

THE ECONOMICS OF SOCIAL SENTIMENTS: THE CASE OF ENVY

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The widespread externality produced by the sentiment of envy is modelled in this paper. We show its specific structural properties, the conditions of its existence, the various reasons for its normative relevance, the basic tool of its analysis ("envy-free preferences") and its crucial relation with the property of equality of liberty ("equity"). We then show that the envy externality respects the efficiency of classical allocative processes in the relevant conditions (competitive markets with not too unequal incomes, cases with indivisibilities, equal-freedom individual choices, etc.). We in particular derive conditions of consistency between efficiency and the vanishing of the envy externality, as well as the effects of actual envy on "divide and choose" processes, preferences to reassignment, multidimensional maximin, "egalitarian equivalence", degrees of envy in society and the rationality of choices.

1. The economics of envy

1.1 Introduction: the puzzle in modelling envy

1.1.1 *The economic importance of social sentiments*

"Social sentiments" such as envy, jealousy, various sentiments of justice and of injustice, the desire to conform or on the contrary to distinguish oneself from others, conspicuous consumption or consumption for dignity or self-respect, sentiments of inferiority or of superiority, benevolence, altruism, charity, compassion, gratitude, malevolence, spite, *schadenfreude*, and the like, are very widespread. They play a major role in social life and in particular an important role in economic life. They substantially influence the behaviour of people, the demands for goods and their output. They are important ingredients of the misery and satisfaction of people, of the value of their relations and of the overall quality of society. They therefore are important for economics, both for explaining consumption, production, or behaviour in organizations, and for the normative evaluation of society. Furthermore, the traditional technical tools of economics can importantly shed light on the effects and consequences of these various sentiments, as it is shown by a few studies (including, we hope, the present one). Indeed, there have been technical economic studies of the structure and consequences of a few of these sentiments, namely altruism, sentiments of reciprocity, envy and conspicuous consumption.¹⁾ However, this field seems to me to be extremely under-studied by economists, given the importance of the topic and the analytical possibilities. If this neglect resulted from a conception of

1) The case of envy is discussed below (see Kolm 1991b and 1993d). The social sentiment most studied by economists is altruism (the literature is too vast to be quoted; see in particular Kolm (1964, 1966) and the most extensive study in Kolm (1984a)). The sentiment of reciprocity is the main topic of Kolm (1984a) (see also an application in Akerlof (1982)). The sense of justice is the topic of Kolm (1966). For conspicuous consumption, see Kolm (1971a).

the division of labour between economists and psychologists or socio-psychologists – the topic would belong to the latter's domain – this would obviously constitute a scientific mistake since economists both are concerned by the effects on their field of study and are the specialists of very relevant conceptual tools. This paper is meant to help redress this imbalance, in presenting the analysis of the particular, yet particularly important, sentiment of *envy*, “the most odious and antisocial of sentiments” (J. S. Mill (1859)).

Now, it may seem that this sentiment constitutes the exception, since an important and well-known economic literature deals with what it often calls “no-envy”. Yet, we will see, this literature essentially misspecifies the sentiment of envy; however, the criterion it considers is very important for another reason, it has a certain relation with envy correctly modelled, and the various properties of this criterion have important, richer and much less trivial equivalents for envy seriously considered.

1.1.2 The nature and structure of envy

Indeed, this literature defines “no-envy” as the fact that no individual prefers any other's allocation to his own.²⁾ It says (or implies) that I envy you if I prefer what you have to what I have. However, I commonly prefer what you have to what I have without experiencing any sentiment of envy in the normal and common sense of the term. I may not be prone to such a sentiment, in particular because I am a “good” person and envy is a nasty sentiment to have, or I may not experience it in this particular case, either because I like you and enjoy that you have or enjoy a good situation, or because I deem that either our persons, or our considered allocations, are not sufficiently comparable. Furthermore, if, on the other hand, I envy what you have, or I envy you for what you have, in the *most common* use of the term *envy* this implies that I experience a sentiment of envy that is disagreeable (sometimes much so) and, therefore, that lowers my utility (or preference) in an “economic” formulation of the phenomenon. My utility or preference would then depend upon *both* your allocation³⁾ and mine, that is to say, this envy is a *negative consumption externality* of your allocation upon me. Much less frequently, “envy” describes more subtle sentiments that do not entail this negative externality.⁴⁾ Yet,

2) The original literature on this topic is however more cautious. The original proposition of this criterion in economics is due to Tinbergen (1946) who calls it the “principle of transfers”. Tinbergen however discusses how nice a society with the resulting absence of envy would be (see Pen (1971)). Foley (1967) just mentions the criterion. The scientific study of this criterion, starting with the book *Justice et équité [Justice and Equity]* (Kolm (1971b)), began by calling it Equity, although this book also discusses its relation to envy and jealousy.

3) We use the following vocabulary. An *individual allocation* is what an individual could have (they are exclusive: an individual can have only one). An *individual's allocation* is the individual allocation that an individual actually has. An *allocation* is a set of individuals' allocations, one for each individual.

4) The existence of these cases where the word “envy” is used and the disagreeable sentiment is absent is fostered by the impossibility for the envious person to have or to be what he envies (e.g., “I envy your youth”), although this is neither a necessary nor a sufficient condition. An instance of this class of sentiments is provided by “admiring envy” (that can even be accompanied by an agreeable sentiment produced by the knowledge of the admired situation or performance – this sentiment may accompany envy but it *is not* envy). In another case, the negative externality of envy that I would experience in comparing a subset of our allocations (e.g., our houses, given that I prefer yours to mine, possibly in the particular sense defined below) does not manifest itself because I prefer my overall allocation or situation to yours.

we focus here on the kind of envy that entails this externality, for several reasons: first, it is the standard and most common case and a very important social sentiment; second, this is the kind of envy that has important normative implications because this is envy that spoils both individual welfare and the quality of society, and that is commonly condemned on moral grounds. We may call this sentiment “strong envy”.⁵⁾ Now, with such an envy externality, first, the “no-envy” studies that omit this externality consider only no-envy by construction, and therefore they cannot take the absence of envy as a criterion; second, with the externality the expression “I prefer your allocation to mine” is not even a priori defined – we will however provide below the adequate definition. Let us also remark that this externality generally implies that an envious individual would be less dissatisfied if the other’s allocation he envies worsens in his eyes (a “malevolence” of particular form and reason that exists only when envy is actual); yet, a moral sentiment that condemns rejoicing from others becoming worse off often superimposes in such cases (the results obtained below do not depend on the presence or absence of such a sentiment and of its effects). We will also consider below the normative uses of all these concepts.

1.1.3 The envy externality and its limits

However, another classical economic literature does consider the negative externality of envy. It often models this structure, and it sometimes uses this model for determining optimal taxation. This includes Mishan (1960) and his reference to Duesenberry (1949), Kolm (1966, and 1971a) for the introduction of optimal taxation, Scott (1972), Brennan (1973), Hirschman (1973), Boskin and Sheshinski (1978), Villar (1988), Nieto (1991). The same view leads Goldman and Sussangkarn (1983) to express that the previously discussed “no-envy” studies misspecify envy, and that, consequently, all their results are mistaken, notably that of the existence of Pareto-efficient and envy-free allocations. This basic criticism is well taken, yet we will see that it does not entail the conclusion that these authors derive from it, because of the particular structure of the envy externality. That is, results similar to those obtained with “misspecified envy” hold when envy is correctly modelled; these actual results are, however, more varied and subtle, much less trivial and more delicate to obtain, and actually meaningful in terms of envy. For instance, the possibility of the envy externality does not prevent the existence of Pareto-efficient competitive equilibria, with otherwise classical conditions; it only adds, for this result, the requirement that incomes are not too unequal (this thus is a case where equality is favourable to efficiency); there is, in fact, no actual envy (only its possibility) at these competitive equilibria. The crucial point is indeed the delineation of the limits of the actual existence of envy, that the “no-envy” studies want to emphasize while they cannot really express it, but that the (more) correct formulation of envy as an externality cannot express directly. Furthermore, the criterion

5) The sentiment of *jealousy* implies strong envy plus other elements that we do not consider here (and that cannot be described by mere preferences). One of these elements is a certain sentiment of debasement of the “self” (akin to Freud’s “narcissistic wound”). Another common element is the sentiment that it would be normal, proper or due to me to have what I am jealous of; indeed, the name “jealous” also sometimes merely describes an attitude of defence of one’s advantage that one deems to be legitimate.

that no individual prefers another's individual allocation to his own will be shown to be very important – and hence its properties are also very important – yet for an altogether different reason.

