August 21, 2008

Working paper: http://www.ehess.fr/kolm/document.php?id=251

THE WAR ON POVERTY: PARADOXES AND POSSIBILITIES

Serge-Christophe Kolm

EHESS, IDEP, CREM.

Abstract

The fight against poverty is made by transfers used in various ways and of both private and public origin. Knowing its possibilities and choosing the right policies require, first of all, understanding the possible reasons and motives of these transfers and their consequences. For instance, private contributions to the universal public good of the poor's welfare are not crowded out by fiscal transfers as they should if they were simple individual altruistic decisions. The various a priori possible explanations reveal a number of surprising impossibilities, contradictions, paradoxes and puzzles. The main remaining possible explanations are based on particular non-individualistic rationalities.

Some motives for giving are in the family of relatively direct preferences about gifts or contributions. They are aroused by the sacrifice or the responsibility; held by the givers or by other people; justified by altruism, other reasons, or no real reason; with a public policy respecting immoral sentiments or not; and possibly with tax rebates, subsidies or matching grants. These motives entail various unexpected consequences, but the most problematic result is that the very large number of people tend to imply that a gift is not crowded out only in the counterfactual situation in which almost all non-poor think that the poor have enough.

Other motives are based on reasons for helping in putative reciprocity, or for cooperating in lateral reciprocity (matching), "Kantian" conducts and social contracts. This social-moral cooperation obtains in particular by following consistent rules of fair contribution that induce efficiency.

Keywords: Poverty, giving, taxes, warm-glow, reciprocity, Kant, social contract, rules.

I. ISSUES AND FRAMEWORK

I.1 An overview

Fighting painful poverty is probably the main economic issue, a foremost duty of societies, and the most valuable use of their income. Understanding this question, however, meets a fair number of puzzles, paradoxes, contradictions and impossibilities which are both a main cause of failures and important and perplexing scientific challenges. Formation, equipment and aid are financed by transfers which, first of all, should be obtained. Since everybody prefers the suffering poor to be better off, this is a universal public good (some aid some poor with the understanding that others aid other poor).[1] These transfers are both by free individual private gifts and forced public taxes. However, from the standard theory of public goods,
1) such a fiscal transfer from an altruistic giver cannot transfer wealth because the giver reacts by reducing her gift by the same amount; hence, a priori it cannot be explained or chosen,
2) nevertheless, everybody can be made better off by such fiscal transfers that fully crowd out (almost) all such private gifts; hence, social and political forces tend to induce these fiscal transfers, and then these gifts cannot be explained.[2]

However, private gifts are far from crowded out. They amount to a few percent of GNP (up to 5%).[3] 9 Americans in 10 report having given over the past year. Half of them claim deduction for giving in their income tax report. Moreover, fiscal transfers provide a larger amount (much larger in some countries). Hence, both explaining private and public transfers and choosing the best policy require a more realistic theory. In particular, people may give either not solely for helping the poor or with some sort of implicit cooperation.

As we will see, however, a number of a priori possible explanations fail. A direct appreciation of one's contribution to helping the poor (by gift and tax), because it is one's sacrifice to this effect, cannot explain giving in the presence of efficient taxation. A direct appreciation of one's gift alone because one is responsible for it might be able to provide the explanation but meets another difficulty: with the large number of people, this explanation of

[1] The French bourgeois ladies used to answer demands for charity with "I have my poor".
[2] See Warr (1982), Roberts (1984), Bergstrom, Blume and Varian (1986), and Kolm (1970a, 1970b, 1971). These gifts and fiscal transfers are non-cooperative (in particular with Cournot-Nash behaviour) and the gifts aim at helping the poor.
[3] In the Netherlands.

