22 April 2013

VERITAS IN CARITATE

(THE PUBLIC GOOD OF FIGHTING POVERTY AND THE PARADOXES OF SOLIDARITY)

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Presented at the Franqui Prize Conference [1]

Brussels, June 2013.

Abstract

The universal, moral and non-excludable public good of fighting poverty is (poorly) provided by both public policies and private initiatives. The former does not crowd out the latter, as public good theory foresees it should, for two types of reasons: specific or intrinsic preferences about the transfers by the person directly concerned ("warm-glows" for short, or deontic preferences – duty, moral, norms –) or by other people ("external preferences"), and implicit cooperation or related social sentiments induced by several possible reasons. These facts imply a large number of surprising results which should be taken into account for explaining transfers and choosing policies. This is very simply seen by the consideration of the marginal equilibrium conditions of the interactions. Opposite effects result from praising sacrifice or responsibility, and for warm-glows or for external preferences. Laundering immoral warm-glows away only reinforces their effects. A large number of givers is often favourable to giving but requires that almost all givers think that the poor need no more. Grants-in-aid and subsidies or tax rebates have no effect with full information. Praise can explain warm-glows from altruisms and create non-crowd out or crowd out of gifts according to its structure. The social reasonings sustaining implicit cooperation can be implicit collective agreements (i.e. "social contracts"), lateral reciprocity (fair matching) or Kantian rationales. Giving from "putative reciprocities" with receivers also avoids the public good conundrum.

Keywords: Poverty, public goods, altruism, warm-glows.

[1] I wish to thank, for enlightening discussions, the participants to the conferences *Caritas in veritate* of the Pontifical Academy of Social Science, and *Karuna-Metta-Mudita* at Soka University.

1. Introduction

1.1 Transfers and the standard model

The main economic and social problem, fighting poverty, is a universal, non-excludable and moral public good very imperfectly provided by both public and non-governmental actions and transfers. Low incomes are not always abject poverty; they are not, for instance, in many balanced traditional societies. In other social contexts, however, poverty is a crippling ontological handicap which forbids living a minimally decent (if not full) life and imposes hardship and suffering. Every human being wants any other's such poverty to be reduced – irrespective of how much she is ready to pay for it (everybody is a large number). Transfers providing consumption, formation or tools can do much. The present situation can only be seen as vastly wanting, due perhaps to selfishness but also to failure in coordination for producing this public good. Nevertheless, every non-poor contributes through taxes and public budgets, which redistribute a relatively large part of GNP in many countries. Explaining and choosing these transfers is an important task of public economics. In addition, many non-poor contribute privately by direct aid or through charities, foundations, non-governmental organizations or solidarity funds.

The very coexistence of these two modes of transfers condemns standard public good theory.[2] This model can neither explain nor choose public transfers since it holds them to be fully compensated by a decrease in private transfers, unless public transfers crowd out private ones but, then, the model cannot explain private transfers. These private transfers exist, however; they usually amount to a few per cent of GNP (up to 5% in the Netherlands); 9 Americans in 10 report having given in the past year; half of them claim deduction for giving in their income tax report. The cause of this failure to explain has to rest on any of the two basic hypotheses of the standard model: individuals care only about their own consumption and the public good (the poor's income or welfare), and they act non-cooperatively between them and with the policy (most often in a Cournot-Nash fashion). This non-cooperation causes the crowding out: it entails Pareto inefficiency which disappears when there is one actual giver only, and the policy can distribute the gain to all contributors by choosing the taxes. However, facts belie both hypotheses. First, for a variety of reasons, people often care more or less about their gifts or contributions and those of other people apart from their global effect on the poor's welfare. This will be denoted as *specific preferences*, and a person's *contribution* to fighting poverty denotes her gifts plus the "distributive taxes" she pays used

[2] See Warr (1982), Roberts (1984), Bergstrom, Blume and Varian (1986), Kolm (1970a, 1970b, 1971).

for this purpose.[3] Second, people often choose their gifts with reasonings that constitute kinds of implicit cooperation – although often an imperfect one – such as "lateral reciprocity" or fair matching, implicit "social contracts", varieties of "categorical imperatives", or "putative reciprocities". These facts have to be taken into account for explaining gifts and for explaining and choosing policy. However, the vanishing of the standard model paradox and failure leaves the place to a number of a priori unexpected, surprising, counterintuitive or paradoxical properties or impossibilities which explain some puzzling facts but raise further questions (they combine the three paradox-inducing fields of: public goods, large numbers and self-reference).

1.2 Topic and method

This paper focuses on the first family of divergences from the standard model, the specific preferences about gifts or contributions.[4] It only very briefly outlines the issues related to the second, the implicitly cooperative conducts.[5] Its central methodological characteristic is the derivation of all its properties very easily thanks to a very simple device: the consideration of the marginal equilibrium conditions of the interaction between the policy and the people who include possible givers. This contrasts, in simplicity, generality of structures, and richness and easy availability of results, with the standard method of the field of considering quantities explicitly. The rest of this introduction shows some of the results. Section 2 presents the relevant model and the further sections derive the various properties.

1.3 Outline of results

[3] The issue of information is discussed in appendix A.

[4] Altruism (empathy, compassion, pity, solidarity) and the neighbouring sense of justice are some of the many possible reasons to have preferences about other people's incomes (see Kolm 1966 and a further abundant literature). Direct preference about one's gift is, in economics, suggested by Olson (1965), Arrow (1972), Becker (1974), Sugden (1982) who points out that "social acclaim" of gifts is a priori based on people valuing the benefit, and consequences were worked out by Menchik and Weisbrod (1981) and Cornes and Sandler (1984a, 1984b, 1986) for a contribution to an ordinary public good, Roberts (1984, 1987), Kolm (1984), extensively Andreoni (1989, 1990) for "warm-glow", Harbaugh (1988a, 1998b) for prestige, and an abundant literature exhaustively reviewed and discussed in several chapters of the *Handbook of the Economics of Giving, Altruism and Reciprocity* (Kolm and Mercier Ythier, eds., 2006). By contrast, the crucial difference between concern about gifts and contributions, direct "external preferences" about other givers' such items, the effects of laundering away immoral preferences, the full analyses of the effects of other people's judgments such as praise, of all the possible implications of the large number of people concerned and of the effects of subsidies and tax rebates, and a number of other reasons for cooperating or for giving, all issues shortly noted, have been either neglected or only partially considered.

[5] This is the topic of Kolm 2008c and 2009.

Altruism will denote a preference for someone else to be happier (rather than, for instance, giving). Without further qualification, it will denote a non-poor's preference for a higher income of the poor. If someone gives because she is an altruist, or because this is a duty, she may feel a "warm-glow" or feel praiseworthy about her benevolent or duty-bound gift. However, if she gives in order to experience such a sensation, this gift's motive is no longer altruism or duty but this selfish objective, and consequently this gift can no longer produce warm-glow or praiseworthiness. Hence such a motive is self-contradictory. The warm-glow may nevertheless exist if the giver is *also* altruistic, which leads her to value and praise the gift's effect on the poor's income and, as a consequence, the gift and the giver (herself). This latter relation can also be due to (other) altruists praising the gift and the giver or to the gratitude of receivers or of altruists (see section 10). In particular, the giver may be altruistic but give not (or not sufficiently) because of altruism, but because of warm-glow which, however, is aroused because of this altruism.

Overall, people have many possible reasons to have specific preferences about their own gifts or contributions. The term *warm-glow* describes well some of them only, but it will be taken to denote all of them for reasons of focus, illustration and some previous use.[6] People can feel proud, praiseworthy, or not guilty or ashamed, or be praised, admired or not blamed, for the *sacrifice* they incur that helps the poor. This, however, turns out not to prevent crowd out of gifts (section 3; basically because it bears on their full contribution, including the tax). By contrast, non-crowd out can be explained by people valuing their *responsibility* when choosing their gift. People caring about their responsibility, rather than only about their sacrifice that helps the poor, in transfers from them that alleviate poverty, is very clearly shown by neurological experiments: their reward system is much more excited when the same transfer is decided by them than when it is imposed to them.[7] However, people cannot so give in order to feel praiseworthy or be praised as genuine altruists since this characteristic is defined by the altruistic motive and this warm-glow satisfaction or image-building (in one's or other's eyes) is not an altruistic motive. This would be a contradiction and an irrationality in this sense. Or, perhaps, the warm-glow giver tries to deceive observers, and possibly herself, about her true motives. Hence this warm-glow giving in the strict sense is impotent (crowded out), irrational or a fake. Such sentiments are also often immoral as being vanity, vainglory or sentiments of superiority towards receivers or co-givers. They may then have to be erased

[6] Various specific motives, present in the literature or not, will be discussed below.
[7] See Harbaugh, Mayr and Burghart (2007). Note also that the receiver's and altruists' gratitude depends much on the giver's responsibility (it also often depends on her concern about the receiver which, however, may be present in both cases).

from a preference-respecting social objective that determines the transfer policy. There are rational technical ways to this. Then, however, quite surprisingly, the effect of the warm-glow is not erased or diminished but it is actually augmented: not only does the effect of the responsibility warm-glow remain, but it is now added the effect of the sacrifice warm-glow (although this is actually the only one that had to be erased from the preferences because it also considers taxes) (section 5). The reason is that deleting an individual's preference for her contribution leads to a lower tax, which lowers crowd out and induces the person to give more.

Specific preferences about other people's gifts or contributions, called *external preferences*, may be held for reasons of comparisons with norms or with others' gifts or contributions including one's own, such as inequality-aversion, keeping in line with others, providing one's fair share, preference for conformity or on the contrary for distinction, envy, jealousy, or sentiments of inferiority or superiority. The resulting effect turns out to be paradoxical in various ways: people end up *giving more* when others prefer them to *contribute less*, and preferences about gifts have no effect. The reason is that an external preference for a lower contribution leads to a lower tax, which lowers crowd out and induces the person to give more. Hence the object of active preferences is the opposite to what it is for warm-glows (contributions versus gifts), and the effect has the noted contrarian character (section 3). Also contrary to the case of warm-glows, erasing such nosy external preferences (externalities) from the preference-respecting social objective just has this effect on the result.

The large number of people raises another series of puzzles. The first one is that if one gift (at least) is not crowded out, then almost all non-poor's valuation of an extra income for the poor should vanish! In spite of this, non-crowd out in large numbers turns out to be a normal possibility. This explains the observation that number is sometimes favourable to non-crowd out. The paradox disappears and this result obtains if and only if standard altruistic people are sensitive to the relative variations of the overall poor's income and hence consider its logarithm (they do not care if the poor's income is augmented by 100 dollars but care if it increases by 1%). This is required by each of these non-poor paying some tax or gift in necessarily finite amount, and their warm-glow giving becomes independent of their altruism. However, when the number of poor increases, these decreasing returns to scale in compassion or solidaristic sentiments imply that there is less concern for the new coming poor. This hardly moral inequality is nevertheless necessary because one cannot empathize the infinity of the world's misery (actually, when the number of non-poor becomes infinite, this is possible for a finite number of them). A possibility is that almost all people care about the "average

poor" or a "representative poor" only; but this is, again, hardly moral ("when you have seen one, you have seen all"). (Section 9).

The same structure also explains the properties of "altruism warm-glows", i.e. warm-glows based on some altruism either of the giver or of people who praise or do not blame her ("praise altruism warm-glows"). The same effect is provided by the giver's altruism towards altruists (second-degree altruism) or by her appreciation of the gratitude of receivers and of altruists. This cannot or can hardly explain non-crowd out if the altruistic sensitivity is about the absolute poor's income variations, but it can if it is about its relative variations (section 10).

A person helps more or less specific poor only, with the implicit assumption that other people help other poor she also cares about.[8] However, the number of joint co-contributors to the same targeted help may nevertheless be high – for example in national fiscal transfers or large charities or solidarity funds. The noted implicit coordination between transfers to different beneficiaries is not a conditional cooperation (if some other stops helping one does not suspend one's aid – possibly, one extends it to the now neglected poor)[9].

This contrasts with helping the same poor. Then, there are a few specific reasons that lead to implicit cooperation (section 12). One is *fair matching* or *lateral reciprocity*: I help given that others help; I do my fair share given that they do theirs.[10] This is not a conditional exchange. However, people are sure that others contribute only if the gifts are public and jointly simultaneous or sequential alternate, or of the other givers are forced to give. In the latter case, everybody is forced, these actions are no longer voluntary (as gifts should be), but they actually are, however, because everyone wants to give what she is forced to since the others provide their share – these constraints may be necessary but are not binding. Another common reasoning is what can be called folk Kantianism: "I help because what if nobody helped?" This kind of answer is for instance the most common one given when people are asked why they care to vote in large elections. Kant's full categorical imperative is, of course, "follow the rule such that, if everybody followed it, you could want the result". However, if applied more thinly than to rough dichotomous choices (lie or do not lie, help or do not help, vote or do not vote, etc.) for instance for determining one's gift as a function of one's characteristics, this leads people applying the principle to choose different rules because they

[8] "*J'ai mes pauvres*" (I have my poor) is the French bourgeois ladies' answer to demands for charity (if you then go ask their husband, you are likely to face the crowd-out complaint: "I just paid taxes for this").
[9] The US would not stop helping US poor if China fails to take care of Chinese poor.
[10] See Sugden (1984), Kolm (1984).

have different preferences, hence their assumption about others' behaviour is false whether these others are also "Kantians" or are not. Both lateral reciprocity and Kantian behaviour are a priori not Pareto efficient, unless they are oriented to choices belonging to a specific set.[11] Finally, another behaviour would be to follow an implicit agreement, that is, technically, a social contract. A notable characteristic of this solution is that, in important cases, the large number of participants is favourable to non-free riding.[12] In the extreme, a Rousseauan social contract leads people to become a "team" (Radner) maximizing the same social welfare function (a "general will") with the result that their Cournot-Nash behaviour becomes Pareto-efficient.[13] Of course, simple sequential giving does not suffice with large numbers since a single giver's choice is not even noticed by others (and abstaining from giving in order to punish a free rider punishes everybody – all altruists and also the poor).