1.1.4 Presentation

This article presents the basis of the economic theory of envy, namely its central concepts and properties. Its analytical core is the structure that constitutes the solution to the apparent contradiction in the logic of envy discussed above, namely: (1) I can envy you only if I prefer what you have to what I have; (2) envy is an externality whereby I am affected by what you have; (3) this externality prevents the direct formulation of condition (1). This structure is the concept of *envy-free preference*. An individual's envy-free preference (or *intrinsic preference*) is derived from its possibly envious preference in assuming that the others' allocations are always identical to his own. Then, an individual can be envious only if he envy-free prefers (or intrinsically prefers) some other's allocation to his own. When envious, he is less happy than if he were not, hence with his envy-free preference. This constitutes a rich and meaningful structure that entails many important properties.

The rest of Part 1 prepares the ground in considering the possible roles and the importance of envy in *normative economics* (Section 1.2), the property of *Equity* as equality of liberty (Section 1.3), and the basic concepts and properties in the *example of two individuals* (Section 1.4). Part 2 presents the general theory of envy, namely the basic concepts, the fundamental lemma, the basic property concerning Pareto-efficiency (envy-free allocations that are Pareto-efficient with the envy-free preferences also are with the actual preferences), the existence of envy-free and Pareto-efficient allocations in a number of cases (in particular convex production sets and preferences), the case of equal-freedom individual choices, the question of unanimous changes from envy-free or equitable allocations, the equivalence with unanimous preference to reassignments, "super-equity" and egalitarian multidimensional maximin, process "divide and choose" with envy, effects of indivisibilities, "egalitarian equivalents", etc. Part 3 considers the structure and conditions of envy, related sentiments and applications. The longest mathematical proofs are relegated to the Mathematical Appendix.

1.2 Envy in normative economics

For its *normative* applications, this analysis provides answers for the five possible ethical views of envy.

(1) Both envy and justice are closely related to equality. Equality prevents envy, and, indeed, equality (identity) of the domains of choice prevents envy (see Sections 2.7 and 2.8). On the other hand, justice requires equality of something (see Section 1.3). Indeed, claims of justice are commonly accused of being motivated by envy, and this is not always calumny. The noblest and the vilest of social attitudes and sentiments thus maintain puzzlingly close relations that should be clearly understood.

(2) No-envy may be a valid ethical criterion because it fosters the absence of recrimination, the quality of society, the manifest virtue of its members, the value of

their relations, and social peace and cooperation. Then, when the possibilities of moral education and suasion are exhausted, it may be valid to advocate allocations that lower or suppress envy, and the various results obtained below concerning *efficient envy-freeness* are normatively important. This can be seen as in line with foremost ethical expertise, since Kant (1797) insists that “only the manifestation of envy makes it an abominable vice”, while “the impulse for envy is inherent in the nature of man”.⁶⁾

(3) Criteria using individuals’ preferences, utilities or welfares may be used (Pareto-efficiency is one of them, but distributional ones may be added). Then, one must consider that envy lowers satisfaction and induces people to care for others’ allocations, and knowing the particular structure of the envy externality is important.

(4) Envy may also on the contrary be considered irrelevant for allocation – for instance, my envying your wealth may not be a good reason for transferring some of it to me. Then preferences or utilities for normative use should be cleaned, “laundered” or “ironed” so as to eliminate the influence of envy (see Goodin (1986) for a general view on preference laundering). Now, these clean preferences happen to be well defined in the case of envy, and they are precisely the envy-free preferences defined below.

(5) Finally, the normative evaluation may be unconcerned with individuals’ sentiments or welfares – it can for instance rest on equal liberties or on group values – and in this case as in all others the theory of envy is necessary for explaining and forecasting behaviour.

1.3 Equity as equality of liberty

In the absence of externality and of envy, the criterion that no individual prefers another’s allocation to his own is a very important one, because it is the central concept of *equality of liberty*, which is the general form of individualistic justice.⁷⁾ We will define below a similar property for the case where envy is a priori a possibility. The basic property is indeed the following theorem, for the case where there a priori is no externality and no envy.⁸⁾

No individual prefers another’s allocation to his own if and only if there exists a domain of individual choice such that, if a domain of choice identical to this one is attributed to each individual, then the considered allocation is a possible resulting outcome of these choices.

Indeed, if such a choice is proposed to each individual, he does not prefer any other’s resulting allocation since he could have chosen an identical one. Conversely, if no individual prefers another’s allocation to his own, there exist domains of individual choice having the considered property, since *the set of unallocated individual allocations* obviously constitutes such a domain.

This criterion is therefore called *Equity*, for *EQUal Independent Instrumental liberTY*, and also because Equity merely comes from the Latin word for “equality”

6) Of course, Kant is also likely to refer to the “autonomy” of controlling one’s own sentiments.

7) See Kolm (1971b, 1973, 1993a and b, 1994a and b, 1995a and b).

8) Same references.

and equal liberty in a broad sense of the term is the general form of justice;⁹⁾ this was indeed the name used in the original analyses of this concept. “Instrumental” means that liberty is valued here for its instrumental value, as a means for what it enables the free agent to obtain; for the individuals, this means that their direct preferences bear only on their allocations rather than also on the domain of choice *per se*; this is a very important case, although certain other freedoms may also be valued for themselves.¹⁰⁾ “Independent” means that an individual’s domain of choice does not depend on other individuals’ choices; that is, we consider here no externality, neither in consumption nor in “production” (i.e., an effect on the domain of choice); the corresponding concept with dependent liberties is in part built up from this simple case (Kolm, 1993b). Equal liberty is a principle that *takes freedom as the end-value of justice*. This is equivalent to considering *individuals’ preferences as irrelevant for justice, contrary to the appearance of the expression of the Equity criterion* (where individual preferences appear because of their effects on individuals’ free choices). This irrelevance is for instance argued by Rawls (1971) and Dworkin (1981), and, although these authors are mistaken in defending it as a universal principle (as counter-examples can show), it is valid in an important set of cases. Finally, the ideal equality or identity of the objects of justice relative to individuals when no other characteristics of these individuals are deemed to have ethical relevance for justice, results from a property of *rationality* in the normal sense of the term rational – for a reason; indeed, no reason could be given for choosing an unequal allocation rather than any of its permutations among the individuals since these allocations are indiscernible, while they differ and the situation of choice implies that one allocation has to be chosen (of course, the possibilities, and the possible additional presence of other relevant criteria, may overwhelm this *prima-facie* reason for equality and lead finally to accepting certain inequalities in these objects).¹¹⁾

This importance of Equity validates the analyses of this concept and of its properties. This validation owes nothing to “envy”. Yet, in addition, Equity is indeed closely related to envy when it is correctly modelled, as it will be shown shortly.

1.4 An example in the economics of envy

The basic concepts are now presented and illustrated with the simplest example. If individual 1 with allocation x_1 (of any nature) and utility function $u^1(x_1)$, prefers individual 2’s allocation x_2 to his own, this is represented by $u^1(x_2) > u^1(x_1)$. But if individual 1 *envies* individual 2 for his allocation x_2 , he *remains concerned by his own allocation* x_1 , yet he is *also concerned by individual 2’s allocation* x_2 , so that his

9) See Kolm (1993a and b, 1994a and b).

10) See the analyses of the value of liberty in Kolm (1982, 1984b, 1995a).

11) Choosing by lottery is not an alternative since (1) it does not constitute a valid reason since it rests on a causality that is irrelevant to the problem (such as the dynamics of falling bodies in the flipping of a coin), and hence it could be only a last resort, and (2) the chosen probabilities have to be justified – equal probabilities can be justified, but by the same general reasoning concerning equality (or there is an infinite regress in lotteries). More detailed presentations of the rational necessity of equality can be found in Kolm (1993a and b, 1994a and b, 1995a and b).

utility function seems to have to be written as $U^1(x_1, x_2)$. This shows the “consumption externality”. We now make precise that this externality is *envy* and not any other kind, and we assume that it is the only externality that is present. We indeed have to emphasize that this section and the whole of Part 2 consider envy and *not* any other kind of externality and social sentiment (other externalities and social sentiments are mentioned in Part 3).

Individual 1 does not envy individual 2 if $x_2 = x_1$, i.e. individual 2’s allocation is the same as his own. We thus *define* individual 1’s *envy-free* (or “*intrinsic*”) *utility function* from his actual utility function $U^1(x_1, x_2)$ as

$$u^1(\xi) \equiv U^1(\xi, \xi),$$

where ξ is any individual allocation. Individual 1 “*envy-free prefers*” (or “*intrinsically prefers*”) individual allocation ξ to individual allocation ξ' when $u^1(\xi) > u^1(\xi')$. This expresses individual 1’s preferences in the absence of any sentiment of envy.