one gift requires that almost every non-poor think that the poor have enough (moreover, one cannot give in order to be praised or praiseworthy as a compassionate altruist since this altruism is not the motive). These conclusions include the effects of the sensitivity to other people's praise or non-blame. However, it turns out that, with the large number, the effects of altruistic praise should in fact vanish, whereas almost nobody can feel praiseworthy for an altruistic reason. With given norms, habits, traditions or comparisons of giving not justified by altruism, the large number may be favourable to giving, but this still implies the almost unanimous view that the poor have enough. Other preferences of other people about a person can lead to an explanation of her gifts, but only if they want her to *contribute less* (by gift or tax, for reasons of comparison), and with probably a similar problem with large numbers. A policy discarding the effects of immoral sentiments (vainglory, envy, sense of superiority) from the social objective leads to adding the effect of vainglory about the value of the sacrifice of the gift to that about its value as denoting the giver's responsibility for helping, but this still leaves the noted effect of large numbers.

Some kinds of cooperation may be present, but they raise a number of puzzles. Some are various types of implicit cooperation between givers. Giving is sequential, but punishing someone who gives too little by giving little next time first of all punishes the poor (and is inefficient in large numbers). "Folk Kantianism" revealed by the expression "What if nobody gives?" is an important motive in spite of its problematic rationality, but making such a choice by maximizing one's utility assuming that other people give similarly leads to an inconsistent and inefficient overall situation. The "lateral reciprocity" of doing one's share when others do or matching others' gifts can be secured by making these others' gifts compulsory; this is a contradiction in terms, however, but, if all individuals share this motive, this constraint is necessary, reached, but not binding, and the transfers are both forced and free, gifts and taxes. Tax rebates, matching grants and subsidies establish a cooperation between givers and the government, but they turn out to have no effect if the givers (and the analysis) take the cost and financing of these public expenditures into account, whatever the motives for giving.

Notable reasons for helping are found in "doing to others what one wants them to do to you", elaborated in the "putative reciprocity" of helping the poor because they would have helped us if the situations were reversed (or they would have helped others if they could or one would have been helped by others if in need). Lateral reciprocity or matching has more scope in sequential giving. Implicit coordination is also supported by the shame or guilt of

shirking an implicit agreement (a "social contract") to give, and, in an important case, the larger the number of givers the lower the necessary morality (then, as in a previously noted case, free riding is less of a problem the larger the number of participants, contrary to a standard view).[4] Kantian conducts become consistent and efficient if people not only act socially-morally but also evaluate in this way with the same evaluation (that is, givers form a moral "team"). Finally, the theory of rules for contributing to public goods analyzes the rules that lead to efficiency for taxation (or pricing), lateral reciprocity and Kant's own rule-Kantianism.

In the end, individual giving for directly altruistic motives is crowded out by efficient fiscal transfers. Giving when there are such transfers can a priori be explained by non-simply altruistic motives belonging to various possible types summarized in table 1. Getting out of the pure public good trap naturally follows two possible ways: individualization of the value of the gift (preference for one's contribution or gift and putative reciprocity, although others' judgments or attitudes towards oneself often become essential), and implicit coordination (Kantianisms, social contract, lateral reciprocity, sequences). A different division opposes motives that are conveniently described directly with individual preferences and those based on a reasoning. The former divide according to different dichotomies that will be explained. The latter sometimes use counterfactuals (putative reciprocity, Kantianisms, social contract), and sometimes are implicit, non-contractual cooperations (Kantianisms, social contract, lateral reciprocity).

[4] When average compassion decreases faster than the logarithm of the relief of poverty (section III.4).

