another. However, when inequality refers to power in social relations, such as in domination and subjection, then equality and liberty are synonyms. Jean-Jacques Rousseau (1755) concludes an elaborate investigation with "Inequality is the source of all evils". Yet he later made precise his objection to income inequality (epitomizing his undignified experience as a servant): no one should have neither so little that she has to rent herself nor so much that she can hire someone. The issue, then, concerns social relations.

[11] There is more to this reasoning. See for instance "Equality", *Encyclopedia of Political Science*, Sage, forthcoming.

Arbitrariness is probably also more offensive when comparing the situations it creates than by itself.

It seems, therefore, that the proper evaluation of inequality has to see it in the broader framework of society, including statuses, the nature of relations, and the type of society notably with regard to the relative value attached to individual and collective responsibility. One can certainly see, as a very long-term feature of mankind, a progress towards the mutual acknowledgment of an equal intrinsic value of each human being (an "ontological equality").[12] The result, a society of equals, is not an egalitarian society. But it certainly implies limits on inequalities. Economists have sometimes been good at making important general ideas precise and operational: can we continue?

## I. DEFINING WELFARE

### I-1. What does that mean, "welfare"?

I have to clarify the meaning of a basic property of the theory of unjust inequalities I presented at the International Economic Association conference on Public Economics in Biarritz in 1966.[13] The issue of this paper that has attracted attention the most is a number of equivalent properties for ranking income distributions according to their inequality. Some of these formal relations were known to mathematicians, many others were not. At any rate, we are social scientists, and therefore we are primarily interested in the social meanings of these properties, including ethical meanings. This clarification turns out to have consequences that extend far beyond the simple issue of inequality comparison or measurement: it justifies a common but as yet ill-founded practice in the theory of optimum distribution and taxation, and also leads one to complement this theory for values demanded by society, such as equal liberty.

If $y_i$ denotes person $i$'s income, one of these properties compares $\Sigma f(y_i)$ for all concave functions $f$. What does this means? Pigou in 1912 and Bentham long before presented remarks that clearly amount to saying that maximizing $\Sigma u(y_i)$ with a concave function $u$ favours equalizing the $y_i$. They think this is utilitarianism, but it cannot be because individuals have different utility functions $u_i$. Utilitarianism maximizes $\Sigma u_i(y_i)$. What is the $u$ without an $i$? We

[12] When the caste system was cristallizing in India, the Buddha admitted in his *Sangha* (community) a *chandala* woman, a bastard of a foreigner and of an out-caste, the worst status of all. This conception was later transmitted to stoicism and then, by Saint Paul, to Christianity ("there is no longer neither slave nor free man, woman nor man, Jew nor gentile") and to the modern Western world (see 1982).
[13] Proceedings edited by H. Guitton and J. Margolis (1966, 1968, 1969).

are not helped by the fact that *any* concave function $u$ yields this property, since the $u_i$ are a priori different. Another of these equivalent properties assumes that it is better to give an extra dollar to a poorer person than to a richer one, and that a "progressive transfer" from a richer person to a poorer one of less than the difference between their incomes diminishes unjust inequality. One of the possible intuitions for this view is that this transfer augments the poorer's "welfare" more than it diminishes the richer's welfare. This amounts exactly to the previous conception in which the concave function $u$ would measure this "welfare". But what if the receiver is a sedate person fully satisfied with her average income and the other is a greedy or sybaritic character who relishes any extra dollar or knows how to make the best of it? Economists classically represent the "welfare" of individuals by their utility functions, which are the different $u_i$. These remarks can be repeated with an aggregate "social welfare function" (SWF) more general than a sum, an increasing symmetrical quasi-concave function, or Schur-concave function, $W(\{u_i\})$, with maximin as a borderline case. Stating that favouring the highest $\Sigma u(y_i)$ is the view of an "ethical observer" does not help much in itself: why would this person hold this view?

Uncertainty may be called upon in order to try to solve the problem. This may be done in two dual ways: the $u_i$ for each income $y_i$ may be uncertain, or the income of each individual may a priori be uncertain.

A possible answer is that we take function $u(y_i)$ because we do not know the actual utility functions $u_i(y_i)$.[14] In order to obtain $\Sigma u(y_i)$ with an increasing concave function $u$, one has to start from a utilitarian $\Sigma u_i(y_i)$ with increasing concave functions $u_i$ (these $u_i$ are those relevant for comparing differences in happiness, a comparison which is in itself problematic, but this is the fate of utilitarianism).[15] Assuming a probabilizable uncertainty, and a "rational" risk evaluation, the corresponding social maximand would be E $F[\Sigma \tilde{u}_i(y_i)]$ where $\tilde{u}_i$ is a stochastic function, $F[\Sigma u_i(y_i)]$ is a specification of the risk-relevant, von Neumann-Morgenstern, cardinal specification of the maximand, and E denotes the mathematical expectation. The only general way for this to be ordinally equivalent to a form $\Sigma u(y_i)$ requires two assumptions, each of which is not a priori and in general satisfied. (1) Function F would be affine (and increasing), that is $\Sigma u_i(y_i)$ would be a specification of the risk-relevant cardinal family. (2) Functions $u_i$ would be independent identically distributed and then one would take

[14] In chapter 1 of the *Handbook of Economic Inequality* (edited by Jacques Silber), this derivation is worked out in the most meaningful possible way.
[15] See *Modern Theories of Justice*, pages 360-366.

$u(y)=\mathrm{E}\,\tilde{u}_i(y)$. However, in any application in which more is known about utility functions (even if they remain uncertain) this would have to be taken into account.[16]

Another proposal would be that preferring a higher $\Sigma u(y_i)$ for all concave functions $u$ expresses this preference for any risk-averse individual who considers that he could have any of the actual incomes with the same probability. Such an individual would indeed have a concave risk-relevant VNM utility function $u$ and this would be her "rational" preference in this uncertainty. However, a social ethical judgment about just distribution has no reason to have the same form as an individual selfish preference in uncertainty. These are different questions. The social ethical judgment is accountable towards society, other people and moral. Moreover, a number of individuals are actually not risk-averse in some range of incomes.

In fact, the actual reasoning assumes more simply that there exists something called individual welfare, representable by an increasing concave function of the individual's income, which is the same function for everybody, and such that the quality of the income distribution can be appraised by the sum of the values of this function for all individuals. However, we do not actually know this function, and hence we are interested in properties that hold for all such functions. What is, however, the relation between this welfare function and the individuals' different utility functions? What is the actual meaning of this function?

Another possible answer is that function $f$ may not refer to individual properties at all. Maximizing $\Sigma f(y_i)$ with an increasing strictly concave function $f$ may just describe attaching some value to equality in the $y_i$ since this is the outcome of this maximization for sharing a given $\Sigma y_i$. This would be so because the $y_i$ are taken as end-values, and an ideal of equal end-values relating to individuals with no other different relevant characteristics is a condition of minimal rationality.[17] The $y_i$ being end-values means that, for this distributive justice, the individuals are deemed accountable for all the psychological and physiological characteristics that utility functions represent.[18] When the $y_i$ are not all equal, the loss in the evaluation induced by this unequal sharing of $\Sigma y_i$ depends on the specific choice of function $f$. Hence, if

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[16] In Amartya Sen's presentation at the Biarritz conference, in order to clarify the famous proposition of Abba Lerner and related discussions by Samuelson, Friedman and Harsanyi, the crucial hypotheses are unambiguously presented as explicit assumptions: "Assumption 3' (Additive Probable Welfare): Probable social welfare is the unweighted sum of the Planner's mathematical expectation of individual utilities", and "Assumption 4 (Equal probability)" assumes the same probability distribution for all $u_i$.

[17] See notably "Equality" in *Encyclopedia of Political Science*, Sage.

[18] The Biarritz paper writes "responsible", as Ronald Dworkin (1981) and John Rawls later said about "tastes", but responsibility implies possibility to influence, which is only limited in this case. Influencing one's own desires is a central topic of the volume *Happiness-Freedom (Deep Buddhism and Modernity*), Presses Universitaires de France, 1982.

all we know is this preference for equal $y_i$ in the sharing, we are a priori interested in properties that hold for any such function *f*.

In fact, "welfare" is a most ambiguous term (which exists in English only – contrary to "well-being"). It is probably most commonly understood as income or as consumption goods. However, economists focus on "psychological welfare" (Rawls), and the noted intuition of this welfare as an impersonal or transpersonal strictly concave function of income exists.

What is at stake here is much more than comparing or measuring inequalities. It is the determination of the distribution of income and wealth that is unanimously considered just by the impartial selves (Adam Smith's "impartial spectators in our breasts") of the nation's citizens.

**I.2. Questions and distinctions**

Consider the following questions.

Should you pay a higher income tax than someone else because she likes dollars more than you do, notably those taken away and one is utilitarian, or less than you do, notably the remaining ones and one is egalitarian (in utility)?

Should you finance someone else's beverage because she only likes expensive wines? This classical "expensive tastes" argument extends in two ways. The other person may have to compensate you for your inability to experience such delicate gastronomic pleasures. And utilitarians meet "cheap tastes": should you finance the other's beverage because she likes cheap beer, and hence generates low-cost utility?

Should I take the 10 dollars you just earned because I like them more than you do?

When everybody shares the same opinion, this includes people who chose a policy such as voters and officials, and a policy based on the opposite opinion cannot be implemented. This is also respect for democracy. This property is the unanimity aspect of "endogenous social choice", i.e. finding the social choice criteria in society itself.

Of course, tastes and capacities to enjoy or hedonistic capacities are prominent when allocating within a family or, more generally, small groups with mutual information and empathy between members. Such capacities are also unanimously taken into account when the issue is the relief of suffering. These are the proximity-empathy and painfull welfarisms ("familism" and "dolorism"). Some people say that this is altruism motivated by empathy or pity rather than issues of justice. If we relate them to justice they would be cases of microjustice concerned with allocations specific with regard to people, reason, goods or circumstances. This opposes the question of *macrojustice* concerned with the general rule of

society and the allocation of the value of the main resources to everybody in general purchasing power. (It is also useful to consider a domain of mesojustice concerned with goods that are specific but particularly important and concern everybody, such as education and health).

The foregoing remarks may be thought of as condemning welfarism for macrojustice (as, for instance, Rawls did). However, we will see that they save welfarism on the contrary, by permitting to define "strict welfarism" using a universal individual welfare function $u$ – different from "utility-welfarism" that considers functions $u_i$ – and which describes the "welfarist intuition" noted at the onset.

**I.3. End-values of macrojustice theories**

These remarks lead one to discard individual tastes and hedonistic capacities, or, more thinly and sufficiently, differences in them, to determine socially possible and desired macrojustice (which includes the income tax and main transfers). Moreover, fully discarding individual utility functions leads to two possibilities. If this function just represents the satisfaction, pleasure, happiness, etc., the individual derives from consumption, this discarding leaves consumption goods. If, rather, this function intends to describe the individual's choice by its maximization, discarding it leaves the domain of the individual's free choice.

Let us denote $x_i \in X \subseteq \Re^m$ a vector of quantities of goods for individual $i$. They may be or include final consumption goods or other goods. In particular, they may be individual $i$'s income $y_i$ or $(y_i,\ell_i)$ or $(y_i,\lambda_i)$ where $\ell_i$ and $\lambda_i$ denote individual $i$'s labour and leisure respectively. Let also $u_i(x_i)$ denote a "standard" utility function of individual $i$ (Pareto's "ophelimity"); $u(x_i)$ the interindividually identical "individual welfare function"; and $L_i$ a domain of free choice of individual $i$. Some freedom of choice is also implicit in the allocation of goods that are not final consumption goods – income $y_i$ is such a case.

The individual end-value of macrojustice social ethics can be $u_i(x_i)$, $u(x_i)$, $x_i$ or the liberty offered by $L_i$. If the noted general opinions about hedonistic capacities and tastes lead to discard utility functions $u_i$, the resulting end-values are $x_i$ or $L_i$. If one considers these opinions as objecting to the relevance of interindividual differences in hedonistic capacities and tastes only, the end-value may be a function $u(x_i)$, the same for all individuals, if one can derive such a function from the functions $u_i$ by erasing the effects on them of different hedonistic capacities and tastes.

From the rationality of equality (equal treatment of equals), the fact that an item is such an end-value is manifested by a preferred, ideal, *prima facie* equality of this item across individuals. When this equality finds, as obstacle, impossibility or the joint relevance of some other value, the solution is often described by the maximization of some social ethical maximand which would yield this equality if these obstacles did not exist – hence, observing this maximand reveals the underlying end-value.