Furthermore, we focus on the most important kind of envy, “*strong envy*”, the sentiment that is usually considered with moral reprobation, that underlies jealousy, and that is disagreeable (sometimes very strongly). This disagreeableness is what can be grasped by preferences or utility functions. It can indeed be expressed as $U^1(x_1, x_2) < U^1(x_1, x_1)$ (since individual 1 cannot – hence does not – envy individual 2 when $x_2 = x_1$), or $U^1(x_1, x_2) < u^1(x_1)$, and this inequality implies envy since this sentiment is the only possible reason for it by assumption. Conversely, $U^1(x_1, x_2) = u^1(x_1)$ if and only if individual 1 experiences no envy with this allocation. In all cases, $U^1(x_1, x_2) \leq u^1(x_1)$. Furthermore, since an individual cannot envy an individual allocation that he does not intrinsically prefer to his own,

$$u^1(x_1) \geq u^1(x_2) \Rightarrow U^1(x_1, x_2) = u^1(x_1).$$

Note that $U^1(x_1, x_2) = u^1(x_1)$ with $u^1(x_2) > u^1(x_1)$ just means that individual 1 has no psychological propensity to the sentiment of envy, at least for these values of x_1 and x_2 (he may have for others¹²⁾). If individual 1 experiences no envy with the allocation (x_1, x_2) , we can say both that “he is envy-free with it” and that this allocation is “envy-free for him”¹³⁾; this amounts to $U^1(x_1, x_2) = u^1(x_1)$ and it is implied by $u^1(x_1) \geq u^1(x_2)$. An allocation is then “envy-free” if it is envy-free for all the individuals.

As a matter of illustration, this shows directly why the conclusions of Goldman and Sussangkarn do not hold (although their basic tenet that envy is an externality is essentially well taken). Indeed, the following property holds:

An envy-free allocation that is Pareto-efficient with the envy-free preferences, is Pareto-efficient with the actual, possibly envious, preferences.

Indeed, define U^2 and u^2 for individual 2 as U^1 and u^1 were defined for individual 1. The envy-free and possible allocation (x_1, x_2) is Pareto-efficient with the envy-free utilities u^1 and u^2 if and only if, for all other possible allocations

12) Section 3 below analyses the conditions of envy.

13) Literally, only an individual can be envious, and hence the expression “envy-free” or “without envy” is informative only if applied to individuals. Yet, the expression “an envy-free allocation” for designating an allocation that does not arouse envy has become common in economics (although not for actual envy), it only amounts to adjoining to the allocation *stricto sensu* the sentiments it arouses, and it is practical, and unambiguous when envy is correctly modelled.

(x'_1, x'_2) , we have either $u^1(x'_1) < u^1(x_1)$ or $u^2(x'_2) < u^2(x_2)$ (or both), or $u^1(x'_1) = u^1(x_1)$ and $u^2(x'_2) = u^2(x_2)$. Then previous relations, taking account that (x_1, x_2) is envy-free while (x'_1, x'_2) may or may not be, show that either

$$U^1(x'_1, x'_2) \leq u^1(x'_1) < u^1(x_1) = U^1(x_1, x_2),$$

or

$$U^2(x'_2, x'_1) \leq u^2(x'_2) < u^2(x_2) = U^2(x_2, x_1),$$

or both, or

$$U^1(x'_1, x'_2) \leq u^1(x'_1) = u^1(x_1) = U^1(x_1, x_2)$$

and

$$U^2(x'_2, x'_1) \leq u^2(x'_2) = u^2(x_2) = U^2(x_2, x_1).$$

This third possibility (the latter two relations) implies in turn either $U^1(x'_1, x'_2) < U^1(x_1, x_2)$, or $U^2(x'_2, x'_1) < U^2(x_2, x_1)$ (or both), or $U^1(x'_1, x'_2) = U^1(x_1, x_2)$ and $U^2(x'_2, x'_1) = U^2(x_2, x_1)$. This proves that the allocation (x_1, x_2) is Pareto-efficient with the actual utilities U^1 and U^2 , hence the proposition.

Now, in the classical case of the division of given quantities of divisible commodities with *non-envious preferences*¹⁴⁾ represented by utility functions u^1 and u^2 , there exist allocations that are both such that $u^1(x_1) \geq u^1(x_2)$ and $u^2(x_2) \geq u^2(x_1)$, and Pareto-efficient (see, for instance, the Edgeworth box in Kolm (1971b), Crawford (1977), Thomson (1982), or Baumol (1986)). Applying this result to the envy-free utility functions of the actual, possibly envious, individuals, with the foregoing proposition, shows the existence of *efficient envy-free* allocations.¹⁵⁾

Furthermore, in this case certain of these "equitable" and efficient allocations with non-envious preferences are also unanimously preferred or indifferent to equal sharing (same references). Applying this result to the envy-free utility functions and calling Q the vector of total quantities provides allocations such that $u^1(x_1) \geq u^1(x_2)$, $u^2(x_2) \geq u^2(x_1)$, $u^1(x_1) \geq u^1(Q/2)$, $u^2(x_2) \geq u^2(Q/2)$ and that are Pareto-efficient. Reverting to the actual, possibly envious, preferences thus yields $U^1(x_1, x_2) = u^1(x_1) \geq u^1(Q/2) = U^1(Q/2, Q/2)$, and similarly for individual 2. There thus exist allocations that are at once *envy-free, Pareto-efficient and unanimously preferred (or indifferent) to equal sharing*.

We show below that these results extend to the case with *production* in the usual sense of possibilities of transformation among various goods, and any number of individuals and commodities (with the required convexities).

14) By "non-envious preferences" we mean the classical preferences that present no envy externality for any allocation. This is to be distinguished both from the "envy-free preferences or utilities" defined in the text (for possibly envious preferences), and from possibly envious preferences in a domain of allocations where the individual experiences no envy. Of course, non-envious preferences coincide with their own envy-free preferences.

15) For this case of two individuals, the result could be obtained in applying the classical results to the following utility functions where $Q = x_1 + x_2$ is the given allocation: $v^1(x_1) = U^1(x_1, Q - x_1)$, $v^2(x_2) = U^2(x_2, Q - x_2)$. Yet, this cannot be done for more than two individuals whereas the presented proof can straightforwardly be extended to this case. Note that Goldman and Sussangkarn work with examples of two individuals and functions U^i with particular forms. Yet, they omit in particular that the envy externality disappears when $u^1(x_1) \geq u^1(x_2)$, for individual 1.

2. The economic theory of envy and applications

2.1 General concepts

There are n individuals indexed by i, j, \dots from 1 to n .

$x_i \in X$ is individual i 's "allocation" (of any nature).

We note $x = \{x_i\} \in X^n$.

Each individual i has a preference preordering R_i of the x 's in X^n . The fact that it is for the $x \in X^n$ rather than only for the $x_i \in X$ expresses the possibility of externality.

P_i and I_i are the anti-symmetric (strict preference) and the symmetric (indifference) parts of R_i (xR_ix' is either xP_ix' or xI_ix').

The topic of this paper is (strong) envy. Therefore, *by assumption, the only externality that is possibly present is (strong) envy.*

$e\xi \in X^n$ denotes an n -tuple each element of which is $\xi \in X$.

The *envy-free* (or "intrinsic") preference preordering of individual i is a preordering r_i of the $\xi \in X$ defined as

$$\xi r_i \xi' \Leftrightarrow e\xi R_i e\xi'.$$

p_i and i_i are respectively the anti-symmetric (strict preference) and the symmetric (indifference) parts of r_i .

If individual i envy-free prefers no allocation of any other individual to his own, he cannot be envious. Hence,

$$x_i r_i x_j, \forall j \Rightarrow x I_i e x_i,$$

and

$$\{x_i r_i x_j, x_j r_j x'_j, \forall j\} \Rightarrow \{x R_i x' \Leftrightarrow x_i r_i x'_i\}.$$

Since envy is not an agreeable sentiment,

$$e x_i R_i x.$$

Individual i 's "envy-free domain" is

$$F_i = \{x: x I_i e x_i\} \subseteq X^n$$

(since only "strong" envy is considered).

Individual i 's "equitable domain" is

$$E_i = \{x: x_i r_i x_j, \forall j\} \subseteq F_i \subseteq X^n.$$

Society's "envy-free domain" is $F = \bigcap F_i \subseteq X^n$, and society's "equitable domain" is $E = \bigcap E_i \subseteq F$. The allocations of F are called *envy-free* and the allocations of E are called *equitable*. An equitable allocation is envy-free (the converse may not hold).

The main economic properties of envy result from the following basic properties.