An explanation of effective warm-glow is that it occurs when giving is a "deontological" (deontic) action rather than a "consequentialist" one, hence focusing not on its consequences (altruism) but on the act in itself (hence on the giver's responsibility for it). This motive may be a duty or following a norm. Still another motive for helping is *putative balance reciprocity* : I help this person because she would have helped me if our situations were reversed, or she would have helped other persons if she could, or other persons would have helped me if I needed it.[14]

Finally, subsidies, tax rebates and grants-in-aid for gifts have no effect on the result if all the effects and notably the origin and the costs of these funds are taken into account by the agents and by the analysis (section 13).

These results apply more or less to other public goods, and other general results apply to the problem of fighting poverty. The general possible warm-glow of "being a good cooperator" also applies to joint giving although it is bound to be a pale sentiment compared to possible effects of alleviating poverty. The universality applies to climate change, although without the equivalent of specialized contributions or gifts. The moral dimension is the strongest for fighting poverty but may more or less exist in other cases such as contributions to culture, to the environment taken as a final value, or to the defense of a group. At any rate, altruism towards co-contributors would play the same role as a moral objective but would be rather secondary in the poverty case (pleasing altruists or people with external preferences, apart from the effect on their praise or blame). Moreover, the poor's welfare may or may not

[11] Kolm (2008d, 2009). For a different modelling, see also Bordignon (1990).
[12] Kolm (1987, 2008d).
[13] See appendix C.
[14] Kolm (2008a).

enter also directly in a social objective function, depending on what one wants to study (both cases are considered here), whereas a direct government preference for a public good would be less usual. Finally, all effects of the poor's welfare or income may be not only for all brands of extended altruism (empathy, compassion, pity, solidarity, sense of justice) but also for fear of social unrest and buying social peace – hence the results apply to the outcome of class struggle and not only to charity or solidarity.

2. The relevant model of society[15]

2.1 Method

Crowd out is in particular due to the fact that non-cooperation between givers and between them and the public policy (e.g. Cournot-Nash behaviour) is Pareto-inefficient. This source of inefficiency is suppressed by crowd out, and the tax burden can be distributed in such a way that everybody benefits. Therefore, the simplest and most straightforward analysis consists of considering Pareto-efficient states. This calls for two remarks. First, the first-order conditions for Pareto efficiency, associated with those resulting from the free choice of gifts by givers, will provide all the information that permit to answer all the main questions posed in very simple and meaningful ways, by considering the marginal utilities only. Second, a Pareto-efficient choice of the distributive taxes results from the public maximization of a preference-respecting social welfare function under the constraints of the situation, including the non-cooperative behaviours. Now, Pareto efficiency has the classical virtue with respect to unanimity, but it is also a consequence of common types of social and political life, for instance electoral democracy: if the state is not Pareto efficient, a contending party can choose a program which is preferred by the unanimity of votes. All the presented properties will

[15] The very simple and general model used here encompasses many models used in the literature which specify variables or relations, explain or justify them by a theory or by observation, and often apply the model to particular important issues. For example, Brekke, Kverndokk and Nyborg (2003) and Konow (2007) consider norms of giving with an ideal level but a lower provision because of self-interest (we will also propose fuller explanations of warm-glows). Holländer's (1990) model of contributions to any public good motivated by the judgment of the relevant people (also Rege and Telle, 2004) applies particularly for the important, moral and universal public good of the relief of poverty. Harbaugh focuses on prestige and applies this model for specific results. Admiration, signal of wealth (Glazer and Konrad, 1996) and gratitude (Amegashie, 2006, with conceptual problems) have been discussed. The properties that turn out to be crucial cut across all the specific manifestations and are general distinctions such as: is the warm-glow for the giver's sacrifice or for her responsibility? Does it require altruism or not? Is there some norm or duty attached to giving per se? Are the givers motivated by some principle of cooperation and which one? Does the social welfare function also directly include the poor's welfare? Does it respect the givers' immoral sentiments (vanity, vainglory, envy, sentiment of superiority) or not?

result from the conditions of Pareto efficiency alone as a consequence of the choice of the policy.

2.2 Society and variables

Society is divided in two classes, n non-poor indexed by i, j or k, and the poor. The focus is on taxes and gifts provided by the non-poor that increase the poor's income x (which is sufficiently considered aggregatively for many of the results). In order to minimize the derivations, the strategy is to present the conditions in the most general case considered here; the conditions in the other cases result just from dropping the irrelevant variables. In amounts of money, non-poor individual i provides gift $g_i \geq 0$, pays redistributive tax $t_i \geq 0$ (the part of her taxes that is redistributed to the poor), hence contributes to the poor's income for the contribution $c_i=g_i+t_i \geq 0$. Her initial income is Y_i, and her remaining income or consumption is $y_i= Y_i–c_i \geq 0$ or, rather, >0, since with $y_i=0$ person i is poor or, rather, would starve. If X denotes the poor's initial income, their final income is $x=X+\Sigma c_i$. For describing the most general case considered here, denote the two $n–1$ vectors $g_{–i}=\{g_j\}_{j\neq i}$ and $c_{–i}=\{c_j\}_{j\neq i}$.

2.3 Sacrifice and responsibility: representation

A *specific* (or intrinsic) *preference* about some item is represented by this item as an argument of the corresponding utility function u whatever it is. If this item is c_i, one has $u(c_i,\ldots)$. What is specific to c_i for eliciting such a preference is the *sacrifice that benefits the poor* incurred by individual i. If there is such a preference about both c_i and g_i, one has $u(c_i,g_i,\ldots)$. Since the sacrifice effect of g_i is already taken into account by its presence in $c_i=t_i+g_i$, the other argument represents the only other specific (intrinsic) character of g_i, to wit the fact that individual i is *responsible* for its free choice (this is not the case for t_i).[16] When g_i varies,

$$du/dg_i=\partial u/\partial c_i+\partial u/\partial g_i.$$

Another, different presentation of the same thing could be to write $u(t_i,g_i,\ldots)$. The tax t_i has only a sacrifice effect and no responsibility effect (it is not chosen by individual i). Then, the variable g_i stands for both the sacrifice and the responsibility effect of g_i. Therefore, $\partial u/\partial g_i$ is the former $\partial u/\partial c_i+\partial u/\partial g_i$, $\partial u/\partial t_i$ is the former $\partial u/\partial c_i$, and the former $\partial u/\partial g_i$ (the pure effect of responsibility) becomes $\partial u/\partial g_i–\partial u/\partial t_i$. We choose the former representation.[17] Function u is u^i for warm-glows and u^j for $j\neq i$ for external preferences.

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[16] See, however, appendix A, which also discusses the issues of information about t_i and for the policy.
[17] All the results are shown with the two representations in Kolm (2008b).

2.4 "Altruism"

Individual *i*'s preferences about the poor's income *x* is represented by *x* being an argument of function u^i. This effect may or may not be through the effect of *x* on some psychic welfare of the poor. The representation of the effect by *x* alone suffices for the first parts of this study. Specific representation of the multiplicity and of the welfare of the poor will have to be introduced in further sections only. Function u^i depending on *x* is considered to represent individual *i*'s altruism. This can come from empathy, pity, compassion, or sense of solidarity or of justice. However, individual *i*'s preferences about *x* may also have other reasons, such as fear of social unrest (hence the model of solidarity is also a model of class strugle).

2.5 Utility functions

Finally, individual *i*'s utility function is the ordinal

$$u^i(y_i,x,c_i,g_i,c_{-i},g_{-i}). \qquad (1)$$

It is assumed to be differentiable with

$u_1^i=\partial u^i/\partial y_i>0$, $u_x^i=\partial u^i/\partial x\geq 0$, $u_c^i=\partial u^i/\partial c_i$, $u_g^i=\partial u^i/\partial g_i$, and, for $j\neq i$, $u_{c_j}^i=\partial u^i/\partial c_j$ and $u_{g_j}^i=\partial u^i/\partial g_j$. We take $u_1^i\to\infty$ when $y_i\to 0$ ("Inada condition"), which guarantees $y_i>0$ under the choices of g_i by person *i* and of t_i by fiscal policy. We also assume that u_g^i and u_x^i are bounded (individual *i* is not ready to lose everything for the pleasure to give one cent more or to see the poor having one cent more).

The poor's welfare is sufficiently represented by an increasing function $w(x)$ with $w'>0$ for most of the results (it could also be $mw_o(x/m)$ where *m* is the number of poor and w_o a function with $w'_o>0$).[18]

2.6 The two regimes

The public policy maximizes an increasing function *U* of individuals' welfare or utility functions. However, there are two different cases according as to *U* includes or not the poor's welfare as an argument: $U=U(\{u^i\}, w)$ in the *redistributive regime* and $U=U(\{u^i\})$ in the *basically giving regime*. In this latter case, the government is something like the "executive committee of the bourgeoisie" (Marx). This structure corresponds to the ordinary case of public goods. In the redistributive regime, the presence of *w* in maximand *U* may be for a

[18] This form is explicit in section 9.

moral reason, because of the poor's demands and political power, or to prevent social unrest (hence because of such a threat). This regime amounts to assuming that the poor's income x is not sufficiently taken care of by individuals i's concern for x (by the presence of x in utility functions u^i). As concerns welfare, in particular, the presence of x in the u^i may manifest and induce some kind of charity whereas that of $w(x)$ in U may manifest and induce some kind of collectively or politically decided justice.[19]

Pareto-efficiency for the whole population, including the poor, is achieved by the highest U in both regimes. It is, in particular, in the basically giving regime, although the obtained state is one of the Pareto-efficient ones least favourable for the poor. This is a particular case of a general property stating that Pareto-efficiency for a sub-population is a priori Pareto-efficient for the whole population (appendix B). This actually holds under one condition which could fail very fortuitously only (in the present case a change in the set of taxes t_i should affect at least one u^i).[20]

The general case is

$$U=U(\{u^i\}, w) \qquad (2)$$

with $\partial U/\partial u^i=\lambda_i\in]0,\infty[$ and $\partial U/\partial w=\mu\in[0,\infty[$, with $\mu>0$ in the redistributive regimes and $\mu=0$ in the basically giving regime.

A higher μ will tend to induce more crowd-out. In particular, without specific preferences, i.e. in the classical case (or with sacrifice warm-glows only), the result for standard public goods, here for $\mu=0$, that there can be at most one giver (who is the only marginal altruist) becomes, when $\mu>0$, the total ban on gifts.

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[19] Almost all the literature on altruism and giving omits the case of the redistributive regime. Hence it bans the poor from the social welfare function and from the definition of Pareto efficiency, and considers them as altruists' "consumption" only. Exceptions in which the poor's welfare is an argument of the government's maximand, because of the receivers' political power, are found in Roberts (1984), Becker (1978) and, somehow, Peltzman (1976).

[20] For a Cournot-Nash relation between the government policy and the givers, this is for given gifts $\{g_j\}$. The noted condition even needs to hold only at states that are Pareto efficient for the non-poor. From such a state, indeed, change the set of taxes $\{t_j\}$. Then a number of levels u^i change (at least one). All these changing u^i cannot all increase, from the definition of Pareto efficiency (for the non-poor). Hence, at least one decreases. But this decreasing u^i is also a decreasing member of the larger set encompassing all u^i and w. Hence, any possible change in the set of taxes t_i from the state in question makes one member of this larger set of the u^i and w decrease. Therefore, no possible change in the set of taxes t_i from this state makes all the u^i and w increase or not change with at least one increasing. Hence, by definition, the state in question is Pareto efficient for the whole population of the non-poor and of the poor. These properties are applications of general theorems (see appendix B).

2.7 The interaction

The n+1 agents, the n non-poor who choose their gifts g_i to maximize u^i and the public policy who chooses the set of taxes t_i to maximize U are assumed to act in a Cournot-Nash non-cooperative behaviour with respect to all others. In particular, the public policy maximizes U under the constraints of the problem which include these behaviours.

3. General property and paradoxes

Proposition 1

Gift g_i is not crowded out only if

$$\lambda_i u_g^i \geq \Sigma_{j\neq i} \lambda_j \cdot (u_x^j + u_{c_i}^j) + \mu w', \tag{3}$$

with sign = if $t_i>0$.

That is,

non crowd-out, $g_i>0$ $\Rightarrow$ $\lambda_i u_g^i \geq \Sigma_{j\neq i} \lambda_j \cdot (u_x^j + u_{c_i}^j) + \mu w'$, (3)

with sign = if $t_i>0$,

$\lambda_i u_g^i < \Sigma_{j\neq i} \lambda_j \cdot (u_x^j + u_{c_i}^j) + \mu w'$ $\Rightarrow$ $g_i=0$, *crowd out.*

Proof

From individual i's choice of $g_i \in [0, Y_i - t_i[$,

$$g_i>0 \Rightarrow \mathrm{d}u^i/\mathrm{d}g_i = -u_1^i + u_x^i + u_c^i + u_g^i = 0 \tag{4}$$

($\mathrm{d}u^i/\mathrm{d}g_i>0$ is precluded by $u^i \to \infty$ when $y_i=Y_i-c_i \to 0$).

From the policy choice of $t_i \in [0, Y_i - g_i[$,

$$\mathrm{d}U/\mathrm{d}t_i = -\lambda_i u_1^i + \Sigma \lambda_j \cdot (u_x^j + u_{c_i}^j) + \mu w' \leq 0 \tag{5}$$

(with $u_{c_i}^i = u_c^i$), with sign = if $t_i>0$ ($\mathrm{d}U/\mathrm{d}t_i>0$ is precluded by $u^i \to \infty$ when $y_i=Y_i-c_i \to 0$).

The sign of $\lambda_i \mathrm{d}u^i/\mathrm{d}g_i - \mathrm{d}U/\mathrm{d}t_i$ provides proposition 1. □

Condition (3) does not depend on u_x^i, u_c^i and $u_{g_i}^j$ for $j\neq i$. If a utility function u^i does not depend on one of its possible argument z, $u_z^i = 0$. In verbal expressions, the sensitivity of u^i to z denotes the sign of u_z^i ("altruism" is $u_x^i > 0$, warm-glows are $u_g^i > 0$ and $u_c^i > 0$, external preferences are $u_{g_j}^i \neq 0$ and $u_{c_j}^i \neq 0$ for $j\neq i$). That is, the noted effects refer to the

marginal effects. If the corresponding derivatives are monotonous, they refer also to the total effects.

Corollary 1

Non-crowd out does not depend on the individual's altruism and sacrifice warm-glow, and on other's responsibility external preferences.