This individual end-value is, for instance, $x_i$ (in particular $y_i$ or $(y_i, \ell_i)$) for standard "egalitarians". It is also for Rawls who concludes, from the observation that distribution is never achieved by maximizing a $W[\{u_i(x_i)\}]$, that functions $u_i$ have to be discarded altogether for what he calls "social justice" – which is our macrojustice (he says "macro" and "not micro", but his term is ambiguous since, for instance, it may be understood as including the care of handicaps whereas this is an issue in microjustice). However, Rawls emphasizes individuals' freedom of choice from and with means allocated to them which are his "primary goods" (plus classical basic liberties). They are his end-values which should ideally, *prima facie*, be equal. There is one economic primary good in 1971, income $y_i$ (or wealth), to which he adds leisure in 1974 at the instigation of Richard Musgrave (this may better be called free time, as time free from labour, for a primary good, in contrast with the consumption-good flavour of the term leisure).

Standard "welfarist optimum income tax" studies use a function $u(x_i)$ with $x_i=(y_i,\ell_i)$. Most of them say that it is because they do not know functions $u_i$. However, the initiator Jim Mirrlees (1971) happens to be more profound and a keener observer by stating that he takes the same function $u$ for all individuals because "differences in tastes raise different kinds of problems". Yet this raises a problem for the theory since individuals maximize functions $u_i$ rather than function $u$, and classical Pareto efficiency is also with functions $u_i$. Mirrlees, then, in 1986, reverts to functions $u_i$ and rejects function $u$. This raises the vast information problem of knowing the $u_i$, but this is not actually an obstacle since, at any rate, society discards differences in tastes and hedonistic capacities for the choice of the income tax, it does not determine it by comparing marginal variations (or overall levels) of individual utility functions $u_i$, and hence by maximizing any function $W[\{u_i(y_i,\ell_i)\}]$. Moreover, we will remark that the standard maximization of $W[\{u(x_i)\}]$, with the same function $u$ and often $x_i=(y_i,\ell_i)$, can be read in two ways: the social individual end-value may be seen as either $u(x_i)$ or $x_i$.

However, if the bundle of goods $x_i$ includes several goods, their ideal multidimensional equality fails in general to be Pareto-efficient (individuals' preferences are a

priori different). This problem is not raised by income taken by itself (income egalitarians, Rawls 1971), but it is if leisure or labour is added (e.g. Rawls 1974) – yet this case is particular because the individual prices, the wage rates $w_i$, are a priori different. One solution consists in letting individuals freely exchange from their equal allocation – this will be suggested shortly for income and labour. Another consists in using the theory of multidimensional inequality (1977) to consider the least unequal of Pareto-efficient allocations (1996a). The outcome is *super-equitable*, that is, no individual prefers any allocation in the convex hull of individual allocations to her own (1973). If all individuals consume some amount of each good, this amounts to equal incomes (with the efficiency prices).

Finally, this discussion of the end-values of a just overall income distribution leads one to consider five general cases – two of which amount to the same, but which will have to be added a further one – for the nature of the social individual end-value, with generally $x_i=(y_i,\ell_i)$:

$u_i(x_i)$ standard welfarism
$u(x_i)$ $u(x_i)$ Mirrlees 1971 and followers
$x_i$
$x_i$ income egalitarians, Rawls
$L_i$ freedom justice.

What is, however, this interpersonal function $u$? Mirrlees and others provide no clue to its determination or, indeed, to its precise meaning.

### I.4. The individual welfare function

The psychological (or physiological) distinction between hedonistic capacities, tastes and individual welfare is an interesting piece of analysis, but not a necessary one here. Indeed, what matters to represent the noted common opinion about the irrelevance of individual differences is erasing these differences in a social ethical evaluation. Therefore, given a standard welfare function $W[\{u_i(x_i)\}]$, define function $u(x)$ as, for each $x\in X$,

$$W[\{u_i(x)\}] = W[e\ u(x)] \qquad (1)$$

where $e$ is a vector of $n$ ones and $n$ is the number of individuals $i$. Adopting the standard assumption that function $W$ is *non-decreasing and increasing in at least one argument at each point*, function $u(x)$ is well-defined. This operation "averages away" the differences in

functions $u_i$. However, for this averaging to be "balanced", function $W$ has to be *symmetrical*, which we assume. And for this property to be meaningful, functions $u_i$ have to be comparable by more or less, that is at least *co-ordinal, the only requirement for defining function u*.[19] This function $W$ is used as "averaging function".

By an analogy with the *equal equivalent income* of inequality theory, function $u$ can be called the **equal equivalent utility function**. It can be taken to represent the common *individual welfare function* when the interindividual differences in tastes and in hedonistic capacities are averaged away.

From relation (1), if functions $u_i$ are all increasing or decreasing, or, if they have the nature of quantities (as in welfarist income tax studies), are concave or convex, and strictly or weakly in all cases, in a domain $X' \subseteq X$, function $u$ also has the same property.

The new maximand, for utilities cleaned for their differences, is $W[\{u(x_i)\}]$. If functions $u_i$ are not known but are known to be increasing and concave (or decreasing, convex, and in all cases weakly or strictly), function $u$ has this property, and it makes sense to compare distributions $\{x_i\}$ by a higher (or not lower) level of $W$ for all functions $u$ having this property.

However, if individual utility functions $u_i$ are uncertain (as they are) and representable by stochastic variables $\tilde{u}_i$, function $u$ can also average uncertainty away by choosing for $W$ a specification $\hat{W}$ of the corresponding von Neuman-Morgenstern evaluation function and defining $u(x)$ by

$$\mathrm{E}\,\hat{W}\,[\{\tilde{u}_i(x)\}] = \hat{W}\,[e\,u(x)].$$

where E denotes mathematical expectation, for each $x$.

Calling $w(a) = W(e\,a)$, form (1) also gives function $u$ as

$$u(x) = w^{-1} \circ W[\{u_i(x)\}],$$

and similarly with uncertainty.

Then, the strictly welfarist maximand is

$$W[\{u(x_i)\}] = W\left(\left\{w^{-1} \circ W[\{u_j(x_i)\}_j]\right\}\right). \qquad (2)$$

If $W = \Sigma g[u_i(x_i)]$, then $u(x)$ is the generalized mean of the $u_i(x)$ with function $g$,

$$u(x) = g^{-1}[n^{-1}\Sigma g \circ u_i(x)]$$

and

[19] Hence there is a "fundamental utility" in the sense of *Justice and Equity* (1971).

$$W[\{u(x_i)\}] = n^{-1}\Sigma_{i,j}\, g \circ u_i(x_j)\,.$$

In particular, for the utilitarian $W=\Sigma u_i(x_i)$, $u(x)= n^{-1}\Sigma u_i(x)$, and the comparison is of $\Sigma_{i,j} u_i(x_j)$.

With $W=\min u_i(x_i)$, $u(x)=\min u_i(x)$, and the comparison is of $\min_{i,j} u_i(x_j)$.

Form (2) shows that the comparison between the loosely welfarist $W[\{u_i(x_i)\}]$ and the strictly welfarist $W[\{u(x_i)\}]$ depends, apart from function $W$, on the $u_i(x_j)$, that is, on the individuals' evaluations of others' allocations (and their own). The comparisons between the $u_i(x_j)$ are the material of a full domain of fairness theory. In particular, $W[\{u(x_i)\}]<W[\{u_i(x_i)\}]$ when one of the two basic criteria holds: *strict equity-no-envy*, $u_i(x_i)> u_i(x_j)$ for all $i, j$; and *strict adequacy* $u_i(x_i)> u_j(x_i)$ for all $i, j$.[20]

The definition of function $u$ has two consequences, one for the theory of unjust inequality and the other, more important, for the theory of social optimality.

For the theory of inequality, we have thus noted the meaningfulness of comparing $\Sigma u(y_i)$ for given utility functions $u_i$, a utilitarian maximand $\Sigma u_i$, and the case $x_i=y_i$. Function $u$ is increasing concave if functions $u_i$ are. Properties valid for all such functions $u$ hold for any set of such individual utility functions $u_i$. Moreover, if not all functions $u_i$ are concave, function $u$ may nevertheless be concave, provided that a sufficiently large fraction of the $u_i$ have this property. The result uses the utilitarian sum and the possible ignorance of individual utility functions $u_i$. However, it is *a consequence of the formulation of the general opinion of irrelevance of differences in hedonistic capacities and tastes for judging the overall income distribution*.[21] It rests, basically, on the endorsement of this general moral stance, which is practically unavoidable for implementing resulting policies. Discarding individuals' hedonistic capacities and tastes destroys welfarism, but discarding differences in them only can save it.

This theory of the averaging of tastes and hedonistic capacities provides also a solution to the problem of measuring and comparing multidimensional inequalities. For

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[20] See Kolm 1971. Adequacy is meaningful because the individual utilities are comparable (co-ordinal, existence of a "fundamental utility"). Equity-no-envy relates the relation between the two welfarist evaluations to the concept of comparing freedoms of choice by the **Theorem**: Strict welfarism does not increase the measure of social welfare if each individual's allocation can be chosen by this individual on an identical domain for all individuals.

[21] We can in fact take any Schur-concave function $W$, hence in the variables $u(y_i)$, which is also Schur-concave in the $y_i$, or in particular any concave function $W$ (in the $u(y_i)$, it is also assumed to be symmetrical), but the sum suffices.

multidimensional $x_i$, indeed, this inequality can be reduced to the one-dimensional inequality in the $u(x_i)$.

**I.5. Strict welfarism and beyond**

Strict (restricted, weak, pure) welfarism is obtained by replacing utility functions $u_i$ by the individual welfare function $u$. Its social maximand is, therefore,

$$W[\{u(x_i)\}]. \qquad (2)$$

Studies that use such a maximand, notably with $x_i=(y_i,\ell_i)$ for "welfarist optimum income taxation", always take a weakly concave $W$ (for allowing for the "utilitarian" sum) and a strictly concave function $u$.

A maximand of form (2) can a priori have, as end-value relating to individuals i, either the $u(x_i)$ or the $x_i$. If such end-values are all that relevantly distinguish the individuals, the maximand should (1) be symmetrical in them, and (2) prefer an equalization of these values. The symmetry is the case for both the $u(x_i)$ and the $(x_i)$ since function $W$ is symmetrical. For the $x_i$, however, this would justify having replaced the $u_i$ by $u$, since function $W[\{u_i(x_i)\}]$ is not a priori symmetrical in the $x_i$. Five reasons suggest preferring the $x_i$ to the $u(x_i)$ as end-values.

(1) Empirically, Bourguignon and Spadaro (2008) find that the progressivity of the actual income tax schedules cannot be derived from a maximand of the form $W[\{u(y_i,\ell_i)\}]$, which suggests that this is not the ethics followed implicitly or explicitly by governments.

(2) Theoretically, the end-value is the item the equality of which is preferred *prima facie* (ideally, intrinsically). With a strictly concave SWF $W$, this can be "pure welfare" $u(x_i)$. Indeed, if the $u(x_i)$ are not all equal,

$$W[\{u(x_i)\}] < W[e\, n^{-1}\Sigma u(x_i)].$$

However, one can do better for "society's welfare". Indeed, if the $x_i$ are not all equal, with a weakly concave $W$ (allowing for the utilitarian sum) and a strictly concave function $u$,

$$W[\{u(x_i)\}]\leq W[e\, n^{-1}\Sigma u(x_i)]<W[e\, u(\, n^{-1}\Sigma x_i)].$$

That is, if we take as end-value the $x_i$ rather than the $u(x_i)$, equality gives a socially better situation. In this sense, the equality of the $x_i$ is a better egalitarian ideal than that of the $u(x_i)$. This may imply that the $x_i$ constitute a priori a deeper end-value than the $u(x_i)$.

(3) In the noted comparisons for choosing the income tax, the differences between the individuals' enjoyments discarded as irrelevant may a priori be due to differences in the $x_i$ and not in the functions $u_i$, and then they could remain with the $u(x_i)$. Then the noted opinions

may discard function $u$ also, or at least its concavity (with $x_i = y_i$) (but egalitarianism in $y_i$, in $u(y_i)$, or the highest $\Sigma u(y_i)$ would elicit the same answers).

(4) Selecting the $x_i$ as end-values joins the solution directly obtained by egalitarians (and by Rawls) which interpret the common view of irrelevance as discarding individual utility functions $u_i$ altogether, or as a result of this discarding.

(5) The equality of the $x_i$, applied to earned income and labour and completed as indicated shortly, will turn out to have many meaningful properties. Then, it amounts in particular to the last of the rationales noted above, the equal liberty of domains of choices. Now, a central tenet of ethical epistemology is that a principle has to be evaluated from all its angles and possible meanings (cf. Plato's "dialectics" in *The Republic* or Rawls's "reflective equilibrium" focussing on consequences).