2.2 The basic lemma

Lemma. Assume $x \in F_i$ and $x' \in X$. Then, (1) $x_i r_i x'_i \Rightarrow x R_i x'$, $x_i p_i x'_i \Rightarrow x P_i x'$ and $x_i i_i x'_i \Rightarrow x R_i x'$; (2) if $x' \in F_i$, $x_i i_i x'_i \Rightarrow x I_i x'$; (3) if $x' \notin F_i$, $x_i r_i x'_i \Rightarrow x P_i x'$ and $x_i i_i x'_i \Rightarrow x P_i x'$.

The proofs are obtained from the foregoing definitions and hypotheses (for instance, for the first of the six relations, xI_iex_i since $x \in F_i$, $ex_iR_iex'_i$ if $x_i r_i x'_i$ from the definition of r_i , and $ex'_i R_i x'$ as noted above, hence $xR_i x'$; and so on).

We are now in position to obtain the main properties of an economy with possible envy.

2.3 Pareto-efficiency

Proposition. *Envy-free allocations that are Pareto-efficient with the envy-free preferences are Pareto-efficient with the actual preferences.*

Proof. Let $S \subseteq X^n$ denote the possibility set for which we consider Pareto-efficiency and x denote an allocation that satisfies the premisses of the proposition. Then, x being "envy-free Pareto-efficient" implies $x \in S$, and for all $x' \in S$ either $\{\exists i: x_i p_i x'_i\}$ or $\{\forall i: x_i i x'_i\}$. Furthermore, $x \in F \subseteq F_i$ for all i . The lemma thus gives $x_i p_i x'_i \Rightarrow x P_i x'$ and $x_i i x'_i \Rightarrow x R_i x' \Leftrightarrow \{\text{either } x P_i x' \text{ or } x I_i x'\}$, for all i . Therefore, $x' \in S \Rightarrow$ either $\{\exists i: x P_i x'\}$ or $\{\forall i: x I_i x'\}$.

QED

Note that there may also exist Pareto-efficient allocations (with the actual preferences) that are not Pareto-efficient with the envy-free preferences, and such allocations may in particular be envy-free (they then are Pareto-efficient on the set of envy-free allocations).

This proposition permits one to obtain conditions for the existence of envy-free, or equitable, and Pareto-efficient allocations with possibly envious preferences, from conditions with non-envious preferences. Let us note that, with several divisible goods, convexity of preferences is often used; now, with possible envy, the envy-free preferences have the same reasons to be convex (or not) as ordinary non-envious preferences have.

2.4 Classical economies

The central case is the classical one of quantities of divisible private goods with possibilities of production and transformation (the case of given total quantities is a particular sub-case). The only non-classical feature is the possible presence of envy, and therefore of individual preferences with the envy externality. An "equal allocation" denotes an allocation with identical individual allocations. "No-satiation" of non-envious preferences means no local satiation in all commodities at once. Then, the following property holds.

Proposition. *With possible envy, convex production possibilities and convex, non-satiated envy-free preferences, there exist allocations that are: envy-free; equitable; Pareto-efficient; such that no individual prefers any possible equal allocation, and any replacement of the individual allocations by linear combinations of the former ones (in particular any permutation of the individual allocations or any replacement by weighted averages); with equal incomes at the efficiency prices; and implementable as a competitive equilibrium.*

Indeed, consider the *classical theory of general equilibrium* – e.g. Debreu (1959) – with individual preferences that are the *envy-free preferences*, and with *equal sharing of all initial resources and firms' profits among the individuals*. A competitive (Walrasian) equilibrium exists, it is *Pareto-efficient with these preferences*, and all individuals have the *same income produced with these resources, hence the same budget set with the efficiency prices*. From the consumers' equilibrium, no individual envy-free prefers any allocation of this budget set to his own. Hence, first, the allocation is *equitable*, hence *envy-free*, and *Pareto-efficient* (with the actual preferences) from Section 2.3. Second, from the first proposition of the lemma, *no individual prefers an allocation that gives him an individual allocation in this common budget set*, to the equilibrium allocation. We consider two types of such allocations.

(1) One type is the case where the individual allocations are *in the equilibrium budget hyperplane*. This is in particular the case of linear combinations of the equilibrium individual allocations (this includes averages – possibly weighted ones – and permutations). (2) Another case is that of *possible equal allocations*. Indeed, the set of individual allocations of possible equal allocations is the homothetic reduction by $1/n$ of the general possibility set, from the origin. This set is convex (as the general possibility set is). Its border (the homothetic reduction of the general possibility frontier) passes by the arithmetic average of the equilibrium individual allocations (the homothetic reduction of the total quantities) which is also in the common budget hyperplane, and this common budget hyperplane is tangent to this set at this point since the efficiency price vector is an outside normal to this set at this point (as it is to the homothetic general possibility set at the total production point). Hence this set is in the common budget set.

QED

Note that only convexity of the overall production possibilities is needed (rather than for each firm), since one can notionally consider that all the production is undertaken by a unique big firm. Furthermore, the above results apply to any problem that can be taken as a particular specification of the general model considered above (number of individuals, given quantities, etc.).¹⁶⁾

A notable consequence of this result is that the envy externality does not interfere

16) In particular, they can apply to problems where there is no wage income and human resources. They can also apply to problems with human resources and wage incomes. Then, if individual capacities are identical, equal division of resources is naturally obtained by "each one owns himself". When these capacities differ, one unit of time of different individuals are different commodities. Equal sharing of resources then implies that each individual receives $1/n$ of the time of each other, that he can sell on the labour market or to this other individual who can also use it as leisure (or that he can keep as direct personal services). This, however, poses problems for the concepts. The leisure times of different individuals are a priori different commodities, each actually consumed by only one individual. Yet, in this case an efficient allocation is trivially equitable and envy-free since an individual's leisure time is allocated only to this individual, who cannot prefer any other's allocation that contains no leisure of his while he must have some leisure (e.g. for sleep). The obtained properties also hold if individual times are compared by their potential output – as they are in production. This may however be seen as unjust since equal leisure then means a shorter time for more productive individuals. One can also indeed compare individuals' leisures by their duration. Then, however, an equal overall income is a priori unjust against the most productive individuals since, for example, the time for satisfying basic vital functions (sleep, etc.) is counted at the wage rate in reckoning this income. Or, from an equal overall income, more capable individuals "buy" their

with the efficiency of competitive equilibria if incomes are sufficiently close to each other. Then, furthermore, competitive equilibrium prevents the actualization of envy. This is a case where lower inequality is favourable to the implementation of efficiency.

2.5 Indivisibilities

Proposition. *With possible envy and allocations consisting in the assignment of indivisible items and the sharing of a given quantity, equity implies Pareto-efficiency.*

Indeed, this property is known for non-envious preferences (Kolm (1971b), Crawford and Heller (1979), Svensson (1983)). Hence, with possible envy, in this problem equity implies Pareto-efficiency with the envy-free preferences. But this implies Pareto-efficiency with the actual preferences from Section 2.3¹⁷⁾.

2.6 Free exchanges or voluntary agreements from envy-freeness or equity

A unanimous improvement from an allocation that is a best allocation of the set E , or of the set F , for one individual, leads to an allocation that is no more in this set. Hence the following result.

Proposition. *Free exchanges or voluntary agreements may create envy when it did not exist, and they may necessarily do so. They may destroy equity, and they may necessarily do so.*¹⁸⁾

The corresponding property of “non-conservation”, for non-envious preferences and “equity”, is often presented as a main drawback of this criterion. This criticism

leisure at a higher price; they thus have “shrunked” budget sets and smaller actual freedom of choice; this situation may not be equitable (with these variables) – for instance it is not equitable if two individuals with different productivities have the same envy-free preferences. Indeed, no Pareto-efficient and equitable allocation may then exist with any sharing of the resources and profits or transfers [Pazner and Schmeidler (1974), Varian (1974)]. However, such Pareto-efficient and equitable allocations exist in the common case where a more productive individual has a lower (envy-free) marginal willingness to pay for leisure-time, or painfulness of working time, for each bundle of this budget set [Piketty (1993)]. Yet, this concept is not very satisfactory either, because hours of different individuals are no more the same commodity for leisure than they are for labour. Another solution consists indeed in keeping the view that they are different commodities and in strengthening the concept of equity into another one that implies it, and that should be, for several compelling reasons, the concept of **super-equity** (see Section 2.10 below).

17) More generally, Pareto-efficient equity and envy-freeness can be impeded by non-transferabilities due to indivisibilities or human resources. These cases can be met by adjusting either the domain or the criterion. A few instances are presented in this paper and others are elsewhere. The domain can be enlarged and “smoothened” by adding a divisible item or monetary compensations among agents, it can be restricted in not considering human resources (see note 16) or indivisibilities; the criterion can be weakened into “realistic equity” (see Section 2.9 and Kolm (1971b and 1991a)), “minimal equity” (not preferring all others’ allocations to ones’ own and the like, see Kolm (1987)), or maximin or leximin in freedom (Kolm (1993b)), and it can be strengthened into “super-equity” (see note 16).