Corollary 2

Non-crowd out is favoured by the individual's responsibility warm-glow and hampered by others' altruism and sacrifice external preferences.

These are the effects of u_g^i, and of u_x^j and $u_{c_i}^j$ for $j \neq i$, on condition (3).

In particular, non-crowd out of gift g_i is favoured by $u_{c_i}^j < 0$ for $j \neq i$, that is for others preferring individual i's contribution to be lower.

Corollary 3. The multiple paradox

Non-crowd out is favoured by the individual preferring her gift (responsibility) to be higher and by others preferring her contribution (sacrifice) to be lower. It does not depend on the individual's preference about her contribution (sacrifice) and on others' preferences about her gift (responsibility).

The basic reason is that others' preferences about c_i and $x=X+\Sigma c_k$ are also about $t_i=c_i-g_i$. If they are higher, this leads the policy to choose a higher t_i (given g_i), which may crowd out g_i . If $u_{c_i}^j < 0$, this leads to choosing a lower t_i, which tends to induce a higher g_i or may present a crowd out.

4. Warm-glows

Consider now warm-glows alone (no external preferences), hence all individual i's utility functions are of the form

$$u^i = u^i(y_i, x, c_i, g_i). \tag{6}$$

Condition (3) writes

$$\lambda_i u_g^i \geq \Sigma_{j \neq i} \lambda_j u_x^j + \mu w' \tag{7}$$

or, denoting

$$v = \Sigma \lambda_j u_x^j + \mu w' \tag{8}$$

the *marginal social value of the poor's income*,

$$\lambda_i \cdot (u_x^i + u_g^i) \geq v. \quad (9)$$

Assume $u_g^i = 0$. Then, condition (7) cannot hold if μ>0 (the redistributive regime). If μ=0 (the practically giving regime), condition (7) holds if and only if, furthermore, $u_x^j = 0$ for all $j \neq i$. Then, *x* is no longer a public good for the non-poor, at least marginally. If moreover $u_x^i = 0$, it is not even a good for all non-poor, at least marginally. If $u_x^i > 0$, no other individual $k \neq i$ can similarly have $u_g^k = 0$ and satisfy its condition (7). Note that, if $u_g^i = 0$, individual *i* may want to give because $u_x^i > 0$ and/or $u_c^i > 0$, but the tax can equivalently realize these transfers or provide higher ones.

The classical public good case presents these results for the case μ=0.

If $u_g^i < 0$, due, for instance, to modesty or a desire to keep in line with a social standard or with others, condition (7) cannot hold.

Exactly the same results hold when there is no sacrifice warm-glow, that is

$$u^i = u^i(y_i, x, g_i)$$

for any *i*.

Proposition 2. Warm-glows. The sacrifice paradox. Regimes. Full and almost full crowd out.

Sacrifice warm-glows u_c^i have no effect on crowd out. For a person without responsibility warm-glow or without warm-glow at all, crowd-out is full in the redistributive regime or in the practically giving regime if some other person is altruist (and hence if there is more than one altruist if she herself is).

5. Irrationality, immorality, and the moral and rational social efficiency: the moral paradoxes

5.1 Irrationality of the responsibility warm-glow stricto sensu

Let us consider the notion of warm-glow for helping the needy in its strictest and most direct sense. Then, the fact that the responsibility warm-glow alone is accountable for possible non-crowd out in condition (7) is puzzling since this sentiment in itself is self-contradictory. *Indeed, one cannot give in order to be praiseworthy or praised as a compassionate altruist since this motive is not altruistic compassion*. This affects particularly responsibility warm-glow because altruistic or duty-bound giving also entails responsibility. One may be praiseworthy or praised for one's sacrifice that helps the poor, but this applies equally to the

distributive tax and, as a result, this cannot explain non-crowd out, as we have seen. The warm-glow objective of giving may also be to deceive other people by making them believe that one is or is more a compassionate altruist (this may be appreciated in itself or because it creates a reputation which may provide various advantages – social, material, political, etc.). This fraud is immoral, nothing to be proud of. The effect may also be self-deception, leading one to have some impression of being the moral compassionate altruist that one is not, or more so than one actually is, as psychoanalysis may be able to explain. This is an irrationality, however.[21] All these effects may be in addition to some existing genuine altruism and altruistic satisfaction ($u_x^i>0$), but this has no direct effect on non-crowd out.

5.2 Immorality and laundering preferences

Another important aspect is that warm-glow is often, in fact, vanity and vainglory, sometimes accompanied by a sentiment of superiority with regard to people who aid less (perhaps according to means or to relation with receivers) or to people aided (condescension). Warm-glow may thus tend to be immoral and irrational, with important consequences. The social criterion may have not to respect individuals' immoral social sentiments (should someone be deprived of something because some other people envy her?).[22] It may also have to discard contradictory aspects of individual preferences, but we will shortly see that this has no consequence on the non-crowd out condition if it concerns responsibility warm-glow, because preferences about g_i have no effect on the public choice of t_i.

The method for laundering preferences for the effects of some variables, with preferences of general form, consists in assuming that these variables have some fixed level. In general, this level matters for the result. If there is no a priori given natural level for this purpose, the consistent and rational solution consists in choosing the level that would result from choices in which its variability has no effect. In the present problem, this means deleting the effects of the corresponding variables (some or all g_i or c_i) in the conditions of the

[21] The non-altruistic giver may also give because she wants to be a (compassionate) altruist and knows that a classical way to try to have a sentiment is to act as if one had it (with the help of dissonance reduction). This is probably still more praiseworthy than being altruistic. Note that the term "rational" has been used here in standard senses (including consistency, non-contradiction and non-self-deception), not in the particular restricted sense of ordering consistency common in economics.

[22] If Pareto efficiency results from political life, people may impose the government to respect their full preferences, including their vices. However, they may also agree, in the public discussion or in a collective agreement, to discard these immoral aspects for the social moral choice. They may even enjoy that the government discards these regrettable aspects of their preferences that they do not have the willpower to abandon by themselves, as Ulysses with the Sirens or as help against their addiction to bad thinking.

government's choice of t_i (conditions (5)). New conditions replacing conditions (3) are then obtained for the corresponding i, and the c_i and g_i result from the solution of the 2 n conditions of the choice of the g_i and t_i. The effect of the variable in question is erased, and the choosen level of the variable is that consistent with the whole situation. The conditions of the individual choices of the gifts g_i do not see their form affected since these choices are free and the present issue is not moral education.

Conditions (5) do not contain u_g^i. Therefore, laundering preferences for effects due to responsibility warm-glows has no consequence. This is remarkable since it is precisely responsibility warm-glows which, in the end, provide the non-crowd out conditions (3). Moreover, the noted irrationality (contradiction) concerns essentially the responsibility warm-glows.

Laundering the effects of sacrifice warm-glows (for instance because they would be vanity and vainglory) consists in deleting terms u_c^i in the corresponding relations (5). This is equivalent to adding a term $-\lambda_i u_c^i$ to the left-hand side of inequality (5), hence to the term $\mu w'$. Since there is no $\mu w'$ in condition (4), the result of combining conditions (4) and (5) is to add, in the resulting condition (3), $-\lambda_i u_c^i$ to $\mu w'$, that is to the right-hand side of the inequality, and this amounts to adding u_c^i to u_g^i in the left-hand side. Therefore, the effect of laundering away the effect of warm-glow just amounts to adding u_c^i to u_g^i in the non-crowd out condition. The resulting *laundered non-crowd-out condition* for gift g_i is

$$\lambda_i \cdot (u_g^i + u_c^i) \geq \Sigma_{j \neq i} \lambda_j \cdot (u_x^j + u_{c_i}^j) + \mu w' . \qquad (10)$$

The expression $u_g^i + u_c^i$ describes *both the responsibility and the sacrifice effects of the gift* g_i (u_g^i and u_c^i respectively). Gift g_i is now valued as *responsible sacrifice*.

This important result is paradoxical in many ways.

- We wanted to erase the effects of the warm-glows in the social objective function.
- So erasing the effects of the responsibility warm-glows is not possible or has no effect, whereas they are the warm-glows which entail the non-crowd out conditions and essentially the psychological irrationality.
- As a consequence, the effect of responsibility warm-glow in the non-crowd out condition is not affected.

- Erasing or deleting the effects of the sacrifice warm-glows is possible, although they play no role in the non-crowd out conditions (3). This is the only warm-glow effect that can be deleted.

- This makes a change if and only if there is a sacrifice warm-glow u_c^i >0: condition (3) becomes (10).

- Then, this addition of u_c^i to u_g^i in (10) compared to (3) means that a priori the non-crowd out necessary condition is enlarged[23]: this laundering warm-glows away tends a priori to be favourable to non-crowd out notwithstanding its being due to warm-glow.

- Therefore, by erasing the effects of an individual's all warm-glows from the objective function, the effect of her warm-glows on non-crowd out is reinforced. More specifically, the role of the effective warm-glow, that for responsibility, is not affected. The laundering can only bear on the ineffective sacrifice warm-glow. And laundering the effect of the sacrifice warm-glow from the objective function ends up adding the effect of this sentiment in the non-crowd out condition.

Proposition 3. The moral paradoxes.

A moral or political discarding of warm-glow satisfaction of vainglory, vanity or superiority (and irrationality) from the social ethical objective does not change the warm-glow effect of pure responsibility but now adds that of sacrifice, thus a priori favouring non-crowd out and the existence and size of the gift in question – in spite of the fact that the discarding actually bears on the sacrifice warm-glow alone.

One consequence is that if, in the whole population, there is no responsibility warm-glow but there is some sacrifice warm-glow, then the moral laundering of the social objective may suffice to permit the possibility of gifts (if μ>0) or of more than one gift with some other's altruism (if μ=0). Condition (10) naturally implies that the gift of a person without warm-glow of any type is fully crowded out if μ>0 or if some other person is an altruist.

5.3 External preferences

There may be several reasons to erase external preferences from the social objective, essentially two types of them. Some such preferences may be immoral, such as envy, jealousy or sentiments of superiority. Another, more general reason may just be that external

[23] However, the variables in all the terms of the conditions no longer have the same value, and hence an opposite conclusion is a priori possible with some form of the utility function.

preferences constitute externalities, they are nosy opinions about others' business, and as a result their possible effects may have to be discarded.

At any rate, the effect of discarding them is thoroughly different from the case of warm-glows. Indeed, deleting a $u_{c_i}^{j}$ for $j \neq i$ from a condition (5) of the policy choice of t_i simply deletes this effect from the non-crowd out condition (3). The reason is that these terms and preferences do not intervene a priori in individuals' choices of their gifts (if the corresponding judgements do not induce kinds of "warm-glow" preferences or aversions).

5.4 Warm-glows alone

In the case of warm-glows alone, i.e. in the absence of external preferences, the laundered non-crowd out condition (10) becomes

$$\lambda_i \cdot (u_g^i + u_c^i) \geq \Sigma_{j \neq i} \lambda_j u_x^j + \mu w' \qquad (11)$$

or

$$\lambda_i \cdot (u_x^i + u_c^i + u_g^i) \geq v, \qquad (12)$$

that is, with condition (4),

$$\lambda_i u_1^i \geq v .$$

5.5 Representation of both cases by the same notation

In all the following – except the explicit formulas of section 7 – the two cases of non-laundered and laundered warm-glows are jointly represented by the same notation: arguments g_i in function u^i can represent either the responsibility warm-glow alone for an absence of warm-glow moral laundering, or both the responsibility and the sacrifice warm-glows when there is such a laundering. In this latter case, u_g^i now stands for the former $u_g^i + u_c^i$.

6. Deontic and normative giving

The classical theory of action distinguishes as opposite two classes of motives or reasons for, and hence explanation of, action (i.e. free voluntary or wilful action): in "consequentialism", an action aims at its consequences. In "deontological" motives or reasons for, or explanation of, action, the action is desired for itself, valued in itself ("deontology" is the study of "duty", but, applied to the action, the adjective would rather be "deontic", and not all motives for desiring an act in itself can be classified as "duty" – e.g. beauty, hence aesthetics, habit, or the "warm-glows" considered above). Norms of action belong to the deontological category. They

may be social or moral or both (shame and guilt are the sanction-sentiments for failing to obey social or moral norms, respectively). The distinction between consequentialist and deontological explanations is both indispensable and highly ambiguous. For instance, "having performed an action" is a kind of consequence of the action, and there may be a duty to want some (other) consequence or just "to induce the best consequences possible", or a norm to have such desires; an action can also have several joint motives or reasons of different kinds. Yet it is important to keep the distinction.

Applied to giving, altruism aiming at improving the poor's income or welfare is clearly a consequentialism (but most moral systems make it a duty or a norm not only to help but also to have altruistic sentiments). In contrast, warm-glow giving is a priori deontic. However, giving because this is a duty or a norm may arouse no further sentiments. The possible pure satisfaction of having obeyed (or obeying) a duty or having followed (or following) a norm need not arouse the kind of rather immoral warm-glow sentiments discussed above. Yet they may also develop into them. The deontological motives in general, and particularly duty itself, focus on the individual's responsibility for the action, precisely the fact needed for non-crowd out.

Having a duty, the guidance of a norm, or sentiments such as altruism or warm-glows, always result from both social influence of various possible kinds (education at all ages, examples, etc.) and individual autonomous choice. This latter factor – autonomy in the Rousseau-Kant sense – implies individual responsibility which thus intervenes at this level also.

When norms or duties intervene, the full explanation requires explaining them also. This a priori requires investigations in the sociology of values (possibly for warm-glows also) or in their rationality such as the reasonings pointed out below (section 12) including Kant's social rationality (which, however, also considers consequences).