With $x_i=x$ for all $i$, maximand (2) takes form (1) which has two remarkable consequences. First, it amounts to maximizing $W[\{u_i(x)\}]$, with the actual individual utilities $u_i$, with their full actual differences. Second, it also amounts to maximizing $u(x)$. This determines the best equality, but depends on the corresponding constraints.

**I.6. Distributing earned income**

The main question is the distribution of earned income. This is the largest part of income, by very far in an intertemporal view in which capital, which is itself produced by definition, is taken as the result of the natural resources, one of which is individuals' given productive capacities used by labour.

Then, $x_i=(y_i,\ell_i)$, or the equivalent $(y_i,\lambda_i)$ for post-1974 (post-Musgrave) Rawls. The equality is $y_i=y$ and $\ell_i=\ell$ for all $i$. If $w_i$ denotes individual $i$'s wage rate (unit productivity), the constraint on the distribution is $ny=\Sigma w_i\ell$ or $y=\ell\,\overline{w}$ where $\overline{w}=(1/n)\Sigma w_i$ is the average wage rate. Hence the best choice is $\ell=k$ that maximizes $u(y,\ell)$ under this constraint or $u(\ell\,\overline{w},\ell)$. For a differentiable function and an interior solution, $k$ satisfies $\overline{w}\,u_1+u_2=0$. This is the choice of labour and earnings by the "average individual" with the average utility function $u$ and the average productivity $\overline{w}$. Figure 1 representing labours and incomes shows the maximum of function $u$ under the constraint represented by the line of slope $\overline{w}$ from the origin, reached at point $K=(k,\overline{w}\,k)$.

**Figure 1. The two-part income: equality and liberty**

This solution, however, has three defects.

1) Liberty. The solution generally violates individuals' freedom since they prefer, from this allocation, to work more (this will be the relevant case) and keep the extra earnings.

2) Pareto efficiency. For the same reason, this solution is generally not Pareto efficient.

3) Self entitlement. One of the questions of section 2 suggests that people attach some value to being entitled to one's earnings. Since these earnings depend on the individuals' given productivity, this implies valuing some self-ownership of these productive capacities.

The same answer solves these three problems: from the obtained egalitarian solution, let people free to work more and to keep the extra earnings. Then, if individual *i* freely chooses to work $\ell_i$, she earns the extra $(\ell_i - k)w_i$, and her resulting disposable income is

$$y_i = k\overline{w} + (\ell_i - k)w_i. \qquad (3)$$

This is a two-part income, sum of an egalitarian income $k\overline{w}$ and of a "classical liberal" income $(\ell_i - k)w_i$. The egalitarian income is the same for everybody, and results from the redistribution of individuals' earnings with the same labour $k$, the "equalization labour" (the informational possibility is shown shortly). Free exchange without redistribution is a property of "classical liberalism". It is applied here to the extra labour $\ell_i - k$. Coefficient $k$ is a degree of equalization. The particular case $k=0$ is full classical liberalism. Figure 1 shows the individuals' budget lines from the common point $K$ with slopes $w_i$, and individuals' choices on these lines.

**I.7. Summary of theories**

The question of the end-value and objective of the various theories, and the way they derive from one another, can now be summarized (table 1). From the individual utility functions $u_i(x_i)$, used in a SWF $W[\{u_i(x_i)\}]$, of basic economics, general opinion about income distribution in macrojustice leads one to discard either the utility functions (as Rawls does, for instance) or the differences in hedonistic capacities and tastes. The latter solution leads to "strict welfarism" with an individual "welfare function" $u(x_i)$, the same for all individuals. This gives meaning to the highest $\Sigma u(y_i)$ with concave $u$ of Pigou, Bentham and the classical property of income inequality theory. However, the quantity of goods $x_i$ is still preferred as an equalizand, which joins the solution of directly discarding utility functions from the $u_i(x_i)$. This is the solution of income egalitarians and of Rawls (1971) who later adds leisure (1974)

or, equivalently, labour. From this equality, adding free exchange of labour permits liberty, Pareto efficiency and some demanded self-ownership of productive capacities.

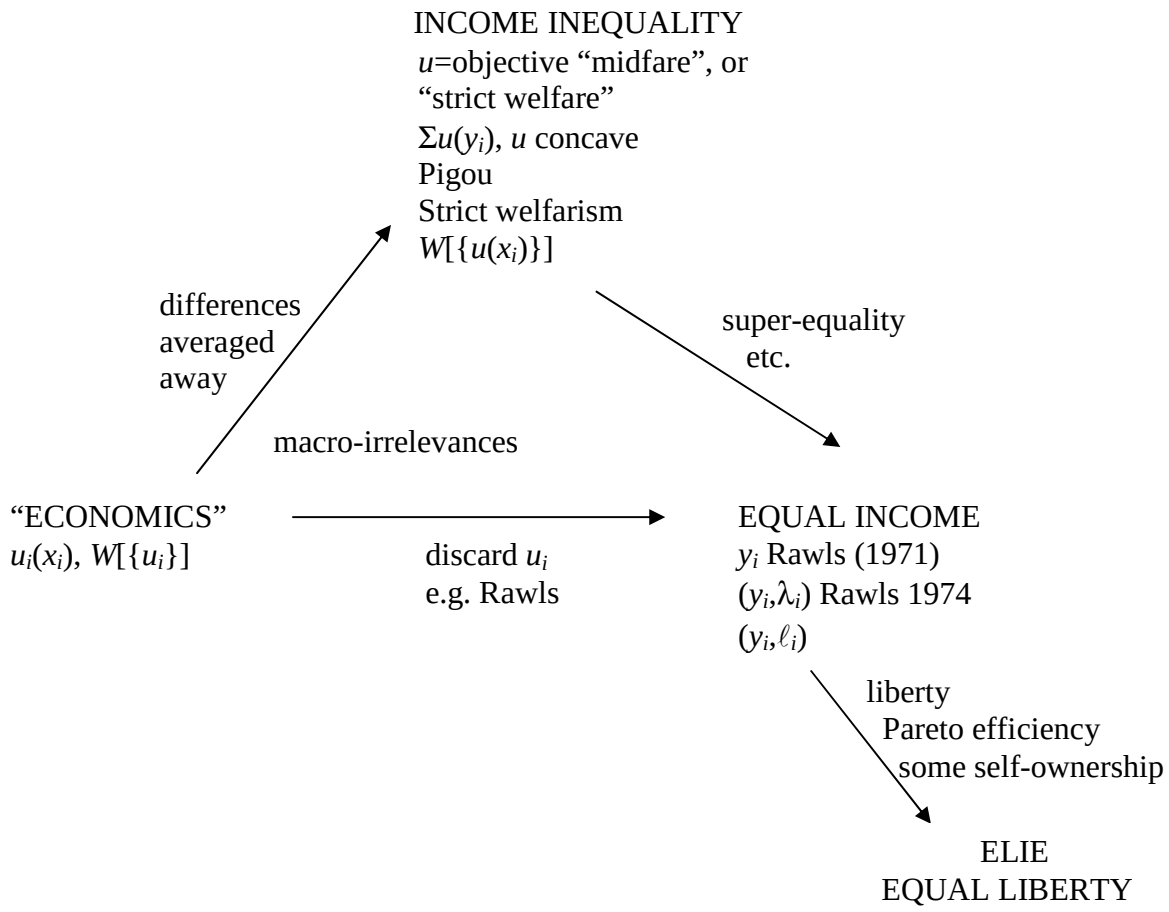


**Table 1. Four theories**

## II. ELIE, EQUAL LIBERTIES

### II.1. Equal-labour income equalization

This distributive scheme is "equal-labour income equalization" (ELIE). We shortly see that it amounts to equal liberties (with different domains of choice). Form (3) also writes

$$y_i = w_i\ell_i - T_i \tag{4}$$

where

$$T_i = k\cdot(w_i - \overline{w}) \tag{5}$$

is a tax or a subsidy of $-T_i$ if $T_i<0$. That is, each hour of person $i$'s labour $k$ is taxed by $w_i-\overline{w}$ if $w_i>\overline{w}$ and subsidized by $\overline{w}-w_i$ if $w_i<\overline{w}$. This de facto implies $k\leq\ell_i$ since taxing leisure is

generally not accepted and providing a wage supplement for hours that produce no wage seems absurd. In fact, ELIE schemes that diminish any reasonable measure of income inequality as much as actual national redistributions have an equalization labour $k$ of 1 to 2 days per week (from the USA to social-democrat Scandinavian national communities). Hence $k<\ell_i$ for normal full labours $\ell_i$. Cases of $\ell_i \leq k$ are reported to the general case by particular theories and devices, such as a theory for involuntary unemployment or, for part-time labour contracts or second wages in families, as in the French tax law presented in the next section. Remaining cases of particularly low labours of productive people have a number of possible solutions that are cases of microjustice. They range from a universal basic income permitting non-earning activities (van Parijs) to drafting people whose labour can save lives, passing through just demanding these able people to pay their consumption – the solution of both Rawls and Saint Paul ("he who does not work does not eat").

A consequence is that $y_i \geq k\overline{w}$ , a minimum income determined along with coefficient $k$. In rather homogeneous societies, there is often a rough consensus about a norm of minimum income which entails a similar opinion about the distributive coefficient $k$.

Individual $i$'s "total income", her income plus the value of her leisure $w_i\lambda_i$, is, taking $\ell_i+\lambda_i=1$ as the measure of total time,

$$v_i=y_i+w_i\lambda_i= k\overline{w} +(1–k)w_i, \qquad (6)$$

or, if $v=\{v_i\}$ and $w=\{w_i\}$ denote the vectors of the $v_i$ and $w_i$,

$$v= k\overline{w}\, e+(1–k)w. \qquad (6')$$

That is, the $v_i$ are a **concentration** (a linear uniform concentration towards the mean) of the $w_i$ with coefficient $k$.[22] This structure of transformation is one of the two simple ones that diminish inequality the most for a given amount of transfers.[23] It amounts to transforming the $w_i$ by a proportional decrease (in proportion $k$) and an equal increase that restores the total sum. A *synthetic inequality index* is an index the absolute form of which $I(x)$ for $x\in \Re_+^n$ is *equal-invariant* ($I(x+\mu e)=I(x)$) and the relative form of which $I^r=I/\overline{x}$ with $\overline{x}=n^{-1}\Sigma x_i$ is *intensive* ($I^r(\lambda x)= I^r(x)$), from which the absolute form is *extensive* ($I(\lambda x)=\lambda I(x)$). Then, for the absolute form of any synthetic index, $I(v)=(1–k)I(w)$, and $k=[I(w)–I(v)]/I(w)$ in which $I^r$ can equivalently replace $I$. Examples of synthetic indexes are the Gini absolute and relative coefficients, the variance and the standard deviation, and $\Sigma(|x_i–\overline{x}\,|)$ for $I(x)$.

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[22] See *Monetary and Financial Choices* (1966) for application to risk.
[23] The other is bitruncation. See chapter 1 of the *Handbook of Income Inequality Measurement*.

This distributive structure has other important meanings. Its transfers are *from each according to her capacities, or equally in labour, to each equally* (in income). It also amounts, for its participants, to an *equal universal basic income* ($k\overline{w}$) *financed by an equal labour of all* ($k$) *or according to capacities*. Moreover, it amounts to a *general balanced labour reciprocity*: each individual yields to each other the product of the same labour ($k/n$). This distribution also amounts to an *equal sharing of the value of productive capacities* when the fraction $k$ is measured in *income value* (output) and the rest in *labour-leisure value*. Each individual receives according to *desert* or to her work for labour $k$ and to *merit*, i.e. to her work and her capacities, hence to her works, for the rest.

These transfers are those of the distributive branch or function of public finance. Other taxes finance other public expenditures. If distribution is optimum in this way, these other financings should be neutral in this respect, that is they should be according to benefit taxation. There are, however, other classical principles of public finance. One is paying according to capacities, which, for earned income, should be capacities to earn, the $w_i$. Another is by "equal sacrifice" which could be by an equal effort or labour. Both come to the same. Individual $i$ finances $bw_i$ of budget $B=n\,b\,\overline{w}$. This is how ELIE finances the basic income of $k\overline{w}$ and, on the whole, individual $i$ pays $(k+b)w_i$, the product of her labour $k+b$. Then, on figure 1, individual budget lines are translated towards higher $\ell$ by $b$, and they all pass through the same point $\ell=k+b,\ y=k\,\overline{w}$. Individuals may also pay an equal amount $a$ (which moves the common point of budget lines to $y=k\,\overline{w}-a,\ \ell=k$). They may also make payments of both types, $a+bw_i$, which provide the budget $n(a+b\overline{w})$ and move the common point to $y=k\,\overline{w}-a,\ \ell=k+b$. Principles of all types may exist jointly.

This simple, core distributive theory is completed in various ways. The first extends it to multidimensional labour (duration, intensity, education-formation-training, etc.). These extensions are the object of two volumes: *Macrojustice* and a collective volume edited by C. Gamel and M. Lubrano.