18) The corresponding property for non-envious preferences is in Kolm (1971b, page 82 of the 1972 edition). The possibility is also noted by Feldman and Kirman (1974) and by Goldman and Sussangkarn (1978).

would therefore also be valid for no-envy and equity in the case of possibly envious preferences. This criticism, however, *does not apply to the allocations that are also Pareto-efficient*, since this precludes unanimous improvements. Now, the property of equity or envy-freeness was proposed in order to restrict the set of Pareto-efficient allocations.

2.7 Equal liberty and envy

In important cases, the individuals choose their own allocations in identical domains of choice. Examples are equality of opportunity, equal rights or equal “basic liberties”, equal resources with equal access to markets, in particular equal-income competitive equilibrium and competitive equilibrium from an equal allocation. We consider cases where an individual’s domain of choice does not depend on others’ actions and choices (this is the only case where equality of these domains is a priori defined¹⁹). We also assume that the individuals know that the others choose from this same possibility set.

Furthermore, with possible consumption externality and in particular envy, the game-theoretic modality of the choice has to be specified. We consider here non-cooperation, or “*individual choice*”, that can be given, in the present case, the specific meaning that each individual i chooses individually his allocation x_i in his possibility set in considering anything about the others’ choices – the only assumption is that he knows each will be in a possibility set identical to his own. For instance, each individual may consider that his own choice can, or cannot, influence others’ choices; he may be sure or uncertain, and right or mistaken, concerning others’ choices or his influence on them; he may believe that others choose individually or have certain cooperations among themselves; another’s choice may be either observed, or estimated, or forecast; there may be Nash or Stackelberg equilibria, etc. Indeed, *the specifics will not matter because the considered externality is envy and because of the identity of domains of choice*.

We thus call *equal-freedom individual choice* the situation where all individuals choose individually from identical possibility sets and know that all choose from identical possibility sets. The result then is the following.

Proposition. *With possible envy, equal-freedom individual choices coincide with Pareto-efficient allocations and are equitable and envy-free.*

The proof is provided in the Appendix.

This implies in particular that in spite of the externality, cooperation cannot provide Pareto-improvement from any individual choice situation. Indeed, even if the individuals cooperate, they certainly choose one of the considered allocations since these choices constitute Pareto-efficient equivalent dominant strategies for all.

2.8 Notional equality of liberty

The justification of equity as a notional equality of liberty, discussed in Section 1.3 for the case of non-envious preferences, is also valid with possibly envious preferences, thanks to the following proposition.

19) In other cases, specific definitions of equality of liberty can be provided (Kolm (1993b)).

Proposition. *With possible envy, an allocation is equitable if and only if there exists a domain of individual choice such that the allocation can be an equal-freedom individual choice with this domain.*

The direct proposition results from the preceding proposition. The converse is obtained in choosing domains of individual choice identical to the set of the actual individual allocations. Indeed, if, then, $x_j P_i x_j$ for all j , individual i cannot do better than choose x_i , whatever the others do, since first, no other choice intrinsically satisfies him more, and second, this choice guarantees that the allocation is in E_i , hence in F_i , and from the lemma (first proposition).

The considerations of Section 1.3 concerning the meaning of this property are also valid here.

2.9 Unanimous preference to reassignments

Proposition. *An allocation is equitable if and only if it is unanimously weakly preferred to all its reassignments, and in particular to all pairwise exchanges of individual allocations.*

Reassignment means a permutation of the individual allocations among the individuals. The proposition is obvious for non-envious preferences.

With possible envy, call $\sigma(i)$ an n -permutation of the indices i , and $\sigma x = \{x_{\sigma(i)}\}$. Then, $x \in E \subseteq E_i$ implies $x_j P_i x_{\sigma(i)}$ for all σ and all i , and hence, from the lemma (first proposition), $x R_i \sigma x$ for all σ and all i . That is, reassigning an equitable allocation makes no individual intrinsically better-off, and if something else occurs it can only be the introduction painful envy.

Conversely, if $x \notin E$, there is an individual i such that $x_j P_i x_i$ for some $j \neq i$. Then, call k an individual whose allocation x_k is one of the most intrinsically preferred by individual i , i.e., $x_k P_i x_l$ for all l . We also have $x_k P_i x_i$. Consider now permuting x_i and x_k between the two individuals i and k and call x' the new allocation ($x'_i = x_k$, $x'_k = x_i$, $x'_l = x_l$ for all $l \neq i, k$). Then, $x' \in E_i$ and $x'_i P_i x_i$, and hence, from the lemma (second proposition), $x' P_i x$. But x' is a reassignment of x , and in particular a pairwise reassignment of x (only x_i and x_k have been changed). Hence x is not unanimously preferred to all its reassignments, nor even to all its pairwise reassignments.

QED

In particular, equity precludes voluntary bilateral exchanges of individual allocations. Hence it implies market equilibrium when the transferable individual allocations are indivisible (but there may also be non-equitable market equilibria).

One reason for the importance of the proposition is that it enables one to propose a second-best equity concept when equity, or Pareto-efficient equity, is impossible. This concept is **realistic equity**, or unanimous (weak) preference to all possible reassignments. In particular, a reassignment can be directly impossible because an individual cannot have another's allocation – for instance cannot perform another's job – (this sometimes prevents his envying it, in particular for sentiments akin to jealousy).²⁰⁾

20) See Kolm (1971b). In Kolm (1991a) we show the existence and the market implementability of realistically equitable and Pareto-efficient wages and job assignments.

2.10 Super-equity

For the allocation of divisible quantities of private commodities (with production-transformation in general), a central property is *super-equity*, i.e. with non-envious preferences, *unanimous (weak) preference to all (possibly weighted) averages of the individual allocations* (i.e. to all bundles in their convex hull) (Kolm (1973, 1987, 1991b, 1993c)). Super-equity has the following properties. It is the *strictest multidimensional maximin*. It *implies equity and unanimous preferences to all specific averages of individuals' allocations*. With Pareto-efficiency, smooth preferences and values measured at efficiency prices, it implies (vs. is implied by the fact) that *each individual's income does not fall short of the value of the bundle, consumed by any other individual, of the commodities of which they both consume some amount (vs. which he likes)*; hence it implies *equal incomes for individuals who consume the same commodities* – thus restricting maximally the set of Pareto-efficient and equitable allocations.

For divisible quantities of private commodities ($X = R_+^m$ where m is the number of goods), we call *average* of several vectors (bundles) a (generally) weighted average (linear convex combination). We then say that:

– An allocation x' is *an average* of the allocation x when *each of its individual allocations x'_i is an average of the individual allocations x_i of x : $\forall_i, x'_i \in Co(x)$ where Co denotes the convex hull operator.*

– An allocation of consumption goods x is *super-equitable* when *no individual prefers any of its averages.*

– An allocation x is *intrinsically super-equitable* when no individual envy-free prefers any of its averages. This means that *no individual envy-free prefers any average of the individual allocations to his own allocation*. This is the concept of *super-equity* for non-envious preferences (Kolm, 1973), applied for the envy-free preferences.

Proposition. *An allocation is super-equitable if and only if it is intrinsically super-equitable.*

Proof. Indeed, if x is intrinsically super-equitable, and x' is an average of x , then, for each i , $x_i r_i \xi$ for all $\xi \in Co(x)$ and in particular, first $x_i r_i x'_i$ since $x'_i \in Co(x)$, and second $x_i r_i x_j$ for all j since $x_j \in Co(x)$, that is $x \in E_i \subseteq F_i$; then, from the lemma, $x R_i x'$. Conversely, if x is *not* intrinsically super-equitable, there exist i and $\xi \in Co(x)$ such that $\xi p_i x_i$. Then, from the lemma and since $e \xi \in E_i \subseteq F_i$, $e \xi P_i x$, and the allocation x is not super-equitable since $e \xi$ is one of its averages.

QED

An average of an allocation which keeps the same total quantities is said to be *more concentrated* than it is. We furthermore say that an allocation x' is *more equal* than an allocation x when it is more concentrated and there exists some bistochastic $n \times n$ matrix $\{b_{ij}\}$ ($\forall_{i,j}: 0 \leq b_{ij} \leq 1, \sum_k b_{ik} = \sum_k b_{ki} = 1$) such that $x'_i = \sum b_{ij} x_j$ for all i .