RATHER DIRECT PREFERENCES

{ sacrifice / responsibility } { justified / shallow } { duty, propriety / preference } { self-praise / others' praise } { own / external }

{ comparative / non-comparative } { few / numerous } { morally laundered / not } { rather direct / reasoning }

REASONINGS

counterfactual {
- **putative reciprocity**
- ***the 3 moral cooperations***
- 1- **Kantianisms**: { folk (act) / Kant (rule) } { full (team) / half (rule) }
- 2- **Social contract** + shame of shirking
- 3- **Lateral reciprocity**, match, fair

} *non-contractual, implicit cooperations*

SEQUENTIAL

SUBSIDIES, REBATES, MATCHING GRANTS

Table 1. The types of non-simply altruistic motives for giving to fight poverty

The associations of the various characteristics that make a difference for the problem at hand lead to a large number of relevantly different types of motives. In addition, each has various different actual manifestations and origins. For instance, the altruistic sentiments, which may intervene indirectly for praising gifts or contributions, may be empathy, compassion, pity, generosity, solidarity, sense of justice or injustice, and so on.[5] Praise and blame, of oneself or by others, also have a number of forms.

The explanations of giving that will be considered apply to contributions to other public goods, all of them or a category of them according to the case, but we shall discuss fighting poverty only here. This public good has particular characteristics: it is the welfare of some people (it may thus be in itself an argument of a social welfare function), it is very important, universal (who does not want the poor to be better off?), moral (the most moral of moral public goods – others can concern culture, the environment, etc. –, which may be

[5] See Kolm 2006.

important for motivations), fully non-excludable (the poor's situation may be hidden, but people do not give in order to know, and this may undercut motives for giving), and provided by both private and public transfers with very numerous givers and still more numerous taxpayers. Hence explanations that may be discarded in the end for some reason, for instance because of the large number of givers, may be valuable for other kinds of public goods.

I.2 Evidence

When asked why they give, people provide a variety of answers in addition to "for helping the poor" for an altruistic motive of pity, compassion induced by empathy, remedying injustice, or solidarity with people belonging to some community of any kind to which they also belong (family, nation, etc., or just mankind). These answers are, notably:

What if nobody gives? ("folk Kantianism");

To follow the rule: "Do to others as you would want them to do to you";

In particular, the needy would have helped me if our situations were reversed; or I would be or have been helped (by anybody) if I needed it; or the needy would help (anybody) if they could (*putative reciprocities*);

I have been helped myself (generalized or direct reciprocity).

I want to be helped if I come to need it (expected reverse or direct reciprocity, or sequential exchange).

The beneficiaries have been helpful in other circumstances (reverse or direct reciprocity).

I would feel bad, guilty, or ashamed if I did not give;

I have to give, I feel I have to (a duty, an obligation); my moral principles, my religion or Reverent X tell me to give;

I like or enjoy giving, it makes me feel good ("warm-glow") or proud;

I help because I am a good person (moral identity assertion); or because I want to be a good person (moral identity seeking); this goodness is generosity and altruism in action, and perhaps also in sentiments along with compassion; I may even seek more or less consciously to become good in sentiment by acting as if I were (by dissonance reduction);

Other people are watching me and judging me; I like approval, praise, esteem (perhaps admiration, glory, prestige or acclaim), and possibly the receivers' gratitude, and I dislike reproaches, reproof, blame (perhaps despise or scorn); I seek to keep or improve my image or my reputation; I have to keep my reputation and that of my group (family, etc.);

Giving is something we do (in our family or other group) (social identity assertion or group warm-glow); it is a tradition or a custom;

I take my share of the burden as others do (*lateral reciprocity*);

I do not shirk the implicit agreement to help (an ethic of the social contract);

I give to show that I have the means to do it;

I want to keep up with other givers; to conform or on the contrary to distinguish myself; not to feel inferior or, on the contrary, to feel superior (*comparative* motives);

I imitate them;

In a number of the previous ways, I follow a moral or a social norm, thus avoiding guilt or shame;

I am just used to give (a habit); my parents gave; I was educated like this;

I receive a tax rebate or there is a matching grant;

All these judgments attach sometimes to the cost I incur for the benefit of the poor in gifts and taxes, or to my responsibility in gifts only;

I give to the poor in order to prevent their taking more and social unrest.