**II.2. Informational possibility: the example and experience of the French tax system**

Tax $T_i$ of (5) can also be written as

$$T_i=(k/\ell^o)w_i\ell^o-k\,\overline{w} \qquad (7)$$

where $\ell^o$ is a benchmark labour. This shows that it amounts to two bonuses, an exemption of the earnings of overtime labour over $\ell^o$, and a uniform tax credit or rebate $k\,\overline{w}$, the same for all individuals, from a linear income tax.

Mirrlees (1971) suggests that the tax base is earned income $w_i\ell_i$ because the government cannot know the wage rates $w_i$, and a vast literature starts with this assumption. He ends the same article, nevertheless, by noting that labour duration $\ell_i$ can also be observed, which yields the wage rate $w_i$, and that we have other means of estimating a person's earning capacities which however – he thinks – would induce much hiding and evasion.

However, the French tax system has the two bonuses of form (7), including the exemption of overtime labour over a low official labour duration.[24] It amounts to basing the tax on the wage rate $w_i$. This duration can be reduced so that most working people do some overtime labour. This applies to wage income which is 9/10 of labour income. There is practically no cheating because this could not be done without the tax administration being informed about it. Inputs of labour other than duration are also taken care of. Education is public and free and financed by the income tax (hence choosing more education elicits higher public costs financed with the future higher tax on the increased wage rate). Productivity premia (for labour intensity) and formation premia will be truncated. In non-wage labour (1/10 of total), self-employed people, professionals and farmers often pay a lump-sum tax. Productivity can be estimated by comparison with wage labour of the same type. All the routine of tax administration with statements, checking, various estimates, verification, penalties, etc. can be and is used.[25] Tax authorities find that, on the whole, evasion is very much lower than when the base is total earned income (then, classically, about 30% of the base evades the tax in all countries).

The gain in revenue efficiency and administration costs is matched by gains in overall economic efficiency, justice and liberty. Marginal labour is not taxed, inframarginal labour units only are. The exemption is both for the income tax and for the "contributions" financing social security, and the marginal wedge so suppressed is a high 65%. The tax base consists in given productive and earning capacities (when all dimensions of labour are taken into account) that is, items individuals are not responsible for. There is no marginal interference with free exchange of labour and, as shortly noted, ELIE schemes induce people to work with

[24] Expressed in hours per week and in days per year for executives whose daily hours are not clear. For part-time labour contracts, the exemption is of the "complementary hours".

[25] There are minor attempts of people who work more intensely than standard to present the extra earnings as due to overtime labour. But since intensity is another input of labour, this is perfect for the theory and instructions are to accept their claim.

their most valuable skills (and thus to reveal them), they respect basic social liberty even for people who pay a tax, and they secure equal real freedom.[26]

### II.3. Social liberty: full "formal" freedom (the ELIE paradox: the higher the tax paid, the larger disposable income and *de facto* freedom of choice are)

The two standard kinds of liberties (apart from mental freedom) are relevant for income distribution. One is social liberty, freedom from forceful interference, also more or less described as negative freedom (Kant, J.S. Mill, Berlin), civic freedom (Mill), basic rights or liberties, or "formal freedom" (Marx). The addition of other means of various possible kinds provides "real freedom" (Marx). In a society with social liberty, individuals are only constrained not to use force against others – in so far as they do not voluntarily abstain from it. When individuals' intentions are incompatible, this is solved by the allocation of the relevant means, often rights and in particular property rights, resulting from the distribution and exchanges. Hence social liberty can be full for all, and it is then equal.

Social liberty implies that people have the right to act (without forceful interference and forcefully interfering), hence to use their capacities: they have the use-rights in their capacities, here the free choice of labour $\ell_i$. Classically, this liberty is conceived from an allocation of given resources (as with Pareto's model of markets). This is the case with ELIE. However, with $k>0$ there is some redistribution of the value of the earning power of given productive capacities – i.e. of the rent of these capacities. Then, some people receive transfers but some others pay a tax. However, the following property holds.

**Theorem 1 (the social liberty theorem)**

*With an ELIE structure of transfers, someone who pays a tax, or a higher tax than someone else, has a higher disposable income for the same labour and more* de facto *freedom of choice (she can both work less and consume more*).

*Proof*

With $k>0$, individuals $i$ with $w_i<\overline{w}$ receive $k\cdot(\overline{w}-w_i)>0$. However, individuals with $w_i>\overline{w}$ yield $T_i= k\cdot(w_i-\overline{w})>0$. Such an individual $i$ with a higher $w_i$ pays a higher $T_i$, but her disposable income $y_i=k\overline{w}+(\ell_i-k)w_i$ is also higher for each $\ell_i>k$. This implies a higher freedom

[26] A welfarist (with function $u$) optimum tax study of 1974 optimized for both the tax schedule and the tax base. It then proposed ways of basing the tax on wage rates, including the exemption of overtime labour, and analyzed the resulting tax structure. However, the present French tax policy resulted from public presentations and discussions of the proposals of the volume *Macrojustice.* The present overtime exemption has been the central policy proposal of a presidential candidate.

of choice of the pair ($\ell_i$, $y_i$), by inclusion of domains of possible choice. The individual can in particular work less ($\ell_i$) and consume more ($y_i$). Moreover, this amount is globally transferred to individuals with $w_i < \overline{w}$ , which augments their freedom of choice. The basic reason for the proposition is that, with an ELIE scheme, someone who pays a higher tax $T_i$ has a higher earning power $w_i$, and the tax takes only part of this advantage.

This possibility shown by ELIE transfers has a major importance for political philosophy. Classical liberalism opposes all non-voluntary transfers. It defines itself by full self-ownership or by social liberty and thinks that they imply each other. Full self-ownership certainly implies social liberty, would it only be from the concept of ownership. The converse rests on the idea that a forced transfer – say a tax – forces the person to work more or to consume less. With an ELIE scheme, however, a tax, and a higher tax, go with the possibility of the taxed person to work less and to consume more. Therefore, classical liberalism has better justify self-ownership of the value of one's given productive capacities otherwise, which it can do with its second kind of justification, the concept of natural right. That is, an individual's capacities fully belong to her (property) because they belong to her (being part of); they are hers because they "are" her. Classical liberalism as full self-ownership is a particular case of ELIE, that with $k$=0.

**II.4. Equal real freedom**

Moreover, ELIE also has a property of equal real freedom, which is provided by different possibility sets, however. Equality in this liberty can be defined in several ways. One is described by figure 1. It is **equal free exchange from an equal allocation** (of income $k\overline{w}$ and labour $k$ or leisure $1–k$). Form (1) projects this property on the income dimension. However, the following result also holds.

**Theorem 2 (the total economic freedom theorem)**

*The ELIE distribution corresponds to different budget sets providing equal freedom of choice.*

*Proof*

The standard way of defining equal consists in deriving it from a definition of more and less. This leads to order domains of choice according to the freedom of choice they provide. This order is sufficiently described by an ordinal "freedom function" $F(D)$ such that $F(D)>$, = or < $F(D')$ expresses this freedom order for two domains of choice $D$ and $D'$. An individual's

choice of labour $\ell$ and income (consumption) $y$ can be described by her choice of $y$ and of leisure $\lambda=1-\ell$ in her budget set $Py+W\lambda\leq V$ where $P>0$ is the price of consumption goods and $W=Pw$ and $V=Pv$ her wage rate and her total income expressed in the same nominal units, respectively (figure 2 for $P=1$). This domain of choice is fully described by this total income $V$ and the prices $P$ and $W$. Hence one can write $F=F(V; P, W)$. If the prices are classically represented by a linear price index $\alpha P+\beta W$ with $\alpha>0$ and $\beta\geq 0$, $F=\phi(V, \alpha P+\beta W)$. Since multiplying $P$,$W$ and $V$ by the same positive number does not change the equation of the domain, and the domain, function $F$ is homogeneous of degree zero, and so is function $\phi$. Hence $\phi=\varphi[V/(\alpha P+\beta W)]$. Since $F$ is ordinal, so are $\phi$ and $\varphi$, and $\varphi$ is increasing since $F$ is increasing in $V$. Hence the ranking is according to $V/(\alpha P+\beta W)$. This is the classical "purchasing power" or real (total) income. The foregoing amounts to the "axiom" that freedom of choice in budget sets is ranked according to purchasing power. Note that linear price indices are the standard use (for instance the Paasche and Laspeyre indices). They represent the value of a bundle of goods the quantities of which are the coefficients. The only other meaningful price indexes are those derived from a given utility function taken in its indirect (Roy) form. They are not relevant here since the reference is not utility but liberty. The linear price index is unit neutral since each price is multiplied by a quantity of the corresponding good, and these obtained money values are meaningfully added – since prices intervene in the same direction (a higher price of any good restricts freedom of choice by inclusion of domains). Any other aggregation of the money values of the goods can hardly have actual meaning.[27]

Then, since $V/(\alpha P+\beta W)=v/(\alpha+\beta w)$, equal liberty means that $v=(\alpha+\beta w)\,\gamma$ for some constant $\gamma$. That is, for all $i$, $\beta\gamma w_i+\alpha\gamma=v_i=w_i-T_i$ or $(1-\beta\gamma)w_i-T_i=\alpha\gamma$. Since $\Sigma T_i=0$, this implies $(1-\beta\gamma)\,\overline{w}=\alpha\gamma$ and, denoting $1-\beta\gamma=k$, $T_i=k\cdot(w_i-\overline{w})$, that is ELIE.

All budget lines $y_i+w_i\lambda_i=v_i=w_i-T_i$ pass through the same point $K(\ell_i=k,\ y_i=k\,\overline{w})$.

**Figure 2. Equal-freedom budget sets**

### II.5. Incentive compatibility

ELIE transfers do not depend on labour $\ell_i$ and, therefore, do not induce the corresponding wasteful disincentives. However, they depend on wage rates and this could induce people to

[27] See also *On real economic freedom* (2006, 2009).

work with capacities that are not their most remunerated ones. Indeed, if $w_i$ denotes the highest wage rate individual *i* can obtain, this individual can also generally earn various rates $w'_i < w_i$ by not using her best (most highly paid) skills at work.[28] She may make such a choice if she thinks that the fiscal authority bases her taxes and subsidies on this actual and observed $w'_i$, in order to diminish the tax or transform it into a subsidy if $w_i > \overline{w}$, or to augment the subsidy if $w_i < \overline{w}$ (hence she would benefit whatever $\overline{w}$ if $k>0$, and therefore she need not know $\overline{w}$ to behave this way). The individual may think that the government would take the observed $w'_i$ as base either because it deems the actual wage rate to be the appropriate basis so as not to tax or subsidize unused capacities of value $w_i - w'_i$ (just as it chooses $k \leq \ell_i$), or because it mistakes it for the value of capacities $w_i$, or for any mixture of these reasons. However, the following property holds.

**Theorem 3**.

*With ELIE transfers, individuals choose to work with their most remunerated capacities.*

*Proof*

Individual *i* thus chooses both labour $\ell_i$ and skills that earn $w'_i \leq w_i$, that maximize some increasing ordinal utility function

$$u^i[1-\ell_i, (\ell_i - k)w'_i + k\overline{w}'],$$

where $\overline{w}' = (1/n)\Sigma w'_j$.[29] Variables $\ell_i$ and $w'_i$ are independent. The derivative $\partial u^i / \partial w'_i$ has the sign of $\ell_i - k + k/n$ if individual *i* takes the $w'_i$ for $j \neq i$ as given (no collusion), but whatever they are. Therefore, individual *i* chooses $w'_i = w_i$ if $\ell_i > k \cdot [1-(1/n)]$. This is the case for macrojustice in which $\ell_i \geq k$.

Hence, *the individuals choose to work with their best skills and thus to "reveal" their capacities and to exhibit their economic value*. The government can understand this (it does not need to know individuals' utilities, but only that individuals prefer higher disposable incomes for given labour). Hence, it does not need to raise questions about basing taxes and

[28] See Dasgupta and Hammond (1980).

[29] Choosing a more remunerated but more painful or disagreeable activity, or the contrary, is considered as working more or less, and a corresponding full analysis has to consider, in a framework of multidimensional labour, the relevant dimension(s) that affect both the productivity and the painfulness or intrinsic attractiveness of labour.

subsidies on the actual values of capacities $w_i$ or on the observed wage rates $w'_i$ since using the latter as base makes them be the $w_i$. And the individuals can in the end know this conclusion.[30]

## II.6. The just degree of equalization

### *II.6.1 The degree of redistribution*

Coefficient *k*, the equalization labour, is a degree of equalization, redistribution, solidarity with respect to the endowment of earning powers, common ownership of the value of productive capacities, balanced labour reciprocity, and concentration of total incomes. It is the fraction of the average wage turned into a minimum income and the equal labour that provides this basic income. This is a highly significant number. Similarly, for any redistribution, the duration such that a complete equalization of incomes during it reduces a measure of inequality as much as this redistribution does, is a particularly meaningful measure of the intensity of the equalization achieved by these transfers.