This concept of “more equal” is the strictest multidimensional extension of the unidimensional property of dominance of Lorenz-curves and of passing from a distribution to the other by a sequence of transfers from richer to poorer (see Kolm (1966, Sections 6 and 7) for the unidimensional case and Kolm (1977) for the multidimensional extension). “More equal” implies this relation for the quantities of

each good on the one hand, and for incomes measuring these bundles with any given prices on the other hand. "More equal" implies "more concentrated". They are identical if the individual bundles x_i are independent vectors (since the preservation of quantities implies $\sum_j (\sum_i a_{ij} - 1)x_j = 0$; this is in particular the case when there are fewer goods than individuals).

Since an equal allocation, which is a certain rational ideal of justice (see Section 1.3) is generally inefficient, one is led to consider the second-best egalitarian solution of a "multidimensional maximin". Super-equity is this solution in the sense of the following proposition.

Proposition. *An allocation is super-equitable if and only if no individual intrinsically prefers a more equal one.*

If an allocation is super-equitable, and hence also intrinsically super-equitable, no individual prefers, and also intrinsically prefers, any average, and in particular any more equal allocation. Conversely, if an allocation x is not super-equitable, and hence not intrinsically super-equitable, there exists one individual, say 1, and one individual allocation $x'_1 \in Co(x)$ such that $x'_1 p_1 x_1$. The individual allocation x'_1 is also defined as $x'_1 = \sum b_{1i} x_i$ with weights b_{1i} ($0 \leq b_{1i} \leq 1$ for all i and $\sum b_{1i} = 1$). We then define other b_{ij} as: $b_{i1} = b_{1i}$, $b_{ii} = 1 - b_{1i}$, $b_{ij} = 0$ for $i \neq 1$, $j \neq 1$ and $i \neq j$. The matrix $\{b_{ij}\}$ is bistochastic. Hence the allocation $x' = \sum_j b_{ij} x_j$ is more equal than the allocation x , and we have $x'_1 p_1 x_1$.

QED

The competitive equilibrium of Section 2.4 is super-equitable and shows consistency of super-equity with Pareto-efficiency, unanimous preference to all equal allocations, competitive implementation and income equality, separately and together, in classical economies.

2.11 "Divide and choose" processes

An allocation between two individuals is made by a "divide and choose" process when one individual constitutes two lots among which the other individual chooses. This process has been extensively studied for non-vious preferences, with consideration of the various cases of information and of divisibility.²¹⁾ Among the properties that hold when envy is an actual possibility, let us point out the following ones.

Proposition. *The sharing of given quantities of various commodities by a "divide and choose" process with possible envy, perfect divisibility, perfect divider's information on chooser's preferences and unsatiated, strictly convex, smooth envy-free preferences, if it does not yield equal sharing,*

- is equitable and envy-free,
- is better than equal sharing for the divider and worse than it for the chooser (so that it is better to divide than to choose),
- is Pareto-efficient if the chooser is sufficiently prone to envy,

21) Kolm (1971b). The Edgeworth box considered there is also useful when envy is a possibility.

– is such that free exchanges from its outcome lead to violations of equity, and may arouse the chooser's envy but not the divider's envy.

The proofs and other properties are provided in the Mathematical Appendix. The fact that envy may render the process Pareto-efficient is in particular notable.

In all cases, Pareto-efficiency can be secured by giving the chooser the extra option of taking half of each quantity, thus of *imposing equal sharing*, as proposed by Crawford (1980) for non-envious preferences. With the help of the result of Section 2.3 above, one can show that the basic properties of the outcome are the same with possibly envious preferences as with non-envious ones: the outcome is *equitable*, hence *envy-free*, *unanimously preferred to equal sharing*, practically *Pareto-efficient*, *better for the chooser and worse for the divider than without this extra option*, but still such that *it is better to divide than to choose*.

2.12 Egalitarian-equivalents

The notion of “egalitarian-equivalence” proposed by Pazner and Schmeidler (1978) for non-envious individuals (and allocations that are vectors of quantities of divisible commodities), has two possible forms when individuals can a priori be envious, i.e. with the envy externality (and any type of allocations defined by the set X). First, the individual allocations $x_i \in X$ should be said to be *egalitarian-equivalent* when there exists an individual allocation $a \in X$ such that $x_i I_i a$ for all i . Second, they can be said to be *intrinsically egalitarian-equivalent* when there exists an individual allocation $b \in X$ such that $x_i I_i b$ for all i . This latter concept is formally identical to the original one extended to any X , and it is relevant if we want to “launder away” envy for ethical purposes. If, for some x , both criteria are satisfied, then $b R_i a$ for all i , with $b I_i a$ if $x \in F_i$ and $b P_i a$ if $x \notin F_i$. Indeed, $x_i I_i b$ is $ex_i I_i eb$, and we have $x_i I_i ea$; furthermore, we have $ex_i R_i x$, with $ex_i I_i x$ if $x \in F_i$ and $ex_i P_i x$ if $x \notin F_i$; hence the result stated. With allocations that are bundles of quantities of divisible commodities, and *with possible envy*, there exist *Pareto-efficient egalitarian-equivalent allocations*, with or without possibilities of production and transformation. The proof of the original paper (which applies to intrinsic egalitarian-equivalence), with similar assumptions when needed, happens to be adaptable to envious preferences because of the specific structure of the envy externality. Indeed, attribute to all individuals identical bundles that move up along a ray through the origin. Since the allocations are identical, *there is no envy*. Hence, *no individual prefers lower bundles* if we assume free disposal. Therefore, in moving up, utility levels (assumed to exist) do not decrease. Hence we necessarily reach a Pareto-efficient point on the utility frontier. Note that the corresponding constructed allocation usually is impossible (the possibility frontier in the space of commodities has been crossed). Hence, the possible utilities that have been reached correspond to another allocation. This allocation has the constructed allocation with which a Pareto-efficient point on the utility frontier is reached as its egalitarian equivalent, and it is Pareto-efficient. This allocation can be with actual envy.

2.13 Other properties

Numerous properties of “equity” with non-envious preferences have been pointed out (the basic reason of the ubiquitousness of this structure is its meaning of equal

liberty discussed above). All these properties have analogous properties with possibly envious preferences, that involve both no-envy, equity and relations between them. This analysis uses the foregoing lemma and results (and sometimes new assumptions and concepts). The results obtained in this manner concern, in particular: equal income resulting from efficient equity with a connected continuum of possibly envious individuals with smooth intrinsic preferences²²); the allocation of indivisible items and a quantity of divisible "money"²³); "net trades", public goods, common resources; variations in resources, population or preferences²⁴); Tinbergen-equitable wages (equity or non-envy with wage-job pairs)²⁵); etc. We do not present these results here. Rather, we think we should first consider more closely the general basic structure and meaning of the envy externality.

3. The structure of envy, related sentiments and applications

3.1 The structure of envy

For individual i to envy individual j with the overall allocation x , four conditions that represent points of view of individual i are necessary:

- 1) Individual i must envy-free (or intrinsically) prefer individual j 's allocation x_j to his own x_i : $x_j p_i x_i$,
- 2) Individual i must have the required propensity to be envious, to experience envy,
- 3) The individual allocations x_i and x_j must have the required comparability (envy comparability of allocations),
- 4) Individuals i and j must have the required comparability (envy comparability of persons).

The last two conditions mean that envy does not arise if the allocations or the persons are too different. For instance, I could envy your house compared to my house, but it would be strange if I envied your house compared to my car. Of course, one may also consider pairs of a house and a car. Or, if your house and my car were both obtained by us as prizes in the same lottery, I could envy your prize compared to mine (the common origin creates comparability). *With sufficient comparability*, individual i cannot prefer that individual j 's allocation x_j is replaced by x'_j such that $x'_j p_i x_j$, other things remaining equal (remember that envy is the only possible externality), and he may regret this change if it makes him envious or more envious (then $x P_i x'$); a symmetrical remark holds when x_j becomes worse in

22) The case of non-envious preferences is considered by Varian (1976), Kleinberg (1980), Champsaur and Laroque (1981), Mas-Colell (1987).

23) With non-envious preferences, results are obtained by Kolm (1971b), Crawford and Heller (1979), Maskin (1987), Alkan, Demange and Gale (1988), Tadenuma and Thomson (1991), etc.

24) A series of studies by Thomson, Moulin, Roemer, Tadenuma, Fleurbaey, Zhou, etc. present the case of non-envious preferences.

25) See Kolm (1991a).

individual i 's intrinsic judgment, and a corresponding intrinsically indifferent change elicits individual i 's actual indifference. Furthermore, if x_i is replaced by x'_i such that $x'_i p_i x_i$, and nothing else changes, certainly $x' P_i x$ since individual i 's envy for any (fixed) x_j cannot increase (normally, it can only decrease and in particular disappear); a reverse change produces the reverse effect, and $x'_i p_i x_i \Rightarrow x' I_i x$.