7. The basic warm-glow efficiency conditions

The basic efficiency condition for public goods with warm-glows permits one to see simply some important properties. If $G=\{i:g_i>0\}$ is the set of actual givers, condition (9) gives, for all $i\in G$,

$$\lambda_i\cdot(u_x^i+u_g^i)\geq v\geq\Sigma_{j\in G}\lambda_j u_x^j+\mu\, w'\,. \qquad (13)$$

Assume that the poor's income is socially valuable, $v>0$, which implies $\mu>0$ or $u_x^j>0$ for at least one j. Condition (13) then implies, for $i \in G$, $u_x^i+u_g^i>0$, and $\lambda_i \geq v/(u_x^i+u_g^i)$. Then (13) for all $i \in G$ implies

$$\Sigma_{i\in G}[u_x^i/(u_x^i+u_g^i)]+\mu\, w'/v \leq 1.^{24} \qquad (14)$$

with $u_x^i+u_g^i=u_1^i-u_c^i$ from condition (4). This confirms that with $u_g^i=0$ for all i, there can be no giver in the redistributive regime ($\mu>0$) and at most one in the basically giving regime ($\mu=0$) (condition (14) writes $|G|+\mu w'/v \leq 1$ where $|G|$ is the number of givers). Condition (14) also shows that if the number of givers $|G|\to\infty$, its satisfaction requires $u_x^i/u_g^i \to 0$ for almost all givers (except, perhaps, for a finite number of them). In order of magnitude and on average, the last unit of a gift should be provided at least $|G|$ times more for the glory of the giver than for the relief of poverty, where $|G|$ may be several or many millions (but duty may replace glory).

For the morally laundered Pareto-efficient fiscal policy, a similar derivation from condition (12) gives the condition

$$\Sigma_{i\in G}[u_x^i/(u_x^i+u_g^i+u_c^i)]+\mu\, w'/v \leq 1 \qquad (15)$$

with the denominators being u_1^i from condition (4), with similar conclusions with reference to both sacrifice and responsibility warm-glows (produced by gifts g_i)[25].

8. Average conditions: The homogeneous case

The case in which all the non-poor are identical is called the *homogeneous case*. This identity is basically in function u^i and initial income Y_i, and it will result here that it will also be in all the individualized variables for each number n. This case is of course not realistic, but its properties provide the relevant averages, and, in the case of large n, properties which actually hold for "almost all i", that is except perhaps for a vanishing fraction of the i (possibly a fixed number of them). In this homogeneous case in the present model, function U is symmetrical in the u^i, and since the values of the u^i are equal, all λ_i are equal : $\lambda_i=\lambda$ for all i and we denote $\mu'=\mu/\lambda$. All the individualized quantities, functions and derivatives are the same for

[24] With sign = if $t_i>0$ for all $i \in G$ (every giver is also taxed).

[25] With $\mu=0$, sign = ($t_i>0$), and writing the denominator as u_1^i, condition (15) is the classical public-good efficiency condition of Dupuit, Wicksell or Samuelson for the case $u_g^i=u_c^i=0$ for all i.

all *i* and will be denoted by dropping index *i*. Without external preferences and pure sacrifice (tax) warm-glow, this utility function is $u(y,x,g)$. Denote as

$$E_x^g = g u_g / x u_x$$

the elasticity of substitution of warm-glow or deontic giving for altruism in function *u*, for $xu_x \neq 0$. $E_x^g > 1$ means that the individuals are more deontic or warm-glow givers than altruists. We consider in this section the basically giving regime $\mu=0$.

Then, condition (7) writes

$$u_g \geq (n-1)u_x \tag{16}$$

or

$$E_x^g \geq (n-1)g/x. \tag{17}$$

This is satisfied if

$$E_x^g \geq ng/x \tag{18}$$

the share of gifts in the poor's income,

which is satisfied if

$$E_x^g \geq 1 \tag{19}$$

since $x=X+nt+ng$ and $ng/x \leq 1$.

The number *n* does not appear any longer in condition (19). If $t>0$, conditions (16) and (17) write with sign =.

Proposition 4

In the homogeneous basically giving regime, giving (non-crowd out) is possible if the utility elasticity of substitution of warm-glow or deontic giving for altruism does not fall short of the share of gifts in the poor's income and a fortiori of one.

With $gt>0$, the conditions for the choice of g and t are

$$u_1 = u_g + u_x = nu_x, \tag{20}$$

with $y=Y-c$, $x=X+nc$ and $c=t+g$. If

$$u = y + f(g,x),$$

$u_1=1$ and

$$u_x = f_x = 1/n$$

$$u_g = f_g = 1-(1/n).$$

With a Cobb-Douglas utility function

$$u = \alpha \text{Log } y + \beta \text{Log } x + \gamma \text{Log } g$$

with constant $\alpha,\beta,\gamma>0$, one obtains

$$nc=(n\beta Y-\alpha X)/(\alpha+\beta)=(n-1)\beta g/\gamma-X$$

which gives g, c and hence $t=c-g$ for each n. When $n\to\infty$, these solutions become $c=\beta Y/(\alpha+\beta)$ and $g=\gamma Y/(\alpha+\beta)$, hence $g/\gamma=c/\beta=Y/(\alpha+\beta)$ and $t=Y(\beta-\gamma)/(\alpha+\beta)$.

9. Large number paradoxes

9.1 General conditions: one gift requires almost everybody's selfishness

Everybody prefers anyone suffering from poverty to be less poor, both being anywhere in the world. These are large numbers. Actual givers are also numerous. These numbers are also large within redistributive constituencies, such as nations or large organizations of charity and solidarity. (Actually the results for large numbers obtain practically for much smaller numbers.) Hence, consider the basic non-crowd out necessary condition for gift g_i (without external preferences):

$$\lambda_i u_g^i \geq \Sigma_{j\neq i}\, \lambda_j\, u_x^j + \mu\, w'(x). \qquad (7)$$

With bounded u_g^i and u_x^j, condition (7) shows that, when $n\to\infty$, its satisfaction requires that $u_x^j \to 0$ for almost all j (specifically, except perhaps for a finite number of them). The number of $\tilde{u}_x^j$ not lower than any positive number has to be finite.

Proposition 6. The large number paradox

If there is one gift (at least) from a large population, almost everybody tends to feel that the poor have enough (to attach no value to a betterment of the poor's situation). There is at most a finite number of marginal altruisms u_x^j not lower than any given positive number.

Moreover, it seems that a warm-glow of giving should normally rest on the view that the relief of need is a valuable thing (or that other people whose opinion the giver cares for have this view). Section 10 will study these induced warm-glows. In the case of direct deontic giving, however, this relation may be in the social formation of the norm (rather than in the individual's abiding by it). Condition (7) for $n\to\infty$ implies that $\Sigma\lambda_j u_x^j$ and $n\bar{u}_x$ where $\bar{u}_x = (1/n)\Sigma u_x^j$ are bounded since u_g^i is bounded. Hence $\bar{u}_x \to 0$. Moreover, $u_x^j \to 0$ not more slowly than $\bar{u}_x$ does for almost all j. If individual i is such an individual and $u_x^i \to 0$ implies $u_g^i \to 0$ as u_x^i does, for $\mu=0$ (the practically giving regimes) condition (7) requires $\Sigma_{j\neq i}\lambda_j u_x^j \to 0$. This condition then tends to $0\geq 0$. However, the left-hand side tends to zero not more

slowly than $\bar{u}_x$ does, and the right-hand side tends to zero as $n\bar{u}_x$, hence more slowly than $\bar{u}_x$ and than the left-hand side, and condition (7) is not satisfied for all sufficiently large *n*.

If u_x^i has the order of magnitude of $\bar{u}_x$, condition (7) when $n\to\infty$ requires that u_g^i / u_x^i increases not more slowly than *n*. This raises two puzzles. First, it may be that $u_x^i \to 0$ because *x* increases, as with standard goods. Then u_g^i should tend to be disconnected from u_x^i. It becomes a kind of "pure warm-glow". This raises a basic puzzle for its explanation (and perhaps in a sense, again, its rationality). Second, the poor's income is not like any other good. Can one be satiated in other people's destitution? When the population increases, for both non-poor and poor, are the poor newcomers less worthy of consideration than the former ones? Is it all right to be like President Spiro Agnew who used to answer proposals to visit ghettos with "when you have seen one, you have seen all"? Or like the French bourgeois ladies who answer demands for charity with "I have my poor"? Section 9.5 will consider this issue.

The consequence of the vastly counterfactual proposition 6 may be that warm-glow is not the right explanation of non-crowd out. The explanation may be, notably, that there is some implicit cooperation or putative reciprocity (see section 12). However, an explanation can be found within the warm-glow explanation, it fits with common observations of large numbers of warm-glow or deontic givers, and it has very striking implications.

9.2 Asymptotic non-vanishing of the non-poor and of transfers

"Lim" denotes limit when $n\to\infty$. For any variable *a*, denote $\tilde{a} = \lim a$. For any magnitude *a*, denote $\bar{a} = a/n$. For any set of *n* quantities a_i, denote $\bar{a} = 1/n\ \Sigma a_i$.

We can assume, for the before-transfers endowments, $\tilde{\tilde{Y}}_i < \infty$ for all *i*, and $\tilde{\tilde{X}} < \infty$ (the number of poor may vary as the total population does, as in a forthcoming model). Then, if non-poor *i* does not become poor and even starve, $\tilde{y}_i > 0$, that is $\tilde{y}_i = \tilde{Y}_i - \tilde{c}_i > 0$ and thus $\tilde{c}_i < \infty$. If this happens for almost all *i*, $\tilde{\tilde{c}} < \infty$ and $\tilde{\tilde{x}} = \tilde{\tilde{X}} + \tilde{\tilde{c}} < \infty$.

Moreover, if $\tilde{c}_i = \tilde{t}_i + \tilde{g}_i > 0$ for almost all *i*, $\tilde{\tilde{c}} > 0$ and hence $\tilde{\tilde{x}} = \tilde{\tilde{X}} + \tilde{\tilde{c}} > 0$.

Therefore, $0 < \tilde{\tilde{x}} < \infty$ if almost all non-poor remain so (and even do not starve), and either are taxed or give.

Proposition 7. Non-vanishing of non-poor and of contributions

In large numbers, the poor's income per non-poor is bounded from the non-vanishing of the non-poor and does not vanish from that of transfers.

9.3 The necessary logarithmic altruism and independent gift choice.

Consider now the homogeneous practically giving regime, with $u=u(y,x,g)$ (since the presence of c for sacrifice warm-glow has no effect on crowding out conditions). Non-crowd out $g>0$ implies $u_1=u_x+u_g$. If $t>0$, $u_1=nu_x$. Then $u_1=u_g+u_1/n$, hence $u_1=u_g$ for large n since $\tilde{u}_1<\infty$ for $y>0$. Also $\bar{x}\,u_1=xu_x$. For each n and g, the policy chooses t that maximizes nu, by comparing the effects of t on the variables $y=Y-g-t$ and $x=X+ng+nt$. It thus considers

$ndu=nu_1dy+nu_xdx=nu_1\cdot[dy+\bar{x}\,(dx/x)]$ or equivalently $u_1\cdot[dy+\bar{x}\,(dx/x)]$ or, for $n\to\infty$ and $u_1=\tilde{u}_1$,

$$du=dy+\tilde{\bar{x}}\,(dx/x). \tag{21}$$

It may compare dx/x with any $f(y)\,dy$ (for instance $f(y)=1$ or $f(y)=1/y$) in domains excluding $y=0$ for which the "non-poor" are actually poor and even starve. Then

$$du=f(\tilde{y})^{-1}\,f(y)\,dy+\tilde{\bar{x}}\,(dx/x)$$

and

$$u=f(\tilde{y})^{-1}\int f(y)dy+\tilde{\bar{x}}\,\mathrm{Log}\,x+\varphi(g), \tag{22}$$

where $\tilde{\bar{x}}\,\mathrm{Log}\,x$ is the "altruistic utility" and $\varphi(g)$ the "warm-glow utility" function.

For instance

$$u=y+\tilde{\bar{x}}\,\mathrm{Log}\,x+\varphi(g) \tag{23}$$

if $f(y)=1$, or

$$u=\tilde{y}\;\mathrm{Log}\,y+\tilde{\bar{x}}\,\mathrm{Log}\,x+\varphi(g) \tag{24}$$

or

$$u=y^{\tilde{y}}\;x^{\tilde{\bar{x}}}\,\psi(g) \tag{25}$$

for $f(y)=1/y$.

Then, the givers choose g that maximizes u, with, from the general form (22) with $dg=dx=-dy$ and $y=\tilde{y}$,

$$du/dg=\varphi'(g)-1+\tilde{\bar{x}}/x$$

and, since $x=n\bar{x}$ and $\tilde{\bar{x}}/n\bar{x}\to 1/n\to 0$,

$$\lim du/dg=\varphi'(g)-1.$$

Hence, in large numbers the gift $g=\tilde{\tilde{g}}$ is chosen by the maximization of $\varphi(g)-g$. This choice depends neither on x (which is natural since $x=\infty$) nor on $\tilde{\tilde{x}}$ or t. It is per se independent of any aspect of altruism – represented by preferences about x (although in addition altruism may be necessary to explain it as explained in section 10). For instance, if the warm-glow utility $\varphi(g)$ is strictly concave, $\tilde{\tilde{g}}=0$ if $\varphi'(0)\leq 1$ and, if $\varphi'(0)>1$, $\tilde{\tilde{g}}>0$ is determined by $\varphi'(\tilde{\tilde{g}})=1$ (gift g has an upper bound since $g<Y-t\leq Y$). Hence, a priori, both asymptotic giving and non-giving out may occur.

For instance, if the givers are concerned with the relative variations of the gift in addition to those of x in the general form (22) and of y in forms (24) and (25), $\varphi(g)=\tilde{\tilde{g}}\ \mathrm{Log}\ g$ and the utility function is Cobb-Douglas

$$u=y^{\tilde{y}}\ x^{\tilde{\tilde{x}}}\ g^{\tilde{\tilde{g}}}.$$

The general *logarithmic form of the altruistic term* $\tilde{\tilde{x}}\ \mathrm{Log}\ x$ in the general form (22) for large n is a particular specific structure. However, it is *not fortuitous*, and hence not a priori unlikely, because it results from the integration of the condition $xu_x=\bar{x}\,u_1$ for large n, which results from the condition $u_1=nu_x$ which results from the case $t>0$. Now $t\in[0,Y-g[$ can reach its lower bound $t=0$ but cannot reach its upper bound $Y-g$ because this would mean $y=0$ (the non-poor would be poor and even starve). In particular, for the function u in the full domain, we would have $u_1=\infty$ when $y=0$. Hence $t>0$ is not a "knife-edge" intermediate case but one of the two a priori possible cases.