In the foregoing, coefficient *k* has been derived from a SWF *W*. However, one should determine this function, as well as any such function used otherwise, and the meaningfulness of coefficient *k* permits a more direct determination of its level. As a general rule, one should determine the degree of redistribution (if any) as well as its structure. Classical economic studies as well as Rawls assume that the optimum degree of inequality in income (plus sometimes labour) is the lowest one with some index (or maximin) limited by disincentives. This conception, taken by itself, bypasses two aspects. A technical one is the optimization of the tax base and the resulting noted actual possibility of strongly limiting disincentive price effects. A social issue is that common opinion attaches some value to the legitimacy of earnings and even of earning capacities (this varies according to the distributive society in question).

Solving this central social ethical problem necessarily rests on two types of sources of information. (1) Rationality applied to social ethics. (2) People's moral opinions in this respect. These opinions permit answering questions for which rationality by itself does not suffice. Taking them into account may be a requirement of democracy. They may also create social constraints on the possibility of implementation. Two successive aspects of these

[30] If the government used the $w_i$ if it could know them, with $t_i = k\cdot(\overline{w} - w_i)$, and each individual *i* could choose her skills used and $w'_i \leq w_i$, her income would be $\ell_i\, w'_i + k\cdot(\overline{w} - w_i)$, and she would also choose $w'_i = w_i$ if she chooses to work at all ($\ell_i > 0$) and hence when $\ell_i > k$.

opinions should be considered. (1) Their being about justice implies that they should be impartial. This a priori creates a problem in the common case in which they are opinions of people whose interests (or those of people they particularly like) are affected by the distribution in question. Hence, in people's actual views, the impartial content should be distinguished from interest. However, a priori there can be various impartial opinions for the same issue (for instance, both self-ownership and income equality can be impartial theories). Therefore, (2) these impartial views should more specifically be just.

### *II.6.2 The neutral citizens*

ELIE distributions present two remarkable properties with respect to impartiality. First, for individuals with a wage rate equal to average, $w_i = \overline{w}$ , $T_i = k \cdot (w_i - \overline{w}) = 0$ whatever $k$, they neither pay a tax nor receive a subsidy for any $k$, and hence its level does not affect their interest. Therefore, their opinion about it reflects impartial social moral views only (discarding effects on individuals with different wage rates these people particularly like). If these views have no reason to be correlated with $w_i$, these opinions constitute an unbiased sample of those of the population. Other people's moral views are also sometimes revealed (note that when individuals are small in a large number, any view they express is bound to have no actual influence).

### *II.6.3 Impartiality by aggregation* (the theory of the ***moral surplus***)

Second, since an ELIE operation is a balanced set of transfers, in the sum of people's money equivalent (or willingnesses to pay) for it the amounts that represent self-interest cancel out since they are equal to these money transfers with sign plus when received and sign minus when yielded. The remaining sum thus is an aggregate of people's non-self-interested views. This applies marginally to obtain the conditions that determine a surplus equilibrium. A surplus equilibrium is a state such that the sums of the willingnesses to pay and of the money equivalents are non-positive to go to any other state and non-negative to come from it (willingnesses to pay and money equivalent are inverted in these two operations). Since the values of self-interest cancel out, this "moral surplus" principle yields a solution even if the measures of other values are much smaller, for instance in case of "lexical egoism" (but the total amount may not be small because of the large number of people). This aggregation of values is specific but classical. For instance, Bentham says both that the utilitarian sum should include "the pleasure people derive from others' pleasure" and that utility should be measured in money "for lack of a better measure, or we must bid adieu to moral". Pareto (1913)

aggregates individuals' utilities each of which is a function of all individuals' "ophelimities" or welfare, for marginal variations. People's appreciation of others' appreciation, and so on, and the insertion of effects in general economic equilibrium, are included in the theory. This impartialization by aggregation applies to all (balanced) redistribution.[31] Given the ELIE structure, it applies to the determination of coefficient *k*.

### *II.6.4 The informational causes of the sense of justice*

There are other ways to reveal, enact or construct people's impartial views, for instance by observing some of their choices in society, by questionnaires adequately formulated, and so on. Basing social choice on these elements resemble "respecting consumer's preferences", but moral opinions are different from tastes on many grounds such as the intrinsic externality of their effects, the difference between moral satisfaction and welfare, and the role of reason, social emotions and particular experiences and influences in their formation. The causes of these moral views are then to be considered. A person's moral opinion may not be more respectable than the self-interested propaganda that created it (then, we may want "autonomous" opinions).

Impartial opinions about distribution are likely to be more similar to one another than actual ones synthesizing all reasons (because self-interests about distributions are by nature opposed). Indeed, a long series of authors hold that there is only one impartial view. This is certainly the case of Adam Smith in the *Theory of Moral Sentiments*, explicitly that of John Stuart Mill (since, for him, impartiality means equal weights in the utilitarian sum), and even probably that of modern scholars such as Tom Nagel. However, income egalitarianism, classical liberalism and full welfarism with symmetrical influence of individual utilities are opposed to one another and can all be impartial a priori. Nevertheless, individual impartial views or, one may say, political opinions, have causes in the opinions and reasons people have heard, the facts of society they have been informed about, and their life experiences, plus, possibly, particular characteristics of their sensitivities. The question thus is what would be the opinion of a hypothetical person who has all the information about these facts (information which should probably include how it feels to have various crucial life experiences and political convictions). At any rate, a person's opinion about the distribution depends on her information about other people, their situation and their views and reasons.

[31] Kolm in Guitton and Margolis, eds. (1966) and 2004.

### *II.6.5 Dialogue: the curse of non-ergodicity*

Practically, this leads to the age-old solution of dialogue. Dialogue may aim at agreement and social peace, but the transfers of information it constitutes raise the hope that it may lead to some objective definition of the just social state. This, however, leads to a major question and distinction, the **non-ergodicity of dialogue** and the distinction between influence and rationality. Indeed, we would like to infer the value of the outcome of a dialogue from that of the *transformation function* that describes how it transforms the opinions of the participants. This function would be given some obvious qualities such as sincerity and individual possibilities to influence others that are either legitimate or equal (*isegoria*, the Athenian term to express equal right to influence public opinion – then in time of speech in the *agora*). However, when the participants all agree for the same reasons, the dialogue a priori does not change this view. This occurs for any such consensus. Hence, this stationarity property does not permit to characterize the outcome of the dialogue from the characteristics of the transformation function only. That is, the process is not ergodic. This implies that *the outcome depends on the initial opinions*. The final "Ideal-speech" (Habermas) depends on the initial prejudices. This means that one should distinguish a *dialogue by influence* such as the one described from a *dialogue by rationality* that questions perspectives and modes of reasoning more deeply.[32]

## III. SOCIAL SOLIDARITY: THE COLLECTIVE VOLUNTARY REDUCTION OF INEQUALITIES

### *1. GENERAL PRESENTATION AND PHILOSOPHICAL UNDERPINNINGS*

#### 1.1 The general issue

Various European countries redistribute about 30% of National Income. This reaches 40% for some of them (Scandinavian social-democracies). Moreover, these redistributions are largely accepted. These are quite large amounts for societies supposedly primarily ruled and motivated by individualism. This is, of course, mostly achieved by taxation. But this, in itself, does not prove that these transfers are forced in essence. Indeed, distributive justice in society is a public good for people who have an opinion about it. The fate of someone in need is a

[32] See Kolm (2000) for a dialogue inducing more similar orders of preference (and shrinking of the Pareto set), and (2004) for an application to the determination of the degree of equalization by decreases in the inequality of those held to be best by the participants.

public good for all solidaristic or compassionate people who care about it. Even someone who argues she is unjustly treated refers to a principle which is impartial in nature and hence can be shared by others. The public constraint of taxation may simply be a way to check free riding. Moreover, even if people are ready to give not under the condition that others pay too – in an exchange – but, more subtly, given that they give, pay or contribute their fair share – in reciprocity –, the only way for them to be sure that others do this is that these others are forced to do it. Then, there is a constraint, but everybody voluntarily abides by it because the others contribute. The constraint is reached but is not binding, although it is necessary. This is one of the main reasons for the not infrequent case of taxes that are accepted and for their being sometimes paid without much enforcement (this happens to be more frequent in countries in which they are the highest).

Actually, distinguishing between the different motivations is not easy, for the basic reason that each individual shelters several different motives, and one or the other can speak through her mouth (or choose her ballot) depending on time or circumstances: the standard person is usually jointly self-centred and impartial, egoistic and altruistic, selfish and solidaristic.

Since social justice or the need of the needy are joint concerns for people's moral views, the basic formal social structure of the question is that of non-excludable public goods. The good can be more specifically, for instance, an index of inequality or of equalization, the poor's welfare or income or a measure of poverty, or a rule of contribution or of helpful or cooperative behaviour. People contribute to this good at a private cost for them. Hence this set of contributions manifests both a common desire realized by cooperation (each values each other's contribution) and a conflict of interest when sharing the cost of the good, which may lead to free riding. Classical – practical and theoretical – solutions separate these two aspects. First, a rule of relative adequacy or fairness solves the conflictual aspect. Then, under this rule people agree on the choice. This can take various forms. The rule can be the adoption of a joint aim aggregating individuals' interests or values, and society becomes a "team" in Radner's sense. People can then stop any further cooperation and act in a Cournot-Nash way for realizing the common objective. This is the more direct general representation of Rousseau's General Will. The rule can also match individuals' actions or contributions one with the other. It can also define each person's share of the total cost (a share which may vary with this amount). Rawls's determination of justice in society by its stability requires rules for sharing the costs or the benefits induced by deviations from equilibrium. Kant's categorical imperative principle demands choosing "maxims"; this theory is basically flawed – as shown

shortly –, but it can be rescued by restricting this choice to some "consistent" set in which "Kantian" agents make the same choice.

The Rousseau, Rawls and consistent categorical imperative models turn out to lead to possible solutions that coincide with the set of Pareto-efficient states. For the last two, however, this result occurs only when the corresponding externalities (contributions appreciated by all contributors) amount to these contributions being to a single, aggregate public good. Pareto-efficiency is important because its absence is a failure to realize something possible that everybody prefers, it manifests, therefore, an avoidable constraint on collective freedom, and as a result it tends to be socially unstable (for instance a contending party can propose a program that wins with the unanimity of votes).

These rules can take various social forms. They may be norms or standards. They may result from habits or traditions. They may be choices considered normal or rational, or be salient options. They may result from social, cultural and normative evolution, competition and selection (some corresponding basic biological propensities similarly arrived at are a possibility but not a necessity). Rules may also result from agreement or arbitration and be binding by force (generally public) or by a moral of respect or of promise keeping. They may also be conventions, result from conceptions of "social contracts" (putative agreements), or from political processes. Matching rules may result from relations of reciprocity and be "reciprocally accepted terms of cooperation" (Rawls). Rules may be valued as being fair, adequate, proper, or a condition for peace. Following them may be voluntary or imposed. When imposed, this may be to enforce an arbitration or an agreement, secure a reciprocity, or apply a moral norm. When voluntary, following rules may be induced by the dictum of reason or the spur of emotions, as moral duties for avoiding guilt or as social norms for eschewing shame. Praise or blame of other people's judgments, actual or imagined, may be influential.

### 1.2 Landmark theories of moral cooperation

How individual morality concerning society can transform individuals with opposed self-interests into unanimous co-operators reaching a socially efficient state, in liberty and hence without the constraint of a Hobbesian absolute monarch, was, of course, the central social concern of Enlightenment philosophers, who produced the deepest thoughts about it. This includes Hume's theory of conventions (with a clear view of the public good issue), Rousseau's Social Contract seen explicitly as the solution to a public good problem, and the categorical imperative of Kant, the claimed pupil of the former two (especially of Rousseau

for this issue). We should add, nowadays, the late-enlightenment philosopher Rawls (for his stability theory of justice claiming inspiration from Kant).