However, questions of envy comparability of allocations may change these results. Indeed, if x_j is replaced by x'_j such that $x'_j p_j x_j$, other things remaining constant, individual i may cease to be envious of individual j , or he may become less envious (that implies $x' P_i x$), because he deems that x_j is envy-comparable with x_i while x'_j is not (or, perhaps, is less envy-comparable); similar remarks hold for the reverse change or for indifference. Still more paradoxical, if x_i is replaced by x'_i with $x'_i p_i x_i$, individual i may become envious, or more envious, of individual j because he deems that x'_i is envy comparable with x_j while x_i is not (or, perhaps, is less so). This may even compensate the intrinsic preference of x'_i over x_i so that, on the whole, individual i is worse-off ($x P_i x'$). Similar remarks hold for the reverse change and for the cases $x'_j p_j x_j$. For instance, assume you have a house and no car and I have a car and no house, and my car is then replaced by a house that I intrinsically prefer to it but that is not as good as your house. Then, I may become envious because comparability is introduced, and this may even make me so miserable that I become on the whole less happy than I was with my car. These phenomena may induce people to choose alternatives that are intrinsically worse for them than others, not as a result of any lack of information, but on the contrary because they are fully aware of all the effects of their choice on their views and sentiments. Indeed, it is certainly more common, in such situations, that people choose what they intrinsically prefer and foresee neither their future envy nor their finally being worse-off on the whole.

(x, j, i) denoting the n -tuple derived from x by replacing x_j by x_i (while the allocation of individual i remains x_i), individual i envies individual j for allocation x , or does not experience such an envy, according as $(x, j, i) P_i x$ or $(x, j, i) I_i x$. The former case requires $x_j p_j x_i$.²⁶⁾

An individual's ordering of the x 's when his own allocation is given constitutes a reverse ordering of envy for him (since this is assumed to be the only consumption externality, and a disagreeable sentiment). More generally, if $x_i p_i x'_i$, $x P_i x'$ means that individual i is more envious when the overall allocation is x' than when it is x . The same conclusion holds, still more generally, if $x'_j p_j x_j$ and $x P_i x'$.

The interest of Feldman and Kirman's work (1974) is that they want to consider situations where envy exists, and to minimize indices of envy. Yet, the individuals they consider cannot be envious since their utilities do not display envy externality. However, their ideas and results can be validated in the following way. The most secure index they consider is an ordinal one: the number of pairs of individuals i and j such that i envies j . Now, when the relevant envy is a genuine possibility, this "index of envy" is

$$N = \text{number of pairs } (i, j) \text{ such that } (x, j, i) P_i x.$$

26) If individual i 's envy of allocation x_j of individual j depends upon allocations x_k of other individuals ($k \neq i, j$) and in particular upon his envy for them, it can only be for particular psychological reasons in the domains of information – such as competing for attention (negative influence) or inducing by awareness (positive influence) – of the dynamics of sentiments – such as getting used to experiencing envy or on the contrary exacerbation – etc.

We can also define

$$N' = \text{number of pairs } (i, j) \text{ such that } x_j p_i x_i,$$

the “no-envy index”

$$\bar{N} = n^2 - n - N = \text{number of pairs } (i, j) \text{ such that } x I_i(x, j, i),$$

and the “equity index”

$$\bar{N}' = n^2 - n - N' = \text{number of pairs } (i, j) \text{ such that } x_i r_j x_j.$$

We have $N' \geq N$ and $\bar{N}' \leq \bar{N}$, with $N' = N$ and $\bar{N}' = \bar{N}$ when $F = E$, i.e. each individual (strongly) envies any other one whose allocation he intrinsically prefers to his own.²⁷⁾

3.2 Related sentiments and applications

An individual can also envy another's wealth, welfare or happiness, or his tastes and his capacities for happiness. Envy can also elicit preferences of further order such as in: I dislike, or I like, that he envies me or that he is jealous of what I have; I dislike, or I like, that he dislikes, or he likes that I envy him or am jealous of what he has; I dislike, or I like, that he envies or is jealous of some other persons; some people have preferences on these preferences of mine.²⁸⁾ All these preferences rest on envy in the first place, so that they are indifference in the envy-free domain, with consequences similar to that considered above for envy. In particular, preferences for being admired or envied, and envy for others being admired or envied, are most common sentiments and sources of emulation in society.

Considering envy is necessary for understanding, explaining and appraising many aspects of economic life. This has been done – with or without considering the envy externality – for the determination of wages (Tinbergen (1946) and a full analysis in Kolm, (1990, 1991a)), for general tax policies (Mishan (1960) relying on consumption externalities considered by Duesenberry (1949)), for deriving the consequences for optimal taxation (Kolm (1971a), Boskin and Sheshinski (1978), Tillman (1984 and 1989), Bös and Tillman (1985)), for sharing costs of public goods or of common resources (Moulin (1990a and b)), for the measurement of poverty (Podder²⁹⁾). Models of organizations and of consumers' behaviour should often take envy and related sentiments and behaviour into account. Such sentiments also influence or determine important collective behaviour of certain groups, in particular nations, and in particular in the economic field. In all these cases, the correct modelling of envy is necessary and its numerous logical properties prove useful. We have also noted that, in a number of important cases, these externalities respect the efficiency of decentralized decision making, free exchange, competition, markets and the price

27) Feldman and Kirman also introduce cardinal individual envy indices. They see clearly that “negative envy” (when $u_i(x_i) > u_i(x_j)$) may not be a correct concept, and in the case where they maintain it they should rather consider a “sentiment of superiority” with which, however, individual allocations would often not be compared with these subjective preferences (see Section 3.2 below).

28) Yet, there is no infinite regress, and two or three iterations is a maximum as is always the case with direct mental iterations of all types (sentiments, information, etc.; this is to be distinguished from knowing the existence of a logical – e.g. mathematical – recurrence).

29) Work in progress.

system, that indeed prevent the actualization of envy. This depends on the structure of the goods and it may require that incomes are not too dispersed.

Finally, the other “social sentiments” noted above are amenable to similar treatments, that more or less parallel that of envy when they are based on interpersonal comparisons. Their economics differs in both the criterion for comparing (often something more “objective” than the individual’s preference) and the structure of the comparison. The most similar to envy is the “sentiment of inferiority”, and, in this case, there exists a possible symmetrical “sentiment of superiority”. This, however, does *not* provide a “symmetrical” to the sentiment of envy (that would validate Goldman and Sussangkarn’s analysis), for the following two types of reasons. There is no implication between any two of the three propensities to experience sentiments of envy, of superiority and of inferiority. Any can exist without the others (and envy has been our topic here). Furthermore, envy on the one hand, and sentiments of inferiority and of superiority on the other hand, do not use the same scale for comparing individuals’ situations. Indeed, sentiments of superiority and inferiority rest on objective and socially defined comparisons rather than on intrinsic preferences (both comparisons may yield the same classification, but they need not, except for certain particular cases like preferences concerning social status). A behaviour like “keeping up with the Joneses” is an avoidance of a sentiment of inferiority; “conspicuous consumption” seeks a sentiment of superiority. Other comparative sentiments are preferences for conformity or for distinction that respectively shun or value deviations on whatever side. The economic analysis of social sentiments doubtlessly lags behind their pervasive importance in social and economic life. This paper is aimed primarily at suggesting that this is not for lack of meaningful and important properties obtainable by this approach.

Mathematical appendix

A1. Proof of the proposition of Section 2.6

Let $Y \subseteq X$ denote the domain of possible choices of each individual (identical domains for all).

Denote as Z_i the set of the maximal elements of the ordering r_i on Y : $Z_i = \{a \in Y \text{ and } (b \in Y \Rightarrow ar_i b)\}$. We have $Z_i \subseteq Y$ and $Z = \prod Z_i \subseteq Y^n \subseteq X^n$ denotes the Cartesian product of the Z_i . Assume that no Z_i is empty.

1) *The allocations $x \in Z$ are equitable: $Z \subseteq E \subseteq F$.*

Indeed, $x \in Z$ is $x_i \in Z_i \subseteq Y, \forall i$, which implies, from the definition of Z_i , $x_i r_i x_j, \forall i, j$.

2) *Z is the set of individual choice allocations.*

Denote as x_{-i} the set of x_j for $j \neq i$ that individual i thinks will hold. x_{-i} may depend on x_i . If it is uncertain, the relations hold for all its possible values. The only condition is $x_{-i} \in Y^{n-1}$.