Hence, non-crowd out in large number is a standard possibility. This result is to be contrasted with the ideas and reasons that hold free riding to increase with the number of participants n. Empirically, such an increase in free riding with n is often observed with low numbers n. However, giving in large numbers is a usual situation. Ribar and Wilhem (2002) provide a number of empirical examples. However, this explanation implies two particular structures: independent choice of gift and logarithmic altruism.

The general form (22) entails $\tilde{u}_x=0$ and $\lim xu_x=\tilde{\tilde{x}}>0$. The general condition for $g>0$, $u_g\geq(n-1)u_x$, or $u_g>nu_x$ for large n, is $\bar{x}\,u_g>xu_x$ and hence, for $n\to\infty$ with form (22), $\tilde{\tilde{x}}\,\tilde{u}_g\geq\tilde{\tilde{x}}$ or $\tilde{u}_g\geq 1$, and $\tilde{u}_g=1$ for $\tilde{t}>0$ ($\tilde{u}_1=1$ by assumption). At any rate, for $\tilde{t}>0$ and large n, $\bar{x}\,u_g=xu_x$, that is,

$$g=\bar{x}\,(gu_g/xu_x) \qquad \text{or} \qquad ng/x=gu_g/xu_x=E_x^g \tag{26}$$

Let us summarize the results.

Proposition 8. Giving in large numbers: standard and independent, logarithmic altruism
In large numbers, for the homogeneous practically giving case with social taxation,
1) Both non-crowd out and crowd out of gifts are a priori possible and likely. In particular non-crowd out is a standard situation.
2) This depends on the warm-glow or deontic utility of the gift only.
3) This warm-glow utility and hence the choice of the gift or of not giving are independent of altruism and self-interest,
4) The utility of altruism is additively separable and proportional to the logarithm of the poor's income.
5) The share of gifts in the poor's income is the elasticity of substitution of altruism for warm-glow or duty.

The logarithmic asymptotic form results from taxes and from the non-vanishing of non-poor's individual income and contribution. It represents a sensitivity for relative variations of *x*. This has a classical psychological meaning, the Weber-Fechner law (sensation varies as the logarithm of the excitement does). In the present case, it says that "compassion decreases like the relative decrease in poverty". For ordinary public goods, this logarithmic form is a plausible structure of decreasing returns to scale. For the relief of poverty, however, it raises a major moral problem. If there were one poor only, or a given number of poor, this may represent decreasing returns to scale in their relief as perceived by the givers, or by the receivers and endorsed by the givers. However, if the number of poor increases as the number of non-poor does, this increase is reflected in absolute variations *dx* of incomes, and, in this respect, decreasing returns would mean that new coming poor are less important than already existing ones, an a priori unjustified inequality. Could one be relatively satiated in the amount of destitution in society?

9.4 The general case

Consider $u^i = u^i(y_i, x, g_i)$ for all i (no external preferences and a presence of c_i for a sacrifice warm-glow has no effect on the crowding out conditions). If $t_i>0$,

$$\lambda_i u_1^i = \Sigma \lambda_j u_x^j + \mu\, w'(x). \qquad (27)$$

For n=∞, this is

$$\tilde{\lambda}_i \tilde{u}_1^i = \Sigma \tilde{\lambda}_j \tilde{u}_x^j + \tilde{\mu} \lim w'(x). \qquad (28)$$

We have $\tilde{u}_1^i \in]0,\infty[$ ($\tilde{y}_i > 0$), $\tilde{\lambda}_j \in]0,\infty[$ and $\tilde{u}_x^j \in [0,\infty[$ for all j, $\tilde{\mu} \geq 0$ and $\lim w'(x) \geq 0$.

Hence, condition (28) implies that $\tilde{u}_x^j \geq 0$ for at most a finite number of j, $\tilde{u}_x^j = 0$ for almost all j in this sense, and $\tilde{u}_x^j > \alpha$ can hold for at most a finite number of individuals j for any given number $\alpha > 0$. If $\tilde{u}_x^j > 0$ holds for at least one j, condition (28) may hold even if $\tilde{\mu} = 0$.

If $\tilde{\mu} = 0$ and $\tilde{u}_x^j = 0$ for all j, one should have $\Sigma\tilde{\lambda}_j \tilde{u}_x^j > 0$. Writing $e_j = x u_x^j$ individual j's *relative marginal altruism*, one has, from condition (27),

$$\bar{x}\lambda_i u_1^i = (1/n)\Sigma\lambda_j e_j = e \tag{29}$$

which defines e. Assuming $\tilde{\bar{c}} > 0$ and since $\tilde{y}_i > 0$ for all i, $\tilde{\bar{x}} \in]0,\infty[$. Hence condition (29) implies

$$\tilde{e} = \lim (1/n)\Sigma\lambda_j e_j = \tilde{\bar{x}}\tilde{\lambda}_i \tilde{u}_1^i > 0 . \tag{30}$$

This implies $\tilde{e}_j > 0$ for a non-vanishing fraction of the j.

Moreover, from condition (27),

$$\tilde{e} = \tilde{\bar{x}}[\tilde{\lambda}_i \tilde{u}_1^i - \tilde{\mu} \lim w'(x)] < \infty$$

whatever possible μ. If $e_j = x u_x^j \to \infty$, it is no faster than x and n do since $\tilde{u}_x^j < \infty$. Hence $\tilde{e} < \infty$ if and only if $\tilde{e}_j = \infty$ for no more than a vanishing fraction of the j, that is, if $e_j < \infty$ for almost all j.

For a j with $\tilde{e}_j \in]0,\infty[$ and sufficiently large numbers, $x u_x^j = \tilde{e}_j$ and hence

$$u^j = \tilde{e}_j \text{ Log } x + \phi^j (y_j, g_j), \tag{31}$$

the sum of an altruistic utility term and of an "income-warm-glow-deontic" utility function ϕ^j.

Individual j chooses g_j that maximizes u^j for given t_j and c_k for all $k \neq j$. We have

$du^j/dg_j = \tilde{e}_j/x + d\phi^j/dg_j$ and, for $n \to \infty$, and hence $x \to \infty$ since $\tilde{\bar{x}} > 0$,

$du^j/dg_j = d\phi^j/dg_j$.

Hence this choice of g_i maximizes function ϕ^j. This function does not depend on $\tilde{e}_j$ and has no relation with individual j's altruism. Denoting $\phi_g^j = \partial\phi^j/\partial g_j$ and $\phi_1^j = \partial\phi^j/\partial y_j$, the condition is $\tilde{\phi}_g^j - \tilde{\phi}_1^j \leq 0$, with $\tilde{\phi}_g^j = \tilde{\phi}_1^j$ if $\tilde{g}_j > 0$, the asymptotic non-crowd out of gift g_j. If $\tilde{\phi}_g^j < \tilde{\phi}_1^j$, $\tilde{g}_j = 0$ and gift g_j vanishes.

Proposition 9. Necessary asymptotic logarithmic altruism and independent gift choice in the general case.

If there is one fiscal transfer (at least), in large numbers

1) There is at most a finite number of marginal altruisms positive or larger than any given positive level.

2) If all marginal altruism vanish and in the practically giving regime, relative marginal altruism should not vanish for a non-vanishing fraction of the non-poor.

3) Relative marginal altruism is finite for almost all non-poor.

4) Utility functions satisfying conditions 2) and 3) are additively separable in altruism which is proportional to the logarithm of the poor's income.

5) These people choose their gift a priori independently of this altruism.

6) Each gift can a priori either exist or not.

The general reasons and remarks proposed for the homogeneous case are specified and confirmed by this result. In both cases, warm-glow may, in addition, depend on altruism as considered in section 10.

The result is factually and morally puzzling. Moral altruistic non-poor with $\tilde{u}_x^i > \alpha \geq 0$ cannot be more than a finite number, a vanishing fraction. All non-poor, except a non-vanishing fraction of them in the practically giving regime, can have a marginal altruism u_x^i decreasing faster than *x* and *n* increase.

The general non-crowd out condition for gift g_i in the practically giving regime,

$$\lambda_i \cdot (u_g^i + u_x^i) \geq \Sigma \lambda_j \, u_x^j$$

with sign = if $t_i>0$, amounts, for sufficiently large *n* with $\bar{x} >0$ since $\tilde{\bar{x}} >0$, to

$$\bar{x}\, \lambda_i \cdot (u_g^i + u_x^i) \geq e,$$

and hence

$$\tilde{\lambda}_i \cdot (\tilde{u}_g^i + \tilde{u}_x^i) \geq \tilde{e} / \tilde{\bar{x}},$$

and, with the most common $\tilde{u}_x^i = 0$,

$$\tilde{\lambda}_i \, \tilde{u}_g^i \geq \tilde{e} / \tilde{\bar{x}}$$

with sign = if $t_i>0$ (hence form (30) since $g_i>0$ implies $u_1^i = u_x^i + u_g^i$, hence $\tilde{u}_g^i = \tilde{u}_1^i$).

9.5 The Agnew paradox: the necessary large number immorality

Caring about variation dx through the relative dx/x may be due to perception illusion for large x, but, then, this is not morally adequate. Morally, the last poor should not be less important than the first one. Then, if, in the homogeneous case, $n\to\infty$ entails $u_x\to a>0$, a constant, for instance if $\tilde{\bar{x}}$ Log x in form (22) is replaced by ax, for large n one has $nu_x=na>u_1=1$ when $n>1/a$. This means that tax t should transfer all non-poor's incomes, towards making them poor to begin with.

Actually, large numbers are the case not only for altruists, non-poor and givers, but also for the poor themselves. And a higher income of the poor can have two causes with opposite effects on poverty: a higher income of specific or average poor, which lowers poverty, and a larger number of poor with similar incomes, which augments poverty (the number of poor is the classical measure of poverty).

For the homogeneous model, consider that the poor are identical, in number πn proportional to the number n of non-poor, with a constant integer π, with the same income $x/\pi n=\bar{x}/\pi$, the same initial income $X_o=X/\pi n$, receiving the same share of gifts and distributive taxes, and each with the increasing differentiable utility function $w_o(x/\pi n)$. A non-poor should be less poor than a poor, that is

$$y=Y-c>X_o+c/\pi \tag{32}$$

or

$$c<\pi(Y-X_o)/(1+\pi). \tag{32'}$$

An increase in n means the corresponding duplication of society's two classes.

For distinguishing the two effects on variable x, it should be replaced, in utility function $u(y,x,g)$, by $z=\pi n w_o(x/\pi n)$ (which denotes a utilitarian concern for the poor), with $u(y,x,g)=v(y,z,g)$. Denoting

$$E[w_o(\bar{x}/\pi)]=(\bar{x}/\pi)\cdot w_o'(\bar{x}/\pi)/w_o(\bar{x}/\pi)$$

the elasticity of $w_o(\bar{x}/\pi)$, one has

$$dz/z=E[w_o(\bar{x}/\pi)](dx/x).$$

Introducing this dx/x in equations (21), one obtains form (22) where Log x is replaced by $\{1/E[w_o(\tilde{\bar{x}}/\pi)]\}$Log z. Since Log z=Log πn+Log $w_o(\bar{x}/\pi)$, the utility has additive logarithmic decreasing returns to scale with respect to both each poor's welfare w_o and the number of poor πn. For very large n, $w_o=w_o(\tilde{\bar{x}}/\pi)$, and the welfare of an additional poor is altruistically valued, but as $\tilde{\bar{x}}/\pi n$, hence less and less when their number increases and towards zero.

If all poor are considered equally, the term including Log z in the equivalent of form (22) is replaced by $b\pi n\, w_o(x/\pi n)$ for some constant $b>0$. Then, however, $u_x=b\, w_o'(\bar{x}/\pi)\to b\, w_o'(\tilde{\tilde{x}}/\pi)$, and $nu_x>u_1=1$ for sufficiently large n. This implies that the optimum tax t transfers all non-poor's income, which contradicts condition (32).

Moreover, the simple fact that function u be increasing in the number of poor πn for given individual poor's welfare w_o violates a concern for lower poverty. The concern for the poor should not have the utilitarian additive form to begin with.

The most straightforward way to take account of the number of poor in a general situation where this number is m would be to replace x by x/m in the altruistic utility function (1). The only effect is to replace u_x by u_x/m in all the analyses. This is favourable to the non-crowd out condition in the practically giving regime $\mu=0$ (it amounts to multiplying u_g by m) and to $t=0$. The number of poor is then preferred to be low, but only because of its effect on the sharing of the total poor's income x. The normal poverty-averse altruistic utility function should be both increasing in the poor's individual welfare or per capita income, and decreasing in the number of poor, with at least two variables short of less aggregated models. However, in the foregoing analysis, π is just a given demographic factor, and n a hypothetic number for studying the effects of large numbers.

Another solution consists in replacing the term including Log z in the equivalent of forms (22) by $\beta w_o(x/\pi n)$ for some constant $\beta>0$. Then, $n\, u_x\to(\beta/\pi)\, w_o'(\tilde{\tilde{x}}/\pi)=\gamma$. If $\gamma>1$, there is again the impossible full transfer by tax t. If $\gamma<1$, $\tilde{t}=0$. If $\gamma=1$ some $\tilde{t}>0$ is possible. Moreover, a logarithmic form for the poor's welfare function w_o is not unreasonable (for instance because of the Weber-Fechner law). Then $\gamma=\beta/\tilde{\tilde{x}}$ and $\gamma >$, = or <1 according as $\beta >$, = or $<\tilde{\tilde{x}}$.

The two last considered solutions, however, amount to the non-poor being concerned with some concept such as the "average poor" or the "representative poor". The number of poor is no longer a concern. This is "Agnew altruism" ("when you have seen one, you have seen all"), a hardly acceptable position. Many poor is worse than few.

Psychologically, however, it is impossible to have a non-vanishing empathy for each of an infinite number of needy. This large number is also the reason that leads Kant to call charity an "imperfect duty": it has to be done but cannot be done sufficiently.

Proposition 10. The Agnew paradox.

In large numbers, the just, equal, non-vanishing concern for all needy leads to instability. Concern for an average or a representative needy is possible but hardly moral.