### *1.2.1 The team theory of the General Will*

The extraordinarily influential Rousseau sees society's problem as a public good question – each gives his efforts to all and benefits from the efforts of all, his free will wants to engage in this cooperation, but his interest may induce him to shirk, hence "he must be forced to be free", and, indeed, he wants to be forced to be free. These individuals engage in a (putative) Social Contract and emerge as citizens who all basically want to achieve the General Will. What the General Will exactly is, however, is not well-known, and majority voting is a way of pooling experts' opinion about it. This is what Condorcet studied. The most straightforward representation of the General Will is a SWF, transforming the citizenry into a "team" (all want to maximize the same thing). However, a minimal interpretation of the General Will is that it indicates what is strictly necessary for cooperation only, the rules of comparative joint actions.[33]

### *1.2.2 The case of aggregate contributions*

We will consider *n* individuals with each individual *i* contributing $x_i \in \Re_+$ (in money value for simplicity). Let $x = \{x_i\} \in \Re_+^n$ denote the vector of the $x_i$. Each individual *i* has an initial income $y_i$ and an ordinal differentiable utility function $U^i(y_i - x_i, x)$. The presence of *x* denotes individual *i*'s concern with each contribution $x_j$ to the quality of society (distributive justice, lower inequality or poverty, etc.). It turns out that an essential distinction concerns the case in which *x* intervenes in all the $U^i$ through the total sum $X = \Sigma x_i$ only. Then, $U^i = u^i(y_i - x_i, X)$, which is assumed to be increasing quasi-concave.

*Definition*

*In the aggregate case, the externality occurs through the total sum of contributions only.*

In the general case, each $x_i$ is a public good by itself (contributed by one person only). In the aggregate case, there is only one public good, the sum *X* (or its consequences). This

[33] There have been a number of interpretations of Rousseau's General Will with game theoretic concepts, including the prisoner's dilemma by Runciman and Sen (1965) and the core for non-excludable public goods by Kolm (1987).

means that the dollars of contributions from different people are perfectly substitutable for their effects deemed relevant by all contributors (apart from their being a direct cost for the contributor). This forbids, in particular, all variants of "warm-glow" effects for contributing: in so far as these contributions are voluntary, they are for moral motives only (we note in section 4.3 the strong limitations of warm-glow explanations). The aggregate case implies that there is no loss due to impossibilities of allocating at best contributions of various origins. It also means that there is an integrated conception of the moral quality of society and an integrated collective action to this end. It participates to these conception and action being those of a justice-seeking genuine community. In this aggregate case, the beneficiaries of aid or transfers have to consider that they are helped by society as a whole rather than by particular individuals. The full pooling of contributions may secure the anonymity of helpers. Relatedly, the aim is social justice rather than individual pity or compassion, although they can be motives for favouring justice. Distinguishing the aggregate case turns out to be essential because this is the case in which the models of Rawls's justice as stability and of the consistent categorical imperative lead to Pareto efficiency.

We will denote

$$V_j^i = [\partial U^i / \partial x_j] / [\partial U^i / \partial (y_i - x_i)]$$

and $v_i = u_2^i / u_1^i$. If $U^i$ has the form $U^i = u^i$, then $V_j^i = v^i$ is the same for all *i*. Conversely, if $V_j^i$ is the same for all *j*, this practically, de facto, implies that $U^i$ has form $U^i = u^i$ since, in the problem under consideration, one can hardly see which other structure could induce this property – the $x_j$ would influence $U^i$ additively marginally but not overall.

For simplicity in presentation, in all the following, derivatives and conditions for maximum and efficiency are written for cases of existence, uniqueness and interior solutions only (the results extend to the other cases).

### *1.2.3 Rawls's stability theory of justice*

The social moral consensus need only solve the question that sets individuals' interest against one another, distribution, in distributive justice. Rawls (1971 chapter 7, 1980, 1982) considers a "well-ordered society" in which people, in a continuing Habermasian social dialogue and with a shared political culture, agree on the distribution (while actually experiencing it). This specific distribution is determined by its property of stability under these conditions. The consensual choice implies that, when more or less of the relevant public good than this equilibrium is produced, the participants agree on a fair, just or proper rule for sharing the

extra cost or the savings realized, directly or by comparing the variations in their contributions. In this way, what each person pays or gains in the variation is an increasing continuous function of the variation in the total amount or in the contribution of any other; this is a *deviation rule* (exactly defined shortly). This rule solves the problem of the divergence of interests and permits unanimity in the choice. Then, a stable consensus is defined as follows.

*Definition*

*A set of contributions is a stable consensus when no deviation rule permits a unanimously approved variation and one deviation rule creates a unanimous preference for no variation.*

Any rule can a priori be chosen, but it can be, for instance, equal variations, variations in proportion to the individuals' income before or after the contributions or to any other property of the participants, or variation for each person proportional to her total contribution.

A *deviation rule* states, for each $i$, the deviation $r_i(x,\lambda) \in \Re$ of $x_i$ from state $x$ depending on a parameter $\lambda \in \Re$ such that $r_i(x,0)=0$, and differentiable and increasing in $\lambda$ with the notation $\partial r_i / \partial \lambda = r'_i(x,\lambda) > 0$. The $r_i$ are matching deviations (each level $r_i$ corresponds to one level $r_j$ with reciprocity and overall consistency – see section 2.3) and they also constitute a sharing rule of the total deviation $\Delta X=\Sigma r_i(x,\lambda)$ since this increasing function of $\lambda$ can be inversed as $\lambda=g(x,\Delta X)$ which gives $r_i(x,\lambda)=s_i(x,\Delta X)$ with $\Sigma s_i=\Delta X$. Denote as $r(x,\lambda)=\{r_i(x,\lambda)\}$ the set of the $r_i$. With deviation $r$, $x$ becomes $x+r$, and

$$U^i = U^i[y_i - x_i - r_i(x,\lambda), x + r(x,\lambda)].$$

If individual $i$ prefers no deviation under this rule, that is $\lambda=0$,

$$\Sigma_j V^i_j (y_i - x_i, x) r'_j(x,0) = r'_i(x,0). \qquad (8)$$

If this holds for all $i$, the $n$ conditions (8) may determine the set of transfers $x$.

In the aggregate case, $U^i = u^i(y_i - x_i, X)$ for all $i$, $V^i_j = v^i = u^i_2 / u^i_1$, and the $n$ conditions (8) imply

$$\Sigma v^i(y_i - x_i, X) = 1,$$

the condition for the Pareto efficiency of the contributions $x$ to the public good $X$. Hence the result:

**Theorem 4.**

*Justice defined by its rule-stability is Pareto-efficient in the aggregate case in which all transfers are considered as contributions to the same public good and practically in this case only.*

We shortly see that conversely, in the aggregate case, any Pareto-efficient set of contributions is a stable consensus, hence in particular with unanimous preference for no deviation according to some deviation rules. These deviations need even be linear only (the $r_i$ are proportional to $\lambda$, are in given proportions, and are constant fractions of $\Delta X$).

### *1.2.4 The consistent categorical imperative (rationalizing Kant)*

*The flaw or failure*

Kant's categorical imperative is the best known rational social ethics. "Act according to a 'maxim' such that, if everybody followed it, you could want the result". Thus, moral duty could secure cooperation in a decentralized way. The evaluation of "wanting the result" may, however, depend on the person. Then, the categorical imperative may lead different people to choose different maxims. With such choices, no maxim prevails universally. In particular not the "good" one if there is one. These people's notional hypotheses turn out to be false. Kant, however, does not see such cases. This is because, in his examples, the choice is in a dichotomy between broad categories of acts, such as "lie or do not lie", "help or do not help". This dichotomy also occurs in the "folk-Kantianism" of people applying the same idea; for instance, the most common answer when someone is asked why she votes in large elections in which one ballot makes no difference is: "What if nobody voted?". In such cases, the same choice may be made by people whose evaluative judgments are not identical. In fact, the question is not even asked: can you want people to always lie, never help or never vote?

*Consistency*

In other cases, however, the options are more numerous and face more or less different individual evaluations. The noted diversity of choices and chaos may occur. A way to prevent this is to restrict the set of available alternatives to maxims such that everybody prefers the same one when assuming they are universally applied.

*Definition*

*A set of alternative maxims is consistent if all agents applying the categorical imperative choose the same one.*

Consistent sets depend on people's modes of evaluation. Their consideration can remedy the "diversity failure" of the categorical imperative. However, this remedy by the restriction of alternatives to a consistent set leaves the selection of this specific set to an external choice, not as a result of the internal logic of the imperative.

*An example: matching and sharing rules*

Assume that the alternatives constitute a set parameterized in a continuous way by a parameter $\lambda \in \Re$. Consider, moreover, that individuals' actions are payments $x_i \in \Re_+$ for individual $i$. Payment $x_i$ depends on $\lambda$, but it may also depend on individual $i$'s characteristics $c_i$ and be $x_i = R(c_i, \lambda) = r_i(\lambda)$ by definition of function $r_i$. Assume $r_i$ to be increasing and differentiable. Note that such a parameterized rule of payments constitutes a "matching rule" between the $x_i$ since, for each $x_i$, there corresponds a unique $x_j$ for all $j \neq i$ with reciprocity and overall formal consistency (see section 2.3.1).[34] It is also a sharing rule of the total amount $X=\Sigma x_i$ since $X=\Sigma r_i(\lambda)$, an increasing function of $\lambda$, can be inversed as $\lambda=h(X)$ and $x_i=r_i(\lambda)=r_i[h(X)]=s_i(X)$, with $\Sigma s_i=X$. Each individual $i$ may be concerned with $x_j$ for $j \neq i$ (the externality crucial for the Kantian problem) and with $x_i$ as her contribution. Hence, each individual $i$ is concerned with the $n$-vector $x=\{x_j\}$, and her evaluation is described by her "utility" function $U^i(y_i - x_i, x)$. Write $V_j^i(\lambda)$ as $V_j^i$ for $x_k = r_k(\lambda)$ for all $k$.

Individual $i$'s application of the categorical imperative is her choice of the alternative "maxim" defined by $\lambda$ when assuming that all individuals act according to the same "maxim", that is, pay $x_j=r_j(\lambda)$ for the same $\lambda$ for each $j$. Then she chooses her *preferred maxim* $\lambda=\lambda_i$ that maximizes $U^i$ with $x_j=r_j(\lambda)$ for all $j$. The condition is

$$\Sigma_j V_j^i(\lambda_i) \cdot r'_j(\lambda_i) = r'_i(\lambda_i). \qquad (9)$$

Denoting as $r=\{r_i\}$ the set of the $r_i$, the matching rule $r(\lambda)$ is *consistent* when all $\lambda_i$ are the same, $\lambda_i=\lambda'$. Then,

$$\Sigma_j V_j^i(\lambda') \cdot r'_j(\lambda') = r'_i(\lambda') \qquad (10)$$

[34] An individual may also prefer that all others contribute maximally and she contributes only little or not at all. However, such a rule is not impartial, reciprocal, consistent or universalizable.

for all *i*. Each $x_j$ is a public good, and its condition for Pareto efficiency is $\Sigma_i V_j^i(\lambda) = 1$. In the aggregate case, $U^i = u^i(y_i - x_i, X)$ for all *i* and $v^i = u_2^i / u_1^i$, $V_j^i(\lambda) = v^i(\lambda)$ is the same for all *j*. Therefore, condition (10) writes

$$v^i(\lambda')\Sigma r'_j(\lambda') = r'_i(\lambda').$$

Summing for all i gives $\Sigma v^i(\lambda') = 1$, the condition for Pareto efficiency of the set of contributions $r(\lambda')$.

**Theorem 5.**

*In the aggregate case, and practically in this case only, the universal application of the categorical imperative with a consistent rule implies Pareto-efficiency.*

We shortly see that conversely, in the aggregate case, all Pareto-efficient states can be reached by the categorical imperative with such consistent rules, and even by particular linear rules (individuals' contributions are proportional to one another and are a constant share of the total) after a lump-sum redistribution (the Lindahl case is that in which this redistribution does not occur). However, a number of standard structures of contributive rules cannot be consistent except with particular similarities in individuals' "utilities" and incomes.[35]

## *2. THEORIES OF COOPERATIVE REDISTRIBUTION*

### 2.1 The team General Will solution

With a team General Will solution in which the citizens seek to maximize the same function, the key property is that individuals need not add further cooperation. They reach the common maximum by playing Cournot-Nash, for instance. The General Will "respects individuals' preferences" when it depends on the state of the world through individuals' utility functions

[35] Another issue concerns the meaning of Kant's crucial criterion that "one could want the result". Kant discards "inclinations" (say preferences, which can include affective altruism) in favour of reason, but he also considers self-interest (for instance help others because you may need help sometimes). Indeed, "Interest is the occasion for reason to become practical". Rationality comes as the universalization in the criterion. Moreover, the "utility" considered here can be valuing aid and justice, and the restriction to consistent sets or rules is a normative addition. However, Bordignon's (1990) interpretation is of the type $x_i = \lambda$, the same for all *i*, and each individual *i* maximizes $\Sigma_j u^i(y_j - x_j, X) = \Sigma_j u^i(y_j - \lambda, n\lambda)$. This maximand could be a more general function of these added elements than a sum, or perhaps a maximin in them.