First, if $x_i \in Z_i$ and $x'_i \in Y$, $x_i r_i x'_i$ from the definition Z_i , $[x_i, x_{-i}] \in E_i$ since $x_{-i} \in Y^{n-1}$, and therefore $[x_i, x_{-i}] R_i [x'_i, x'_{-i}]$ from the lemma.

Second, if $x_i \in Y \setminus Z_i$ (i.e. in Y and not in Z_i), choose any $x'_i \in Z_i$. Then $x'_i p_i x_i$ from the definition of Z_i , $[x'_i, x'_{-i}] \in E_i$ since $x'_{-i} \in Y^{n-1}$, and hence $[x'_i, x'_{-i}] P_i [x_i, x_{-i}]$ from the lemma.

Therefore each individual i chooses any $x_i \in Z_i$, and the outcome of the individual choice process can be any $x \in Z$.

3) Z is the set of Pareto-efficient allocations.

We will show that each $x \in Z$ Pareto-dominates all $x' \in Y^n \setminus Z$ and is unanimously indifferent to all $x' \in Z$. Hence, no $x' \in Y^n$ Pareto-dominates it – therefore it is Pareto-efficient –; and each $x' \in Y^n \setminus Z$ is Pareto-dominated by it and hence is not Pareto-efficient.

We have $Z \subseteq E \subseteq E_i, \forall i$.

$x \in Z$ and $x' \in Z$ imply, for all $i, x_i \in Z_i, x'_i \in Z_i$, hence $x_i I_i x'_i, x \in E_i, x' \in E_i$ and therefore $x I_i x'$ from the lemma.

$x' \in Y^n \setminus Z$ implies $x'_i \in Y$ for all i and $x'_i \in Y \setminus Z_i$ for at least one i . Hence, $x \in Z$ and $x' \in Y^n \setminus Z$ imply $x_i R_i x'_i$ for all i and $x_i P_i x'_i$ for at least one i . Therefore, since $x \in E_i$, the lemma implies $x R_i x'$ for all i and $x P_i x'$ for at least one i .

A2. Proof of the proposition of Section 2.10 and other properties

1) *The framework*

We have $n = 2$, x_i is a vector of quantities of divisible commodities allocated to agent i . $x_1 + x_2 = 2g$ is given. Let 1 denote the divider and 2 the chooser. Let x_1 and x_2 stand more specifically for the outcome of the process.³⁰⁾

The behaviour of the chooser is fully described by $(x_1, x_2) P_2(x_2, x_1)$.

Since the divider can choose equal sharing (g, g) , $(x_1, x_2) R_1(g, g)$, and since we consider the other cases, $(x_1, x_2) P_1(g, g)$.

The divider divides in forecasting the behaviour of the chooser. The constraint that this behaviour imposes on his choice is fully described by $x_2 P_2 x_1$, because of the following lemma, proven below.

Lemma

$$(x_1, x_2) P_2(x_2, x_1) \Leftrightarrow x_2 P_2 x_1.$$

2) *Equity, non-envy, comparison with equal sharing and between the roles.*

From the lemma, $x_2 R_2 x_1$ holds.

The condition $x_1 R_1 x_2$ also holds, since, if it were not the case, $x_2 P_1 x_1$ would hold, and this implies $g P_1 x_1$ from the strict concavity of the relation r_1 and $x_1 \neq x_2$. Now, $g P_1 x_1$ is by definition $(g, g) P_1(x_1, x_1)$, and $(x_1, x_1) R_1(x_1, x_2)$ always holds, so that $(g, g) P_1(x_1, x_2)$, which contradicts the divider's choice.

Hence, the outcome is equitable, and envy-free.

The condition $x_2 P_2 x_1$ is the only constraint on the divider's choice of (x_1, x_2) . It defines a compact, open set with border defined by $x_2 I_2 x_1$. Because of non-satiation, the divider's choice is very close to his preferred allocation on this border. But $x_2 I_2 x_1$ implies $g P_2 x_2$ from the strict convexity of chooser's preferences and $x_1 \neq x_2$.

30) The Edgeworth box for two commodities can be used for illustration as in Kolm (1971b) and Crawford (1977, 1980). The analysis without envy is valid for the envy-free preferences, and for the possibly envious preferences when envy is not actually present, hence, for each individual, when he is not envious and in particular when he does not envy-free prefer the other's allocation to his own, and, for both individuals, in the envy-free domain F and in particular in the equitable domain E. When envy is present, its geometrical effect is to push the indifference curves toward intrinsically preferred allocations (so as to compensate the disutility due to envy).

This is $(g, g)P_2(x_2, x_2)$ by definition, and, since $(x_1, x_2)I_2(x_2, x_2)$ because $x_1i_2x_2$, $(g, g)P_2(x_1, x_2)$. There result the conclusions concerning the preferences with equal sharing and between the two roles.

3) *Proof of the lemma*

3a) Assume $x_1r_2x_2$. Then $(x_1, x_1)R_2(x_2, x_2)$ by definition. Then, also, $(x_1, x_1)I_2(x_2, x_1)$ (note that agent 1 has x_2 and agent 2 has x_1 in the allocation (x_2, x_1)). Furthermore, $(x_2, x_2)R_2(x_1, x_2)$. Therefore, $(x_2, x_1)R_2(x_1, x_2)$.

In consequence, $(x_1, x_2)P_2(x_2, x_1) \Rightarrow x_2p_2x_1$.

3b) Assume $x_2p_2x_1$. Then $(x_2, x_2)P_2(x_1, x_1)$ by definition. Then, also, $(x_2, x_2)I_2(x_1, x_2)$. Furthermore, $(x_1, x_1)R_2(x_2, x_1)$. Therefore, $(x_1, x_2)P_2(x_2, x_1)$.

Hence the lemma.

4) *Other properties.*

The same outcome (x_1, x_2) would obtain if the game were played with the envy-free preferences.

Indeed, the constraint on the divider's behaviour, $x_2p_2x_1$, is the same from the lemma. Furthermore, since $x_1r_1x_2$, $x_1p_1\xi_1$ implies $(x_1, x_2)P_1(\xi_1, \xi_2)$ for any allocation (ξ_1, ξ_2) . Indeed, $x_1p_1\xi_1$ is $(x_1, x_1)P_1(\xi_1, \xi_1)$. But $(x_1, x_1)I_1(x_1, x_2)$ since $x_1r_1x_2$, and $(\xi_1, \xi_1)R_1(\xi_1, \xi_2)$. Therefore, $(x_1, x_2)P_1(\xi_1, \xi_2)$.

Envy can make a difference concerning the Pareto-efficiency of the outcome. Without envy, the outcome is not Pareto-efficient. Indeed, at a Pareto-efficient allocation, the two indifference hypersurfaces coincide locally. But at the outcome, the divider's indifference hypersurface coincides locally with the hypersurface defined by $x_1i_2x_2$. And the latter cannot coincide locally with the chooser's indifference hypersurface by construction and from the hypotheses on the preferences.

Now, since $x = (x_1, x_2)$ is practically a best allocation for the divider on the set of allocations x' such that $x'_2r_2x'_1$, then, for an allocation ξ , ξP_1x implies $\xi_1p_2\xi_2$, and ξR_1x implies $\xi_1r_2\xi_2$ (since the possible outcome may not be unique). If ξ Pareto-dominates x , by definition one should have ξR_2x if ξP_1x , and ξP_2x if ξR_1x . Now, if $\xi_1p_2\xi_2$ creates sufficiently painful envy for the chooser (2), this may entail $xP_2\xi$ even if $\xi_2r_2x_2$, and $xR_2\xi$ even if $\xi_2p_2x_2$. That is, with sufficient chooser's envy, the outcome may be Pareto-efficient (with the actual preferences).

However, if the envy effect is *smooth* for the chooser, the outcome is not Pareto-efficient. This smoothness indeed implies that the extra disutility differential caused by the envy effect in the neighbourhood of x is of a lower order of magnitude than that of the distance to x . Then, in considering allocations ξ sufficiently close to x , the situation with envy-free preferences holds for the actual preferences.

When x is not Pareto-efficient, a unanimous move from it is possible (because it is not Pareto-efficient with the envy-free preferences and given the assumed structure of preferences), to reach a ξ such that ξP_1x , ξP_2x , and $\xi_1p_2\xi_2$ from the definition of x . Hence the chooser (2) may become envious. Yet this is not possible for the divider (1) since $\xi_1p_1\xi_2$ (because ξP_1x implies $\xi_1p_1x_1$ since $x_1r_1x_2$, and x_1p_1g since the divider envy-free prefers x to (g, g) , hence ξ_1p_1g , and therefore $\xi_1p_1\xi_2$ because of the convexity of preferences).

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