9.6 External preferences

If we now introduce the possibility of external preferences, the non-crowd out condition (3) includes terms $u^{j}_{c_i}$ which tend to improve the condition if they are negative and may be as numerous as the number of non-poor. A priori, $\Sigma_{j\neq i}\lambda_j \cdot (u^{j}_{x} + u^{j}_{c_i})$ may be limited even if $\Sigma_{j\neq i}\lambda_j u^{j}_{x}$ is not. However, this implies that, on average, people dislike an extra contribution by others about as much as they altruistically approve its contribution to the poor's relief. Although the importance of sentiments of envy, inferiority or superiority in society should not be underestimated, it seems rather unlikely that they could have this effect. One reason is that, for each person, such comparisons are often limited to persons of some group for reasons of both estimated relevance and information, and this group tends to be small compared to the large number of people. However, from a moral point of view for the choice of optimum taxes t_i, the information issue may be irrelevant, and this may also be the case of the sentiment of relevance of comparisons insofar as it also depends on information about other people. Yet moral may rather demand erasing the effects of most of these sentiments rather than extending them.

10. Altruism warm-glows and praise warm-glows

10.1 Endogenous warm-glows

We have seen that in standard cases in large numbers, non-crowded out gifts are provided independently of any a priori altruism. However, the most common reason for attaching a warm-glow or moral value to one's giving to the poor rests on or is supported by some notion that it is a good thing that the gifts help the poor, and such a notion is what is called altruism here. This altruism can be that of the giver herself, or that of other persons. As a general term, we call *praise warm-glow* the effect, on the giver, of other people's praise, approval, or absence of or lower reproach, despise or scorn, for her giving. Warm-glows that ultimately rest on some altruism are called *altruism warm-glows*. They are *self altruism warm-glows* when this altruism is that of the giver and *praise altruism warm-glows* when it is that of other

persons who, as a result, judge the gift, the giving and the giver.[26]. We are concerned with the effects of the last units of gifts. Formally, then, the issue is the dependency of u_g^i (or u_c^i) on u_x^j, for *j=i* (self altruism) or *j≠i* (praise), and more simply their being jointly zero or not. In these warm-glows, the giver is appreciated because of her giving (or fiscal) contribution to the poor's welfare, and not because she is altruistic, compassionate or dutiful.

Two other effects will be studied implicitly with the praise altruism warm-glow because they have the same logical structure and hence raise the same issues and yield similar conclusions: *gratitude* and *second-order altruism* (altruism towards the altruists). Both the receivers and other altruists may be grateful towards the giver who may appreciate it. The structure of this effect is similar to that of praise, and gratitude may be a cause of praise (along with more objective approval, which may be more important for other non-poor, but both are intricately mixed). The logic of this effect, as concerns the other altruists, is considered here along with praise (and under this heading). Moreover, the effects of appreciating altruists' satisfaction have the same structure as those of appreciating their praise. This may be important in some cases. As a general rule several motives can be present about one gift and concur to it (with addition of the corresponding marginal effects u_g^i or u_c^i).

The effect of praises on the giver depends on how her sensitivity to it aggregates the (actual, supposed or potential) judgments of others. In the two practically limiting cases she is sensitive to some kind of average praise, or the effects are added (and the number of praisers counts). Three types of altruism warm-glows are thus considered: "self", "average praise" and "total praise". "Self" and "average praise" altruism warm-glows give roughly similar results, and "total praise" give different ones. The case of large numbers will again give startling results. Realism will again lead to replacing moral altruism concerned with absolute variations *dx* by altruism concerned with relative variations *dx*/*x*. This will give four types of results. The non-crowd out condition is not respected for "self" and "average praise" altruism warm-glows with absolute altruism and is respected for "total altruism warm-glow" and relative altruism. For the other cases, the result depends on the sensitivity of the giver's "utility" to the praise induced by some altruistic increase in praisers' utility due to the gift (or of the giver's warm-glow utility induced by her altruistic utility caused by her gift). The non-crowd out condition

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[26] Praise or disapproval about a gift g_i may also result from other people's external preferences $u_{g_i}^j$ or $u_{c_i}^j$ and not or not only from their altruism u_x^j, notably for comparisons of gifts or contributions or for norm following. This is not further studied here.

requires this sensitivity to be sufficient. However, this condition is rather extraordinary for absolute altruism and total praise warm-glow: for instance, the giver should be more sensitive to the praise induced by a praiser's altruism than to her own altruism, for each of a possibly large number of actual or potential praisers. The fourth case alone, self or average praise altruism warm-glows with relative altruism turns out to give neat conditions for the two possible cases of crowd out and non-crowd out. The homogeneous and the general case are considered successively.

10.2 The homogeneous case

Consider first the simplest case, the homogeneous case. The non-crowd out necessary condition is

$$u_g \geq (n-1)u_x + \mu' w'(x). \qquad (33)$$

Self-altruism warm-glow implies $u_g=\alpha u_x$ for some $\alpha>0$. Then condition (33) is not respected when $n>\alpha+1$ (in particular in large numbers no matter how large given or bounded α may be).

For praise altruism warm-glow, two cases may be considered. In one, the person considers others globally, perhaps along the lines of the "generalized other" of the sociologist G.E. Mead. The resulting u_g^i depends on some average of the u_x^j for $j\neq i$. In the homogeneous case, this writes again $u_g=\alpha u_x$ for some $\alpha>0$, with the same result as for a self altruism warm-glow (crowd out of gift for sufficient n).

However, the number of praisers may matter for the giver. There may be several of them. They may even be numerous or very numerous. This may happen for some gifts. It is even the standard case in the not unfrequent situation in which the giver is motivated by what other people's opinions would be if they knew about this particular gift or absence of it. In the homogeneous case again, if the effects of others' praise are additive, the overall effect is proportional to the number. The effect may also be, more moderately, an increasing but strictly concave function of the number, in particular with an upper bound. The proportional case is a priori the most favourable to creating an important warm-glow (convexities could describe some addiction to praise, which is not uncommon).

The proportional case would be $u_g=(n-1)\alpha u_x$, with some $\alpha>0$. Coefficient α, the *praise multiplier*, is the increase in utility induced by the praise per unit of the increase in the praiser's utility that induces the praise. The praise multiplier is said to be *expansive* when $\alpha\geq 1$ (strictly for sign >). The amount $\alpha-1$ is the *praise surplus*, that is, the marginal utility added

by the praise to the marginal utility that induces it. Note that, in this homogeneous case, u_x is both a praiser's and the praised giver's altruistic marginal utility.

Then condition (33) gives

$$(n-1)(\alpha-1)u_x \geq \mu' w'(x) \geq 0. \tag{34}$$

If $\mu'=0$ (the practically giving case), condition (34) is satisfied for any $\alpha \geq 1$ and n (since $n>1$). If $\mu'>0$ (the distributive regime), condition (34) requires $u_x>0$ and $\alpha>1$. For large n, condition (34) is

$$n(\alpha-1)u_x \geq \mu' w'(x) \tag{35}$$

which implies

$$\alpha - 1 \geq \tilde{\nu}\ w'(x) \geq 0 \tag{36}$$

since $nu_x \leq u_1$ from $t \geq 0$ and writing $\nu = \mu'/u_1$. This is a condition on the praise surplus $\alpha-1$. For this w in the social welfare function U, one can for instance have $w=x$, $w'=1$ and $\alpha - 1 \geq \tilde{\nu}$ for condition (36), or $w(x)=\pi n\, w_o(x/\pi n)$, $w'(x)=w_o'(\bar{x}/\pi)$ and, for $n \to \infty$, condition (36) becomes

$$\alpha - 1 \geq \tilde{\nu}\ w_o'(\tilde{\bar{x}}/\pi) \geq 0. \tag{37}$$

The condition $\alpha>1$ means that a giver is more sensitive to the praise of one altruist because of her gift than she is to her own altruism. This high sensitivity to praise is plausible for a small number of praisers but becomes rather strong and unlikely if it held for each of a large number of praisers.

However, since asymptotic non-crowd out requires that individuals' relevant sensitivity be for the relative dx/x, the altruism warm-glows are, rather, the following. For "self" or "average praise" altruism warm-glows,

$$u_g = \alpha x\, u_x \tag{38}$$

for some number $\alpha>0$, or more generally

$$\gamma(g)u_g = \alpha x\, u_x \tag{39}$$

for some function $\gamma(g)>0$, and in particular

$$gu_g = \alpha\, x\, u_x \tag{40}$$

because of the meaningfulness of the elasticity of substitution of altruism for warm-glow gu_g/xu_x.

For "total praise altruism" warm-glow,

$$u_g = \alpha n\, x\, u_x \tag{41}$$

or

$$\gamma(g)u_g=\alpha n\, x\, u_x \tag{42}$$

for some function $\gamma(g)>0$, and in particular

$$g\, u_g=\alpha\, n\, x\, u_x. \tag{43}$$

Cases (39), (40), (42) and (43) are simply derived from cases (38) and (41) respectively by replacing α by $\alpha/\gamma(g)$ or α/g. This is possible since the condition considered is for $g>0$ and $\tilde{g}>0$.

For self or average praise altruism warm-glow, with form (38), the non-crowd out condition (33) is

$$\alpha x u_x \geq (n-1)u_x+\mu' w'(x) \tag{44}$$

or

$$(\alpha x-n+1)u_x \geq \mu' w'(x)\geq 0. \tag{45}$$

For $n\to\infty$, since $nu_x\leq u_1$ from $t\geq 0$, condition (45) implies

$$\alpha\tilde{\tilde{x}}-1\geq\tilde{\nu}\ w'(x)\geq 0. \tag{46}$$

With $w(x)=x$, this is $\alpha\tilde{\tilde{x}}\geq 1+\tilde{\nu}$, and with $w(x)=\pi n w_o(x/\pi n)$, this is

$$\alpha\tilde{\tilde{x}}-1\geq\tilde{\nu}\ w_o'(\tilde{\tilde{x}}/\pi)\geq 0. \tag{47}$$

For $\mu'=\nu=0$ (the practically giving case), the condition is simply

$$\alpha\geq 1/\tilde{\tilde{x}}\,. \tag{48}$$

With form (40), α is replaced by $\alpha/\tilde{g}$ in conditions (47) and (48), and in particular (48) becomes

$$\alpha\geq\tilde{g}/\tilde{\tilde{x}}\,, \tag{49}$$

the asymptotic share of gifts in the poor's income.

For total praise altruism warm-glow, form (41) in the non-crowd out condition (33) gives

$$\alpha n x u_x \geq (n-1)u_x+\mu' w'(x)$$

or

$$(\alpha n x-n+1)u_x \geq \mu' w'(x)\geq 0$$

or, when $n\to\infty$,

$$(\alpha x-1)nu_x \geq \mu' w'(x)\geq 0 \tag{50}$$

If $t>0$, $nu_x=u_1$, and condition (50) is

$$\alpha n\tilde{\tilde{x}}-1\geq\ \tilde{\nu}\ w'(x)\geq 0$$

which is always satisfied for $\tilde{\tilde{x}}>0$. For form (43), α is replaced by $\alpha/\tilde{g}$ and the same conclusion holds.

Proposition 11. Altruism warm-glows in the homogeneous case

In the homogenous case and large numbers,

1) With an absolute (and moral) altruistic appreciation,

1a) Self altruism warm-glow and average praise altruism warm-glow imply full crowd out.

1b) Total praise altruism warm-glow permits non-crowd out only if an individual is sufficiently more sensitive to each altruist's praise for her gift than to her own altruism, and only if she is not less sensitive in the practically giving regime.

2) With relative altruistic appreciation,

2a) Self and average praise altruism warm-glows permit non-crowd out only if an individual's warm-glow sensitivity to her own altruism or to others' average altruism through praise, α, sufficiently exceeds $1/\tilde{\tilde{x}}$ (or the share of gifts in the poor's income $\tilde{g}/\tilde{\tilde{x}}$ if the warm-glow appreciation is also relative), and does not fall short of it in the practically giving regime.

2b) The non-crowd out necessary condition is always satisfied with total praise altruism warm-glow.

10.3 The general case

10.3.1 Absolute altruism

The general necessary condition (7) for $g_i>0$ writes

$$\lambda_i\cdot(u_g^i+u_x^i)\geq\Sigma\,\lambda_j u_x^j+\mu\,w'(x). \qquad (7')$$

Individual *i*'s self altruism warm-glow can be $u_g^i=\alpha u_x^i$ for some number $\alpha>0$. This amounts to multiplying individual *i*'s marginal altruism u_x^i by $1+\alpha$ for her individual choice.

For large *n*, $\Sigma\,\lambda_j u_x^j$ has the order of magnitude of $n\bar{u}_x$. When $n\to\infty$, since the left-hand side of condition (7') is bounded, so should $n\bar{u}_x$ be, and hence one should have $\bar{u}_x\to 0$. This implies $u_x^j\to 0$ not more slowly than $\bar{u}_x$ for almost all *j*. Assume it is the case for *j=i*, and that individual *i* has a *self altruism warm-glow*. Then $u_g^i\to 0$ as u_x^i does, by hypothesis. Hence $\lambda_i\cdot(u_g^i+u_x^i)\to 0$ not more slowly than $\bar{u}_x$. Condition (7') implies $n\bar{u}_x\to 0$ and $\mu=0$ (the practically giving regime). At the limit, it becomes $0\geq 0$, which is satisfied. However, $\lambda_i\cdot(u_g^i+$

$u_x^i) \to 0$ not more slowly than $\bar{u}_x$ and $\Sigma \lambda_j u_x^j \to 0$ as $n\bar{u}_x$ does, hence more slowly than $\bar{u}_x$, and condition (7') is not satisfied.

With *praise altruism warm-glow*, in the *average* case u_g^i has the order of magnitude of $\bar{u}_x$ by hypothesis, and the foregoing result holds: since $\lambda_i \cdot (u_g^i + u_x^i)$ is bounded, when $n \to \infty$, condition (7') requires that $n\bar{u}_x$ also is, hence $\bar{u}_x \to 0$ and $u_x^j \to 0$ not more slowly than $\bar{u}_x$ for almost all *j*. If this is the case for *j*=*i*, and individual *i* has an average praise altruism warm-glow, $u_g^i \to 0$ not more slowly than $\bar{u}_x$ does and $u_g^i + u_x^i \to 0$ not more slowly than $\bar{u}_x$ does. When $n \to \infty$, $\Sigma \lambda_j u_x^j \to 0$ as $n\bar{u}_x$ does, hence more slowly than $\bar{u}_x$, and hence condition (7') is not satisfied even if $\mu = 0$.