And so on.[6]

These motives for giving lead to different behaviours, reactions to public transfers and consequences, which are never fully obvious, are often opposed to one another, and are often counterintuitive. These effects are shown by modelling these motives and the resulting behaviour.[7] The relative importance of these various motives depends on the particular circumstances. Various motives may be present and work jointly, the same gift or contribution may be involved in several of them.

I.3 "Warm-glows" and reasons

[6] Some of these motives are pointed out in inquiries surveyed by Batson (1998) and Schokkaert (2006). See also Clotfelter (1980, 1985), Woodward (1985), Kahneman and Knetsch (1992), Schokkaert and Ootegem (2000), Spash (2000) and Yamagishi (1986).

[7] A large part of the results will derive from an application and developments of the model of V. Pareto (1913), in which a social welfare function depends on individuals' "utilities" each of which depends on any individuals' "ophelimities" (i.e. welfares). Other people are restricted here to the poor, and giving, fiscal transfers and various motives are introduced. The most general model of the distribution of utilities, welfares, incomes or goods resulting from people's concern about these distributions is proposed in Kolm (1966), in general equilibrium and with transfers.

Individuals' preferences about gifts or contributions that are not only preferences about the poor's wealth or welfare will be denoted as "warm-glows", thus extending, for convenience, focus and illustration, Andreoni's term for a type of joy of giving, although semantically this term describes well a fraction of these motives only.[8] This term will generally be applied more specifically to motives directly described by preferences.[9] However, it proves indispensable to distinguish several types of such warm-glows and to explain them rather than to take them as brute facts. Nevertheless, one common feature is that most of the important results are obtained here very simply by the comparison of the marginal conditions for giving and for efficient fiscal transfers. The following distinctions prove necessary.

– According to their object, there are two types of warm-glows: *sacrifice warm-glows* are concerned with the costs incurred by the person that benefit the poor, hence her gift and the distributive taxes she pays, whereas *responsibility warm-glows* are concerned with her responsibility in this transfer, which, a priori, concerns her decision to give only.[10]

– According to their cause or reason, there are three types of warm-glows: for *altruism-based warm-glows*, the gift or contribution is valued because of the altruism of the giver or of someone who praises her (or still more indirectly by some altruism, for instance that of people who support some institution which influences the giver) in addition to its adding to the poor's welfare; *shallow warm-glows* have no justification and include norms of giving per se, habit, imitation, or some traditions; on the contrary, *justified warm-glows* include altruism-based ones and others which are justified otherwise, for instance by the noted ethics of cooperation, reciprocity or putative reciprocity (purely competitive giving or contributing from the shortly noted comparative warm-glows are to be attached to the "shallow" case).

– *Duty*, *propriety* or *preference warm-glows* refer to the corresponding nature of the motive.

– A *praise warm-glow* results from the giver's appreciation of other people's (or society's) praise (esteem, admiration, non-blame, etc. – it does not refer to self-praise) and is opposed to

[8] For instance, there is avoidance of reproach, guilt or shame, Kant sees duty as necessarily involving suffering, and so on.

[9] In economics, introducing the gift directly in the giver's utility function was suggested by Olson (1965), Arrow (1972), Becker (1974), Sugden (1982) who points out that "social acclaim" of gifts is a priori based on people valuing the benefit, and the consequences were worked out by Cornes and Sandler (1984a, 1984b, 1986, for a contribution to an ordinary public good), Roberts (1984, 1987), Kolm (1984), extensively Andreoni (1989, 1990) for warm-glow, Harbaugh (1998a, 1998b) for prestige, and an abundant literature exhaustively referred to and discussed in several chapters of the *Handbook of the Economics of Giving, Altruism and Reciprocity* (Kolm and Mercier Ythier, eds., 2006).

[10] Qualifications in this respect are presented in appendix B.

the giver's *own warm-glow* – a distinction which is sometimes clear and sometimes less so when the reference is to some shared norm.

– *External warm-glows* may denote the various types – by extension – of preferences about other people's gifts or contributions not directly because they improve the poor's situations.