$U^i$ whatever they are only, by seeking to maximize an increasing function of them, $W(\{U^i\})$. Then the maximization of $W$ secures Pareto efficiency.

In the aggregate case in which functions $U^i = u^i(y_i - x_i, X)$ for all $i$, and denoting $W_i = \partial W / \partial u^i > 0$, the maximum of $W$ requires that each contribution $x_i$ satisfies the condition

$$W_i u_1^i = \Sigma W_j u_2^j = \alpha > 0$$

or $W_i = \alpha / u_1^i$. This implies condition $\Sigma\, v^i = 1$ of Pareto efficiency for the set of contributions $x_i$.

## 2.2 The theories of the stability of justice and of consistent sharing and matching rules

### 2.2.1 General theory

***Matching or comparative rules***

Denote as $a_i \in \Re$ the $i$ th dimension of vector $a = \{a_i\} \in \Re^n$. Let $x^1, x^2 \in \Re_+^n$ be two vectors of contributions related by

$$x^2 = x^1 + \xi(x^1) \qquad (11)$$

where $\xi$ is a vector of *consistent one-to-one matched contributive variations* defined by: for given $x^1$ and all $i, j, k$, $\xi_j = f_j^i(\xi_i)$ where the $f_j^i$ are $n^2$ differentiable functions such that $f_j^{i\prime} > 0$, $f_i^i = 1$ and $f_j^i \circ f_k^j = f_k^i$ (hence $f_j^i = (f_i^j)^{-1}$) – increasing, reflexive, symmetrical, transitive. When vector $\xi$ varies, it can be written as $\xi = \varphi(x^1, \mu)$ where $\mu \in \Re$ is a parameter and $\partial \varphi_i / \partial \mu > 0$ for all $i$. Indeed, $\xi_i$ for any i, or any increasing function of it, can be taken as $\mu$. Define $\mu_i$ by $\varphi_i(x^1, \mu_i) = 0$ (corresponding to $\xi_i = 0$). Then define

$$r_i(x^1, \lambda) = \varphi_i(x^1, \lambda + \mu_i).$$

We have

$$\xi = r(x^1, \lambda)$$

where $r \in \Re^n$ is a matching rule for variation $\xi$, with a parameter $\lambda \in \Re$, $r(x^1, 0) = 0$, $r'_i = \partial r_i / \partial \lambda > 0$ (hence all $r_i$ have the same sign for given $x^1$ and $\lambda$).

***Equivalent sharing rules***

The matching rule $r$ is also a sharing rule of the difference $\Delta X = X^2 - X^1$ where $X^1 = \Sigma x_i^1$ and $X^2 = \Sigma x_i^2$ since $\Sigma r_i(x^1,\lambda) = \Delta X$ is an increasing function of $\lambda$ and hence, inversing, $\lambda = g(x^1, \Delta X)$ and therefore

$$r(x^1,\lambda) = s(x^1,\Delta X),$$

with $\Sigma s_i = \Delta X$ and $s_i$ is an increasing function of $\Delta X$ for all $i$.

***Preferred application, efficiency***

Write now for each $i$ function $U^i$ for $x = x^2$

$$U^i = U^i[y_i - x_i^1 - r_i(x^1,\lambda), x^1 + r(x^1,\lambda)].$$

For given $x^1$, the $x^2$, $\xi = r(x^1,\lambda)$ or $\lambda$ that individual $i$ prefers correspond to $\lambda = \lambda_i$ that satisfies

$$\Sigma_j V_j^i[y_i - x_i^1 - r_i(x^1,\lambda_i), x^1 + r(x^1,\lambda_i)] \cdot r'_j(x^1,\lambda_i) = r'_i(x^1,\lambda_i). \qquad (12)$$

In the aggregate case, hence with $U^i = u^i(y_i - x_i^2, X^2)$ and $V_j^i = v^i = u_2^i / u_1^i$ for all $i, j$, if the $\lambda_i$ are the same $\lambda_i = \lambda^*$ for all $i$ relation (12) for each $i$ becomes

$$v^i[y_i - x_i^1 - r_i(x^1,\lambda^*), X^1 + \Sigma r_j(x^1,\lambda^*)] = r'_i(x^1,\lambda^*) / \Sigma_j r'_j(x^1,\lambda^*). \qquad (13)$$

Summing (13) for all $i$ gives

$$\Sigma v^i[y_i - x_i^1 - r_i(x^1,\lambda^*), X^1 + \Sigma r_j(x^1,\lambda^*)] = 1, \qquad (14)$$

the Pareto efficiency condition for the public good $X^2 = \Sigma x_i^2$.

Specifications of this theory and result yield a number of important results for different problems.

### 2.2.2 The stability theory of justice

***Definitions***

*Deviation rule*

Write $x^1 = x$ in the foregoing formulas. Then a rule $r(x, \lambda)$ is a *(matching) deviation rule from x*. A deviation from $x$ according to such a rule is a *ruled deviation*. We have $r(x, 0) = 0$ and $\partial r / \partial \lambda > 0$. A deviation rule $r(x, \lambda)$ may or may not actually depend on $x$.

Deviation rule $r(x, \lambda)$ is linear when it writes $r(x, \lambda)=\lambda\rho$ where $\rho \in \Re^n / \{0\}$. Then $r_i / r_j$ for all $i, j$ and $s_i=r_i/\Sigma r_j$ for all $i$ do not depend on $\lambda$. It will be sufficient to consider linear deviation rule in the aggregate case (due to the quasi-concavity of functions $u^i$).

*Examples*

Examples of (linear) deviation rules are

$r= \lambda e$, equal deviation,

$r= \lambda y$, deviation proportional to initial income,

$r= \lambda \cdot (y - x)$, deviation proportional to disposable income,

deviation proportional to other characteristics of the individuals,

$r= \lambda \cdot (x - \hat{x})$, proportional deviation from basis $\hat{x} \in \Re^n$,

$r=\lambda x$ (i.e. $\hat{x}=0$), proportional deviation.

*Stability*

The role of matching rules in the theory of stability (section 1.2.3) is to permit or induce unanimity under their actualization. The corresponding matching stability refers, therefore, to unanimity.

A state $x$ is *stable under rule* $r(x,\lambda)$ if all participants prefer an absence of deviation according to this rule (unanimous preference for non-deviation). That is, in the foregoing notations, $\lambda_i=0$ for all $i$ under rule $r(x,\lambda)$.

A state $x$ is *unstable under rule* $r(x,\lambda)$ if there is a $\lambda= \lambda' \neq 0$ such that all participants prefer $x+r(x, \lambda')$ to $x$.

**Theorems**[36]

***6.*** *There is a stable state under any deviation rule.*

*In the aggregate case:*

***7.*** *A stable state under any deviation rule is Pareto-efficient.*

***8.*** *For any Pareto-efficient state, there exist deviation rules under which it is stable.*

***9.*** *For any Pareto-efficient state, there exists no deviation rule under which it is unstable.*

***10.*** *For any non-Pareto-efficient state, there exists no deviation rule under which it is stable.*

***11.*** *For any non-Pareto-efficient state, there exist deviation rules under which it is unstable.*

[36] Some solutions may be corner solutions.

***12.*** *With quasi-concave differentiable functions* $u^i$ *, in each Pareto-efficient state there is one and only one (in direction) linear deviation rule under which it is stable. Its coefficients are proportional to the* $v^i$ *.*

***13.*** *The stable state(s) under a deviation rule proportional to* $x - \hat{x}$ *for a given basis* $\hat{x}$ *is the Lindahl solution relative to* $\hat{x}$*, i.e.*

$$x_i = \hat{x}_i + v^i(y_i - x_i, X) \cdot (X - \hat{X})$$

*where* $\hat{X} = \Sigma \hat{x}_i$ *, for all i. For* $\hat{x}$ *=0, in particular, this is the Lindahl solution* $x_i = v^i X$.

*Proofs*

With $x^1 = x$ and $\lambda_i = 0$ for all *i*, equation (12) becomes

$$\Sigma_j V_j^i [y_i - x_i, x] \cdot r'_j(x,0) = r'_i(x,0) . \qquad (15)$$

These *n* equations for all *i* determine in general the *n* $x_i$ (theorem 6). In the aggregate case, relations (15) become

$$v^i(y_i - x_i, X) = r'_i(x,0) / \Sigma r'_j(x,0) . \qquad (16)$$

which imply $\Sigma v^i = 1$. Conversely, for any Pareto-efficient state, the deviation rules under which it is stable have $r'_i(x,0)$ proportional to the $v^i$. If $r(x,\lambda) = \lambda \cdot (x - \hat{x})$, relations (16) give $x_i - \hat{x}_i = v^i \cdot (X - \hat{X})$. The other properties are easily obtained (see an illustration figure 3).

**Figure 3. Justice as stability**

### 2.2.3 The theory of consistent sharing or matching rules

***Definitions***

Given a matching rule $r(x^1,\lambda)$, $x = r(x^1,\lambda)$ for a specific value of parameter $\lambda$ is an *application* of it.

A matching rule $r(x^1,\lambda)$ is (socially) *consistent* if all individuals prefer the same specific application of it.

Given a matching rule $r(x^1,\lambda)$, from relation (11) the matching rule obtained by replacing $x^1$ by $x^{1'}$ is

$$r(x^{1'},\lambda) = r(x^1,\lambda) + x^1 - x^{1'} .$$

In particular, for $x^{1'} = 0$,

$r(0,\lambda)= r(x^1,\lambda)+x^1$.

Therefore, with these transformations the study of rules $r(x^1,\lambda)$ can be replaced by that of rules in which $x^1$ is replaced by another $x^{1\prime}\neq x^1$, and in particular by rules $r(0,\lambda)$ which are written, for short, $r(\lambda)$.

***Individuals' consistent choices***

With rule $x=r(\lambda)$ and $U^i = U^i(y_i - x_i, x)$, individual *i* prefers $\lambda=\lambda_i$ that satisfies

$$\Sigma_j V^i_j[y_i - r_i(\lambda_i), r(\lambda_i)]\cdot r'_j(\lambda_i) = r'_i(\lambda_i). \qquad (17)$$

If rule $r(\lambda)$ is consistent, $\lambda_i=\lambda'$, the same for all *i*, and (17) becomes

$$\Sigma_j V^i_j[y_i - r_i(\lambda'), r(\lambda')]\cdot r'_j(\lambda') = r'_i(\lambda') \qquad (18)$$

for all *i*. The *n* equations (18) permit in general to determine n parameters of the consistent rule $r(\lambda)$. By symmetry, this is a priori *n* numbers $\alpha_i\in\Re$ such that $r_i(\lambda) = \hat{r}_i(\alpha_i,\lambda)$, for functions $\hat{r}_i$ and all *i*.

In the aggregate case, $U^i = u^i(y_i - x_i, X)$ and $V^i_j = v^i = u^i_2/u^i_1$. Then relations (18) become

$$v^i[y_i - r_i(\lambda'), \Sigma r_j(\lambda')]\cdot \Sigma r'_j(\lambda') = r'_i(\lambda') \qquad (19)$$

for all *i*. Adding relations (19) for all *i* gives

$$\Sigma v^i(y_i - x_i, X) = 1$$

for $x_i = r_i(\lambda')$ for all *i*, the condition for Pareto efficiency of contributions *x*.

This result can be completed by others easily shown.

**Theorems**

**14.** *If each individual contributes assuming that each other's contribution matches hers according to the same consistent rule, the outcome is the unanimously preferred application of this rule. Consistent rules have in general n real-number parameters determined by the n conditions (18).*

By symmetry, this implies that the general from of a consistent rule is $r_i(\lambda)= \hat{r}_i(\alpha_i, \lambda)$ with $\alpha_i \in \Re$ for all *i* determined by conditions (18).

*In the aggregate case:*

**15.** *The unanimously preferred application of a consistent rule is Pareto efficient.*

**16.** *Any Pareto-efficient state is the unanimously preferred application of consistent rules.*

For the Pareto-efficient state *x* these consistent rules *r* have $r'_i(\lambda')$ proportional to the $v^i(y_i - x_i, X)$.

**17.** *In particular, any Pareto-efficient state x is the unanimously preferred application of the consistent rule constituted by a Lindahl rule in which individual i contributes $v^iX$ from an income redistribution by a set of balanced transfers giving to each individual i the net* $t_i = v^i X - x_i$ *with* $\Sigma t_i=0$.

**18.** *Consistent affine matching rules $r_i=a_i\lambda+b_i$ with $a_i>0$, for all i, have, as unanimously preferred application,*

$$x_i = v^i(y_i - x_i, X)\cdot(X - \Sigma b_i) + b_i .$$

*They are Lindahl solutions from the contributions $b_i$, for the rest of the contributions. The Lindahl solution is the case $b_i=0$ for all i. The case of theorem 17 is with $\Sigma b_i=0$.*

In figure 4, the line describes a consistent rule (for instance one of theorem 17).