If the effects of all praises are *additive*, for individual *i*, $u_g^i = \Sigma_{j \neq i} \alpha_j^i \, u_x^j$, where α_j^i is the extra satisfaction that individual *i* derives from individual *j*'s praise for an increase of her gift, per unit of individual *j*'s altruistic satisfaction increment. α_j^i is the marginal *praise multiplier* of praiser *j*'s satisfaction into praised *i*'s satisfaction. Then condition (7) writes

$$\Sigma_{j \neq i} (\lambda_i \alpha_j^i - \lambda_j) \, u_x^j \geq \mu \, w'(x) \tag{51}$$

with the possible case $\mu = 0$. If function *U* has a utilitarian form $U = \Sigma u^i + \mu' w(x)$, so that marginal variations of the u^i are comparable, condition (51) writes

$$\Sigma_{j \neq i} (\alpha_j^i - 1) \, u_x^j \geq \mu' \, w'(x), \tag{52}$$

with the possible case $\mu' = 0$. A praise multiplier with such marginally comparable utility functions is said to be *calibrated*. It is *expansive* when $\alpha_j^i \geq 1$, that is, the marginal utility induced by the praise does not fall short of that which induces it, and $\alpha_j^i - 1$ is the corresponding *praise surplus*. Condition (52) says that there should be sufficiently many $\alpha_j^i > 1$ by a sufficiently large surplus for sufficiently high u_x^j.

Proposition 12. Altruism warm-glows

1) Self and average praise altruism warm-glows entail crowd-out of almost all gifts in large numbers.

2) Non-crowd out with additive praise altruism warm-glow is possible only when sufficiently many calibrated praise multipliers are expansive with sufficient praise surpluses for sufficiently altruistic praisers, so as to compensate the opposite effects.

The praise itself, consisting of spoken or written words, attitudes (applause, approving smile), and more or less perceptible feelings is not easily represented and is not represented here. The praise multiplier is actually the "multiplication" of two facts due to each of the participants: the praise as depending on the praiser's satisfaction (here altruism), and the praised person's appreciation of the praise (the warm-glow). Insofar as marginal utilities can be compared, expansive praise multipliers describe a particularly high sensitivity to praise. The same model also describes the different phenomenon of *second-degree altruism*, to wit giving to the poor in order to satisfy other people who value the poor's welfare. Then $\alpha^i_j > 1$ means that the giver is more satisfied by an increase in this third person's utility than this person herself is. This is particularly strong altruism. For both praise warm-glow and second-degree altruism, it is not sufficient, for condition (52) to be satisfied, that individual *i* be very sensitive to the praise or the satisfaction of a few people she knows well. The negative terms with $\alpha^i_j < 1$ (such as $\alpha^i_j = 0$) have to be compensated.

10.3.2 Relative altruistic appreciation

Therefore, given the a priori importance of self- altruism warm-glow and the importance of praise, proposition 12 does not a priori seem to be a satisfactory explanation of reality. It rests on absolute marginal altruistic valuations (*dx*). However, although this is a priori the moral valuation (see section 9), we have seen that, applied to the structure of the utility functions, it leads to a contradiction in large numbers (suppression of the non-poor), and that the only standard altruistic sensitivity which permits non-crowd out in this case is the relative one (*dx*/*x*). This form may also apply to the altruistic motive for self and praise altruistic warm-glows.

For self and average praise altruism warm-glows, u^i_g may depend on u^i_x as

$$u^i_g = \alpha_i \, x \, u^i_x \tag{53}$$

and

$$u^i_g = \alpha_i \, x \, \bar{u}_x \tag{54}$$

with some constant $\alpha_i > 0$, respectively. However, the person *i*'s relevant sensitivity to g_i may also be different, which can be expressed by multiplying u^i_g by some function $\gamma_i(g_i) > 0$. In

particular, the meaningfulness of the elasticities of substitution of g_i for x in utility functions suggest a relevant comparison between the relative variations dx/x and dg_i/g_i, with forms

$$g_i u_g^i = \alpha_i x\ u_x^i \tag{55}$$

and

$$g_i u_g^i = \alpha_i x \bar{u}_x \tag{56}$$

with some constant $\alpha_i>0$.

Consider the non-crowd out condition (7) in the practically giving case ($\mu=0$), replace u_g^i by one of the forms so defined, multiply both sides by $\bar{x} = x/n>0$, denote $e_j = x u_x^j$, $e=(1/n)\Sigma\lambda_j e_j$, $\varepsilon = x\bar{u}_x$, $r_i=\lambda_i e_i/e$, $\rho_i=\lambda_i\varepsilon/e$, and let $n\to\infty$. The following conditions obtain for conditions (53), (55), (54) and (56) respectively, for the asymptotic values $\tilde{\bar{x}}$ and $\tilde{g}_i>0$ of $\bar{x}$ and g_i,

$$\alpha_i \tilde{r}_i\ \tilde{\bar{x}} \geq 1, \tag{57}$$

$$\alpha_i \tilde{r}_i \geq \tilde{g} / \tilde{\bar{x}}, \tag{58}$$

$$\alpha_i \tilde{\rho}_i\ \tilde{\bar{x}} \geq 1, \tag{59}$$

$$\alpha_i \tilde{\rho}_i \geq \tilde{g} / \tilde{\bar{x}}. \tag{60}$$

These conditions give the lowest α_i for the non-crowd out conditions in the four cases.

For total additive relative praise altruism warm-glow, the relation writes

$$\gamma_i(g_i)\ u_g^i = \Sigma_{j\neq i}\ \alpha_j^i x u_x^j$$

with some function $\gamma_i(g_i)>0$ which can be, in particular, 1 or g_i, for $\alpha_j^i \geq 0$ and $\alpha_j^i>0$ for at least one j (individual i may be insensitive to some other altruistic people's actual or implicit praise – or not be second-degree altruistic towards them). This amounts to replacing α_j^i by $\alpha_j^i x/\gamma_i(g_i)$ in relations (51) and (52). Then, for $\bar{x}\to\tilde{\bar{x}}>0$ when $n\to\infty$, and with $0<\tilde{g}_i<\tilde{Y}_i<\infty$, the non-crowd out conditions (51) and (52) so transformed are satisfied when $n\to\infty$.

Proposition 13. Relative altruism warm-glows

With relative altruism warm glows, in large numbers,

1) Self and average praise altruism warm-glows satisfy the non-crowd out condition if the sensitivities α_i satisfy inequalities (57) to (60),

2) Total additive praise warm-glows always satisfy the non-crowd out condition.

The praise motive may depend on the personality of the praiser and on her relationship with the giver. It requires a priori two types of information: of the praiser about the gift for the praise to exist, and of the giver about the praise. However, we have noted that the praise motive commonly extends to putative praise, to praise that would be if the praiser knew about the gift (one may even give because one's late grand-mother would have liked it or approved of it). This putative approval is also commonly seen as being by practically everybody, in fact by society at large. This gives it a kind of objective value. It also applies to gifts by other people. The specific altruism of particular evaluators is lost of sight. This becomes a social norm, and usually also a moral norm because of the moral nature of helping the poor (these norms tend to sanction non-giving by shame or guilt, respectively).

11. Specializations

People actually support, by any means, a small fraction only of the needy they think should be supported (in the world). They do this with the understanding that other people support other needy, although they do not stop their support when they see that this other support fails (they may then support these other needy also, perhaps by transfer of some of their former support). Moreover, actual supports are usually within some kind of community such as the family, the nation (notably for fiscal transfers), or communities that are more or less local, professional, ethnic, religious, and so on. There are several reasons for this. The means of transfers may already exist (e.g. public budgets) and the transfers may be easier within such a community (e.g. locally). A common general opinion is that one has a particular duty to help needy members of such a community one belongs to (an opinion held notably by people who help and by people who judge them). One is held to have a particular responsibility for such support. This is a notion of a kind of organic solidarity. Moreover, the duty or desire to help may be fostered by a better information about these needs. In addition, one may particularly like other people in these categories. However people nevertheless think that other people in need should be supported. The previous analyses may have to be completed by thinner ones distinguishing both the welfare of various types of needy people and the gifts or contributions that contribute to it. Yet the universal aspect also remains. At any rate, some of these communities are cases of large numbers by themselves – such as nations.

This differs from Duncan's (2004) "impact" giving in which a giver wants to "make a difference", although there is some relation. Then, if $W(\Sigma g_i)$ is a receiver's welfare, an "impact giver" *i* cares neither about *W*, nor about her own gift g_i but about the difference

$W(\Sigma g_i) - W(\Sigma_{j \neq i} g_j)$ she is accountable for (given others' gifts). This would be g_i if W were linear and the difference comes from the concavity of W. As a consequence, there is an advantage in specializing each gift to a particular needy, as much as possible, although it seems that the issue of information about the effect of one's gift and the emotion it arouses may be the main reason for this choice (the effect is lost if I know I have some specific impact but am not informed about it). Duncan also considers preferences for others' gifts being lower (but not the paradoxical effects through policy noted above), and the effects of both cooperation and non-cooperation between givers.

Giving with other people giving as a hope but not as a condition is common for differently specialized givers but it does not appear in so simple a way for contributors to the same poor's welfare directly motivated by this welfare. Implicit cooperation and unilateral giving exist in this latter case but with somewhat more elaborate thinking.

12. Moral-social reasons: implicit agreements, lateral and putative reciprocities, categorical imperatives.

Motives for giving to fight poverty refer sometimes to some moral reasoning or theory more or less elaborate. Three types of them can apply to all public goods: implicit agreement, lateral reciprocity or fair matching, and reasonings of the "Kantian" family. On the contrary, the motive of putative reciprocity is specific to giving (irrespective of the public good issue). These reasonings tend to induce giving or contributing, and to arouse other people's praise for such behaviour. These effects can a priori be described as kinds of "warm-glows" in the structure of preferences or utility functions, but only after the modelling of the corresponding theory. Then, these propensities can be mitigated by the effects of g_i or c_i on y_i (self-interest).

An *implicit agreement* between the non-poor for giving or contributing belongs to the theoretical family of social contracts (Rousseau and Hume are clear about the public good nature of social contracts – each contributes and benefits from all others' contributions, for Rousseau). There may be some psychic moral cost of shirking (total or partial free riding), including the effects of other people's judgments, compared with the material advantage taking others' reaction into account. *Lateral reciprocity* or *matching* is reciprocity with co-givers or contributors (not the usual meaning of the term reciprocity as providing a standard return-gift), that is: given that they give or contribute, then so do I; given that they provide their fair share, I provide mine.[27] The coordination between the participants is realized either

[27] For application to public goods see Sugden (1984) and Kolm (1984).

by a public enforcement of these gifts (they are no longer voluntary in a formal sense but everybody voluntarily abides by this constraint, which thus is not binding, because others' contributions are then guaranteed), or by a sequential dynamics.[28] *Reasons of the Kantian family* include folk-Kantianism ("I give because what if nobody gives" – the most common "reason" given for voting in large elections), or ideas' closer to Kant's ("follow the rule that you could want to be followed by everybody"). Kantian conducts raise a problem of consistency: each individual may assume that the others act or follow rules different from those they actually choose, notably if they also have the same Kantian reasoning and conduct (people have in general different preferred general rules because they derive them from their different overall preferences).[29] Moreover, the ideals of both fair moral matching and Kantian conducts may a priori not be Pareto efficient; they are for "consistent rules" belonging to a particular set only.[30]

Helping because one wants to be helped when one needs it is an explicit example given by Kant of a universal rule and of a reason to want it. The more specific motive of *putative reciprocity* is also common and leads to give without the public good problem. This is: "I help them because they would have helped me if our situations were reversed, or I would have been helped by others if I needed it, or they would have helped others if they could" (respectively direct, extended and reverse reciprocity, applied hypothetically)[31]. This choice of one's behaviour given that of the others can be described by the maximization of one's utility function and a model of the interaction. However, there are two types of genuine reciprocity.[32] In *balance* (or *matching*) *reciprocity*, each gift tends to establish some kind of balance with the other. This leads to a warm-glow structure. In *liking reciprocity*, by contrast, one comes to like the benevolent person who gives to oneself (or to others) and to give to her because one likes her. Then the relation is simply a cause of altruism. An actual return gift of a putative balance reciprocity is no longer a contribution to a public good. It is a personal (two

[28] People may give at time t because they have seen others giving at time $t-1$ or at any past dates, and this may in particular support a steady state. Sequential contributions with or without reciprocitarian motives are analyzed in Kolm (1987), Admati and Perry (1991), Fershtan and Nitzan (1991), Varian (1994), Marx and Matthews (2000) and Masclet, Willinger and Figuières (2007).

[29] Kant did not see this because he actually considered very crude rules only (lie or do not lie, help or do not help). Specific contributions to a public good would be different.

[30] The theory of "consistent" rules of fairness or of universal conduct the unanimous application of which yields Pareto-efficient states is presented in Kolm (2008c and 2009). A particular form of such a general philosophy is the theory of "moral teams" presented in appendix C.

[31] I discovered putative reciprocity when I heard a little girl say, about the tsunami in South-East Asia: "we should help them because they would help us if this happened to us" (a very counterfactual hypothesis given the place she was living in, the Alps).

[32] See Kolm (2008a).

by two) relation with the receiver. The number of givers is irrelevant. The induced u_g^i need not vanish if the u_x^i do.[33]

Finally, cooperation resulting from straight repeated or sequential giving meets diriment obstacles. Giving less or not at all in order to punish another giver who failed to give at the expected level first punishes the poor still more, it also punishes all the other altruistic co-givers, and, with the large number, at any rate the actions of a "small" giver are not even noticed by other people.

13. Rebate and matching-grant neutrality or dual effects (cost or benefit)

In many places and cases, philanthropy is subsidized by tax exemptions or rebates, or encouraged by matching grants. The basic thing about these policies is that, a priori, they have no effect, if all is considered by the analysis and the agents, including with all kinds of warm-glows, external preferences, etc.[34] Notably, the financing of the cost of these policies should not be forgotten. Other things equal, they are financed by taxes. This product could have been directly provided to the poor, that is, what the poor receive from taxes is diminished by this amount. Their income is in this way diminished by the matching grant they receive. Or it is diminished by the rebate or subsidy received by the giver, and the gift minus the rebate is both the cost for the giver and the *final* receipt of the poor for which the giver's choice is responsible. Hence, in all cases, when the giver chooses her gift by balancing the cost for her and the benefit for the poor, both are equal, and this amount is also what the giver or other people may directly value as her gift or as a part of her contribution.