– *Comparative warm-glows* are own or external warm-glows motivated by comparisons between people's gifts or contributions (comparisons between others' or between others' and one's own).

– The individual preferences defining the efficient policy may be cleaned for immoral sentiments (vanity, vainglory, envy, sentiments of superiority) or not.

– The large number of people concerned is the relevant case.[11]

– With all motives, there may be tax rebates or matching grants for the gifts.

These distinctions underlie the a priori surprising properties that appear straightforwardly. A sacrifice warm-glow cannot explain giving. A responsibility warm-glow can but meets several obstacles. One is the noted logical impossibility to give in order to be praised as a compassionate altruist. At any rate, it turns out that the non-crowding out of one gift because of warm-glow implies, because of the large number of people concerned, that almost all individuals' (marginal) altruism vanishes. This means that almost everybody thinks that the poor have enough, which does not seem to be the case. Moreover, this would just prevent altruism-based warm-glows to explain almost all giving. Shallow warm-glows could provide this explanation, however, and possibly even more favourably with the large number,... but provided that almost all altruism vanishes particularly rapidly – and such behaviour has a low rationality. As noted, also, a person's giving can be explained by other people's "external warm-glows" about her, but if they are for her to *contribute less*; and laundering immoral sentiments such as vainglory or vanity from the government's criterion ends up *extending* their relevance to a sacrifice warm-glow – which now adds to the responsibility warm-glow. However these effects generally leave the same large-number problem. Moreover, tax rebates and matching grants would not help without mistaken views.

[11] Fiscal transfers are usually at national levels. As noted, when people give to particular poor, it is usually with the implicit understanding that other people help other poor. This may even be the case internationally, at the world level. At any rate, the scope of concern is just world-wide: practically everybody is concerned about the pain of poverty wherever it occurs.

Various types of warm-glows are not altruism-based, although the fate of the poor and some concern about it are usually still at the origin or at some background of the value of the items, and they may have a larger or smaller effect. Actual and putative reciprocities lead both to valuing one's gift in itself and to altruism. Some shallow warm-glows and the "external glows" are found in comparative "glows" about giving or contributing: keeping up with others, competitive giving, conforming or on the contrary seeking distinction, egalitarian fairness in contributing, avoiding the shame of giving less than others or seeking superiority in giving or contributing more. Conspicuous giving may also simply exhibit wealth (in addition to all the other possible noted effects).

Implicit cooperation intervenes through a variety of reasons. They are similar to those found for other public goods, but, for some of them, particularities of the nature of the good in question make a smaller or larger difference. A warm-glow just for being a good cooperator cuts a sorry figure compared to a warm-glow elicited by the morality of generosity. Free-riding aid to the poor meets the two noted paradoxes of sequential joint giving (punishing a free rider by giving less first punishes the poor) and of the hypothetical general agreement of some social contract to give (non-free-riding may be favoured by large numbers because it may require a vanishing morality only); they are, respectively, a detrimental effect due to the nature of the public good and a favourable one due to the large number of givers.

Other cooperative reasonings apply as for all public goods. The "Kantian" "Do as if others did the same as you", and the lateral reciprocity of giving given that others give (matching, doing one's share) are anti-Nash principles by excellence. The very common answer "What if nobody gives?" reveals the "Kantian" irrational reason assuming a magical causality. Kant applies his principle to the choice of a rule of action. However, if people choose in this way an act or a rule by maximizing their different utility functions, the result is generally inconsistent and inefficient. However, moral people should have more coordinated evaluations in "Kantian teams" maximizing the same social function, or by following "consistent rules" that lead to Pareto-efficiency, apply also to lateral reciprocity and to taxation or pricing (for which they generalize the Lindhal concept), and constitute the moral

coordinating "mechanism" for public-good provision. Lateral reciprocity can be implemented by the forced and free tax-gifts noted above or in a framework of sequential giving.[12]