**Figure 4. Consistent rules**

***Consistent rules with similarities***

Families of rules with less than *n* free parameters include no consistent rule a priori and in general. This includes, for instance, equality or duplication or equal sharing ($r_i=\lambda=X/n$), contributions proportional to initial income $y_i$ ($r_i=\lambda y_i$), to final private disposable income $y_i - x_i$ ($r_i=\lambda\cdot(y_i - x_i)$) or to any other characteristic of the agents, equal final private disposable income obtained with rule $x_i = y_i - \lambda$ (any monotonic transformation of the parameter keeps the logic), and so on. Yet these cases include classical rules. However, similarities in the individuals' relevant characteristics (functions $U^i$ or $u^i$ and $y_i$) may permit consistent rules with less than *n* free parameters. In particular, the *n* equations (17) may be the same. This happens in the aggregate case in some cases in which the functions $u^i$ are ordinally the same. The only remaining difference between the individuals and between their respective equations is that in their income $y_i$. This difference can be eliminated in three ways, two of which are with more specific given structures. Denote $v= v^i$ for all *i*.

**Theorem 19.**

*If individuals' utilities are ordinally the same with* $u^i = u(y_i - x_i, X)$ *as common specification, denoting* $v=v^i$ *for all i, there is a consistent rule in the following cases.*

*1) The chosen contributions equalize the remaining incomes, which can be written as* $x_i = r_i = y_i - \lambda$ *for all i, and*

$$v(\lambda', Y - n\lambda') = 1/n \tag{20}$$

*where* $Y=\Sigma y_i$ *is total income.*

*2) If the* $y_i$ *happen to be the same,* $y_i=\eta$ *for all i, equal* $x_i=\lambda$ *provide the solution, with*

$$v(\eta - \lambda', n\lambda') = 1/n \tag{21}$$

*(Laffont, 1975).*

*3) If function u is quasi-linear,* $u = y_i - x_i + w(X)$, $v= w'(X)$ *and an additive rule* $x_i=b_i+\lambda$ *for all i gives*

$$w'(\Sigma b_i + n\lambda') = 1/n\,. \tag{22}$$

With a strictly quasi-concave increasing function *u*, equations (20), (21) and (22) have a unique solution $\lambda'$ (equations (20) and (21) amount to maximizing this function $u(z, X)$ under the constraint $nz+X=Y$).

**2.3 The aggregate public-good specificity of the unanimity-efficiency coincidence.**

Theorems 4 and 5 show the following result.

**Theorem 20**. *The coincidence between unanimity and efficiency under variational (stability) or consistent matching or sharing rules is characteristic of the aggregate public-good structure of the interaction.*

**2.4 Other behaviour**

***2.4.1 Strategic stability: the core***

Another concept of stability is the core. For joint solidarity, this is the core for public goods without exclusion. Each person appreciates the contributions provided by others, whether or not they belong to the same coalition. There is efficient cooperation within coalitions and non-

cooperative relations between coalitions (e.g. Cournot-Nash, Stackelberg). This has been studied fully.[37] The outcome is Pareto-efficient by nature. The concept of the core is not moral by itself, but it can apply to contributions for a moral objective.

### *2.4.2 Other matching behaviours*

Matching contributions to public goods, of a type different from the one-to-one intrinsically consistent (section 2.3.1) structure considered above, exist and have been studied. Sugden (1984) considers that individuals reciprocate to others' contributions by contributing an amount which is a function of the set of these contributions whatever they are, from a moral motive of contributive fairness, with a Cournot-Nash contributive equilibrium. This moral behaviour avoids free riding but does not implement Pareto efficiency.

Guttman (1978, 1987), Danziger and Schnytzer (1991) and others consider a two-stage contributive process (a non-ethical one, but applicable to ethical public goods). People first announce that they will match others' contributions at some rate, and then contribute (or buy some quantity of the good) in addition to paying the matching sums. Under some conditions, the outcome is the Lindahl equilibrium, hence a Pareto-efficient one. However, this outcome constitutes a particular distribution (but this is also the case for market outcomes). More importantly, each individual is more satisfied by playing differently than as indicated (see, for instance, Jackson and Wilkie, 2002). Therefore, the fact that people play this game has to be secured by public force or by some norm of behaviour of whatever origin.

### *2.4.3 "Warm-glows"*

The foregoing has considered cooperative reduction of inequality or poverty. Public transfers are more or less accompanied by private gift-giving. Such gifts by agents who are just both benevolent and self-interested (without specifically cooperative behaviour) are crowded out by efficient public transfers. The existence of private gifts can be explained by various motives including social norms, cooperative duty, and a preference for one's own gift called "warm-glow" although this does not describe all cases. However, (1) *one cannot be praiseworthy or praised as a moral person if seeking this judgment is the motive for the contribution*. Nevertheless, (2) *some value may be attached to the fact that some of one's wealth contributes to the virtuous end*. Then, however, (3) *this should include jointly the gift provided and the distributive tax paid by the person*, and (4) *this is shown to prevent the*

[37] Kolm, 1987.

*"warm-glow" explanation of non-crowding out.* [38] Non-crowding out by efficient public transfers can only be explained, paradoxically, by the fact that *some other people* prefer the person's *total contribution* – gift plus tax – to be *lower* (for reasons of comparative sentiments such as envy, sentiments of inferiority or superiority, inequality aversion, preference for conforming, etc.).[39] The difficulties in "warm-glow" explanations show the importance of moral motives concerning both the objective and cooperation.

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[38] Kolm (2008b).
[39] Idem.

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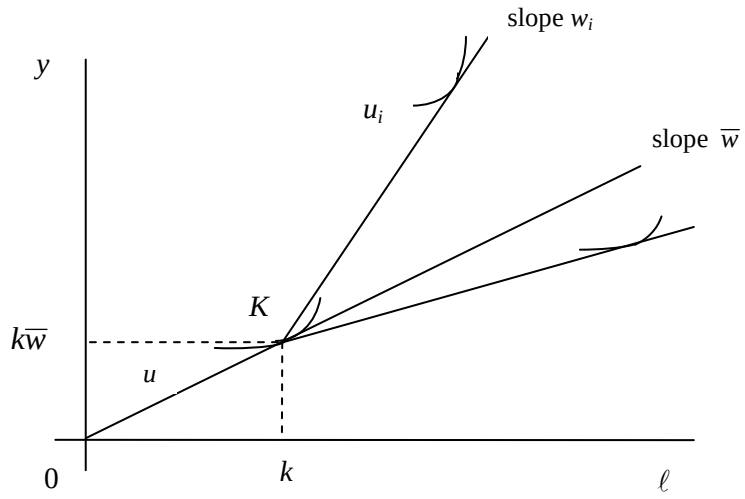


**Figure 1. The two-part income: Equality and liberty**

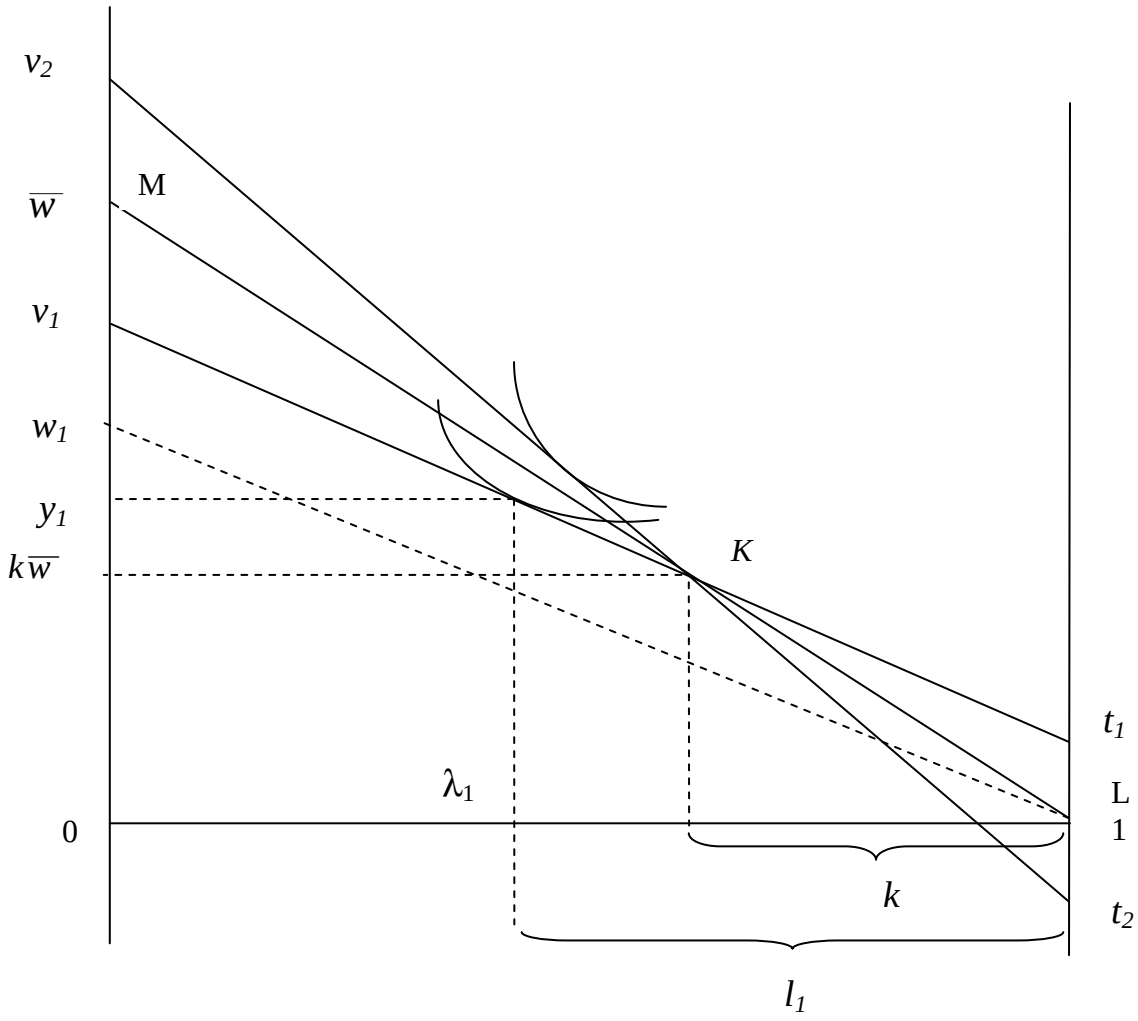


**Figure 2. Equal-freedom budget sets**

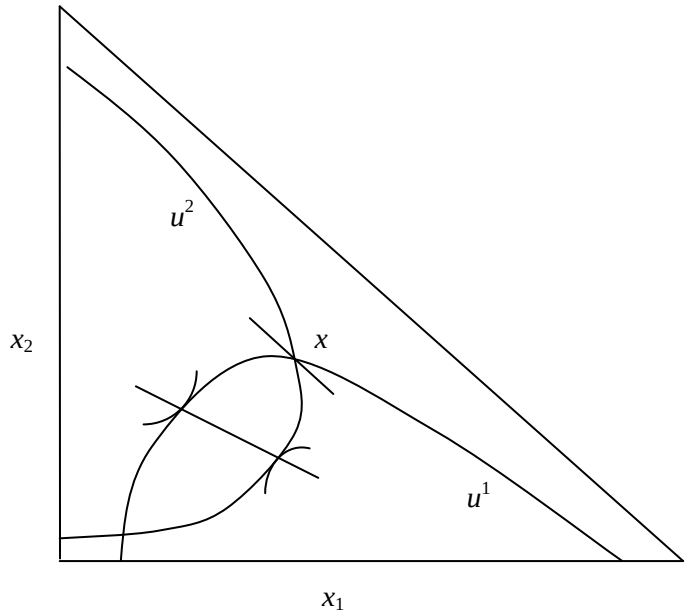


**Figure 3. Justice as stability**

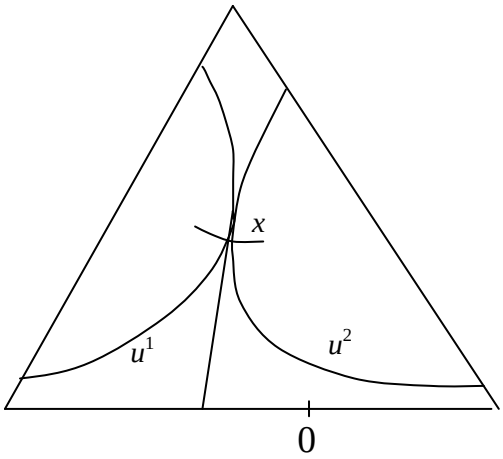


**Figure 4. Consistent rules**