In all the foregoing models, including with all the possibilities of warm-glows, external preferences – utility functions with a priori the most general form (1) – and laundering, if the gift g_i of giver i is augmented by the matching grant $m_i(g_i)$ (with $m_i(0)=0$), the poor receive $g_i+m_i(g_i)$, but the taxes Σt_i finance $m_i(g_i)$ and are diminished by this amount when transferred to the poor. Hence, the poor receive $\Sigma[g_i+m_i(g_i)]+\Sigma t_i-\Sigma m_i(g_i)=\Sigma g_i+\Sigma t_i$. For rebates or subsidies, if the giver i, giving g_i, receives a rebate or subsidy of $r_i(g_i)$ (with $r_i(0)=0$), this is financed from the taxes Σt_i (perhaps, for tax rebates, by a transfer to the income tax fund for leaving other things equal), this amount Σt_i is diminished by this amount $r_i(g_i)$ when it is transferred to the poor, and the poor's benefit due to the gift g_i is only $g_i-r_i(g_i)$,

[33] However, if the object of reciprocity is considered to be the gift relative to the need of the receiver, the return gift may depend on the receiver's means and therefore on other people's gifts to her.

[34] This conclusion in the presence of warm-glows differs from the views of Bernheim (1986) and Andreoni (1990).

which is the cost to giver i. The poor receive, on the whole, $\Sigma g_i+\Sigma t_i-\Sigma r_i(g_i)=\Sigma[g_i-r_i(g_i)]+\Sigma t_i$. All is identical to giver i deciding to give $g'_i=g_i-r_i(g_i)$. In all cases, since the taxes are the same whatever their use, the same result holds if they are not lump-sum.[35]

Of course, if grants, rebates or subsidies are financed, in total or in part, from outside this system, and one forgets about their cost, or if the givers suffer from "gift illusion" and forget about this financing and its effects, other results obtain, with generally increases in the gifts.[36] Then, such a given amount generally enriches the receivers more when it is used for financing matching grants, rebates or subsidies increasing with the gift. In these cases, the cost for the giver differs from the corresponding benefit for the receivers, actually or as they are perceived. This raises, for concerns about a gift or a contribution in themselves, the problem of whether what matters is the giver's actual sacrifice, or the increase in the poor's benefit due to her action, or both, or some combination of both. This choice may more or less differ according as whether the issue is the gift g_i or the contribution c_i.[37] It may also depend on who evaluates (the giver herself or someone else – relevant for induced warm-glows, such as by praise, and for Pareto efficiency). The results may also depend on the hypotheses about the origin of the funds (possibly part exogenous and part endogenous, etc.). They include the determination of the optimum subsidy or matching-grant schedules. The same remarks hold for moral efficiency (along the lines of sections 12).[38]

14. Conclusion

[35] This is the reason of the result of Bernheim (1986) for the case of "pure altruism" (also Andreoni and Bergstrom, 1996). Andreoni (for lump-sum taxes and proportional subsidies) sees well the general logic for the case of "pure altruism", in 1988, but obtains in 1990 a different result for the general "impure altruism" case because he writes (p. 469) that the warm-glow is concerned with the individual gift g_i rather than with $g_i\cdot(1-s_i)$ – where s_i is the subsidy rate for individual i –, which is both the cost for the giver and the benefit for the receivers if the subsidy s_ig_i is financed from taxes and hence deduced from the government's transfers to the poor. This assumption probably results from the three hypotheses that the individual thinks that: the poor will receive g_i; the subsidy $s_i\,g_i$ is given from outside ("as manna from heaven"); and the relevant base for warm-glow is the poor's benefit g_i (hence not reduced by the payment of $s_i\,g_i$ by taxes) and not the sacrifice the individual incurs for it $(1-s_i)\cdot g_i$ – for the items the individual is responsible for (i.e. not t_i). This differs from the assumptions of both the article of 1988 for pure altruism (concerning the financing of the subsidy), and a note mentioning a warm-glow for total sacrifice $g_i\cdot(1-s_i)+t_i$, with the neutrality resulting from the presence of the tax t_i.

[36] The givers do not "see through" the government budget in the expression of Boadway, Pestieau and Wildasin (1989).

[37] For instance, more weight may be put on the cost for the giver for the contribution $c_i=g_i+t_i$ than for the gift g_i by itself, because this cost is emphasized when the relevance of the contribution is justified by the argument that the cost for the giver of the tax paid should be included.

[38] The effects of all these questions are shown in Kolm (2008b).

Fighting incapacitating poverty, perhaps the first economic duty of society, requires, first of all, the organization of the necessary transfers.[39] This is done by both public and private actions, a fact that belies the standard public good model. Both specific preferences about gifts and contributions and implicit cooperation provide the relevant bases of explanation. In this study, the effects of the former have been shown with the very simple but highly revealing device of considering the marginal conditions of agents' interaction. This revealed a number of crucial phenomena, necessary distinctions, and surprising and often paradoxical important results which had escaped previous attention and explain some puzzling facts or raise new questions.

This is the case, for instance, of: external preferences; the distinction between sacrifice and responsibility for warm-glows and for external preferences; the fact that, for warm-glows, one is ineffective (sacrifice) and both are contradictory because a gift (or contribution) for warm-glow is not a gift directly from altruism, which would justify a warm-glow; the reverse and doubly contrarian effect of external preferences (preferring a lower sacrifice induces a higher gift, and preferences about gifts have no effect); the self-defeating laundering of immoral warm-glows (the responsibility effect is augmented by the sacrifice effect which is the one actually erased from the social evaluation); gifts in large numbers implying that almost all non-poor's valuation of decreases in poverty vanishes; nevertheless large numbers often favouring giving; the necessity and immorality of relative (logarithmic) altruism; the shameful necessity of practically caring about an average or a representative poor; the necessarily finite number of minimally moral altruists; the variety of effects of altruism-based warm-glows, according as they are self or average praise on the one hand or total praise on the other hand, and concerned with absolute or relative poverty variations; the necessity of explaining norms of giving; the powerlessness of fully informed grants-in-aid, subsidies and tax rebates. Various types and reasons for implicit cooperation have been noted, notably specialized giving, lateral reciprocity, Kantianisms and implicit agreement (with, again, sometimes a favourable effect of large numbers of givers), plus putative reciprocities.

The facts and possibilities of fighting poverty depend very much on the social, political, institutional and mental structures of the society in question. As a result, for instance, crowd out of private gifts by public policy is lower in the US than in most other societies, notably in those with some "welfare state" ("aid is what we pay taxes for"), and the rationales seem to be more charity and warm-glows, whereas some sense of justice and of

[39] This issue is much broader than the specific one of the working of charitable organizations, which has been much and well studied.

solidarity may be more influent in more homogeneous nations. The characteristics of the various modes of transfers are socially important. Even when they manifest cooperation for producing a public good, public transfers have their shortcomings with the imperfections of the political system, bureaucracy or anonymity. More direct aid also has its shortcomings, however. Altruism is oriented towards the receivers' needs, but its effects are crowded out. Efficient motives rest, for instance, on warm-glows for which the poor's welfare is only a mean to a self-serving objective – the worst Kantian sin. At any rate, giving is an inegalitarian relationship. Charity is often condescending. Not uncommonly, it jeopardizes the dignity of the assisted person. Giving may demand or arouse gratitude when the transfer may just be a very partial correction of an unjust – unfairly unequal or exploitative – wealth distribution. Deontic actions are possible explanations. Kant-like rationality intends to make the individual choice coincide with the social requirement and could refer to altruistic judgment about "the result one can want", but it has the noted shortcomings. Actually social norms are important but have to be explained.

Nevertheless, giving is also the best of worlds when it is empathy and social rightfulness using liberty to remedy destitution and unjust inequalities thanks to a spirit of solidarity both towards receivers and between givers.

Appendix A – Responsibility and information

In the real world, however, the responsibility issue may not be so clear-cut, and questions of information may play a role. Social pressures and even interiorized strong norms of giving may attenuate the person's responsibility for her gift. Moreover, a person might sometimes be considered having some responsibility for the distributive taxes she pays. This happens if these taxes have to result from a collective unanimous agreement (each person's veto gives her full responsibility for the whole of the outcome). One principle of public finance ("liberal social contracts"), in particular for financing public goods, consists of imposing the outcome of such a hypothetical collective agreement. The taxpayer's responsibility is lower if she is only a voter in a vote requiring lower majority, but it comes back if she fully endorses this system.

Moreover, there may be differences in information about g_i and c_i, for individual i and for other people whose opinion influences her warm-glow. Differences in information may not be relevant from a normative point of view, but they are for actual preferences and actions. The giver generally knows her gift g_i. She easily knows the distributive taxes she pays if they are separated from the rest of taxes. If not, she knows her direct taxes, may

estimate her indirect taxes, and may derive t_i from an information about the share of the budget used to help the poor (including by public education, subsidized health care, or other programs). For the effects of the praise or blame of other people on the person's warm-glow, they may just be imagined by her, in particular for the case in which they would know what she knows. Other people may also estimate the person's gifts (she may boast about them), and the distributive tax she pays from some idea about her general taxes (perhaps from her lifestyle) and about the share of the public budget used for helping the poor.

With respect to information for the policy, basing the tax on non-elastic items is much more possible than is commonly said, because nothing is easier than to base a tax on wage earnings on the wage rate practically without cheating (except, perhaps, in extremely small firms), for instance by just exempting overtime labour, and wages provide 9/10 of labour income – the effect on induced education can also be compensated (see Kolm, 2010). The policy information problem then concerns the correlation between the tax base and utilities.

Appendix B - Pareto efficiency for sub-populations

The relation between Pareto efficiency in the two regimes is a particular case of more general properties. Let *z* denote a state, *Z* the set of possible states, $U^i(z)$ the utility function of *any* individual *i*, and *I*, I' and I'' sets of individuals *i*. Say that $z= z^*$ is *strictly Pareto efficient* for the set *I* of individuals *i* if $z^* \in Z$ and, for any $z' \in Z/\{ z^* \}$, $U^i(z') < U^i(z^*)$ for at least one $i \in I$. Strict Pareto efficiency implies ordinary Pareto efficiency. Then, if z^* is strictly Pareto efficient for the population I', it is also strictly Pareto efficient for any population $I'' \supset I'$, and therefore it is Pareto efficient for this population. Note that if $W(\{U^i\}_{i\in I'})$ is a strictly increasing function which has a unique maximum on *Z* at z^*, then z^* is strictly Pareto efficient for population I', and therefore for any larger population $I'' \supset I'$, and it is also Pareto efficient for these populations. In particular, if $|I'|=1$ and I' is made of a single individual *i*, both strict Pareto efficiency for I' and this unique maximum mean a unique maximum of function U^i. This implies strict and usual Pareto efficiency for any population including individual *i*. This can result from individual *i*'s choice of z^* in the set *Z*.

Appendix C – Moral teams

A team is a set of persons with the same aim, notably seeking to maximize the same function (R. Radner). An agreement can be about a function to maximize. A lateral reciprocity

(matching) can be seeking to maximize some function if the other participants do the same. The a priori inconsistency of conducts of the Kantian family are due to the fact that people choose "universal" acts or rules with different objectives, notably maximizing different functions such as their own utility functions. However, since "Kantian" individuals act morally, it may be inconsistent that they do not also evaluate their choice according to a moral criterion. Indeed, Kant insists that moral conduct is not driven by the individual's tastes, that he calls "inclinations" and which include the individual's altruism. These three social ethics can thus lead to all participants wishing to maximize some social welfare function U of form (2) with individual utility functions which are a priori of the most general form (1).[40, 41] The outcome is Pareto efficient with respect to utility functions. Each individual i chooses her gift g_i, and the g_i are independent variables. Therefore, the maximum of U is a Cournot-Nash equilibrium of the game so defined.

However, distributive taxes t_i can also be chosen. The forms of u^i and U entail, with $c_i=g_i+t_i$ and noting $U_{g_i}=\mathrm{d}U/\mathrm{d}g_i$ and $U_{t_i}=\mathrm{d}U/\mathrm{d}t_i=\mathrm{d}U/\mathrm{d}c_i$,

$$U_{g_i}=U_{t_i}+\alpha_i$$

with

$$\alpha_i=\lambda_i u^i_g+\Sigma_{j\neq i}\lambda_j u^j_{g_i}=\partial U/\partial g_i.$$

For the g_i and t_i that maximize U, with the foregoing assumptions,

$U_{g_i}\leq 0$ with sign = if $g_i>0$,

$U_{t_i}\leq 0$ with sign = if $t_i>0$.

If the u^i_g and $u^j_{g_i}$ are not a priori assumed all zero, for the set of g_j and t_j that maximize U, $\alpha_i\neq 0$ except fortuitously. Then U_{g_i} and U_{t_i} cannot both be zero. Hence of the gift g_i and the tax t_i, *if one exists it crowds the other out* at the highest U. Specifically, $g_i>0$ and $t_i=0$ if $\alpha_i>0$, and $t_i>0$ and $g_i=0$ if $\alpha_i<0$. The case $\alpha_i>0$ occurs in particular if $u^i_g>0$ and $u^j_{g_i}=0$ for all

[40] Tastes intervene, then, but not in a self-centered way. In an elaborate work, Bordignon (1990) considers in particular individuals who evaluate with their own different social evaluations, each of which assumes that other people have the evaluator's utility function (tastes) and is, then, utilitarian (it could also be a more general aggregation function, for instance a maximin which would demand ordinal utilities only). The outcome is not Pareto efficient (it is compared with an inefficient political provision). Brekke, Kverndokk and Nyborg (2003) suggest evaluation with "social welfare as I perceive it".

[41] These individuals naturally reveal their utility functions to the others.

$j \neq i$; this is another aspect of $u_g^i > 0$ permitting $g_i > 0$. If $\alpha_i = 0$, which can be seen as non-fortuitous only under the "classical" assumptions implying $u_g^i = u_{g_i}^j = 0$ for all j, g_i and t_i may both be positive; in fact, they are substitutable in all respects (only $c_i = g_i + t_i$ intervenes); this amounts to individual *i* freely paying her distributive tax. In all these conditions, u_c^i and $u_{c_i}^j$ play no role.

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