I.4 The framework[13]

For fighting poverty, non-poor individual i gives an amount $g_i \geq 0$, pays distributive taxes $t_i \geq 0$ (the part of her taxes used for this purpose) and hence contributes $c_i = g_i + t_i$. Her initial wealth of X_i becomes $x_i = X_i - c_i$. By nature, $x_i \geq 0$, and in fact $x_i > 0$ since, if $x_i = 0$, individual i would be poor or, rather, would starve. The poor receive Σc_i and hence, with an initial wealth of X, have finally $x = X + \Sigma c_i$. They are sufficiently taken aggregatively and have a utility function $u(x)$ with $u' > 0$ (alternatively, there are N identical poor who receive each x/N and have a utility function $u(x/N)$). Individual i has a utility function u^i. It depends on x_i with $\partial u^i / \partial x_i = u_1^i > 0$. When $x_i \to 0$, $\partial u^i / \partial x_i \to \infty$, so that any choice of g_i by individual i or of t_i by the fiscal authority guarantees $x_i > 0$. This implies that, for these choices, $g_i \in [0, X_i - t_i[$ and $t_i \in [0, X_i - g_i[$. Function u^i may also depend on x with $\partial u^i / \partial x = u_2^i \geq 0$ (this may result from an implicit dependence on the poor's welfare $u(x)$, or possibly from a dependence on both x and $u(x)$, and then u_2^i stands for $\partial u^i / \partial x + u' \partial u^i / \partial u$). We shall call the case $u_2^i > 0$ altruism, although it might also result from other reasons such as fear of the poor's social unrest, or comparative national pride in having a lower poverty.

---

[12] Aspects of the "Kantian" provision of public goods are analyzed by Bordignon (1990) and Bilodeau and Gravel (2004) after a particular remark by Laffont (1975), related efficient rules for public goods are considered in Kolm (1970a, 1970b), and aspects of lateral reciprocity are studied by Sugden (1984) and Kolm (1984).

[13] The very simple and general model used here encompasses many models used in the literature which specify variables or relations, explain or justify them by a theory or by observation, and often apply the model to particular important issues. For example, Brekke, Kverndokk and Nyborg (2003) and Konow (2007) consider norms of giving with an ideal level but a lower provision because of self-interest (the relevant issue here will be the distinction between norms justified by altruism and other norms of giving). Holländer's (1990) model of contributions to any public good motivated by the judgment of the relevant people (also Rege and Telle, 2004) applies particularly for the important, moral and universal public good of the relief of poverty. Harbaugh focuses on prestige and applies this model for specific results. Admiration, gratitude and signal of wealth (Glazer and Konrad, 1996) have been discussed. The properties that turn out to be crucial cut across all the specific manifestations and are general distinctions such as: is the warm-glow for the giver's sacrifice or for her responsibility? Does it require altruism or not? Does the social welfare function include the poor's welfare? Does it respect the givers' immoral sentiments (vanity, vainglory, envy, sentiment of superiority) or not? Do the givers abide by some moral reason or by some norm of cooperation, and which ones?

The government chooses taxes t_i to achieve Pareto efficiency given the constraints (in particular the type of agents' behaviour such as Cournot-Nash). This may be for a moral reason and/or for a political one since democracy tends to induce Pareto efficiency.[14] This is described by the maximization of a preference-respecting social welfare function

$$U(\{u^i\}, u), \tag{1}$$

With $\partial U/\partial u^i = \lambda_i > 0$ and $\partial U/\partial u = \lambda \geq 0$. The presence of u in this function formally differentiates this problem from the case of other public goods for the contributors i. However, there are two types of "regimes". In the cases of the *basically giving regime*, function U does not depend actually on u and $\lambda=0$. The highest U nevertheless achieves Pareto efficiency for all individuals *including the poor* in this case if, for each i, *a change in* t_i *affects* u^i (t^i may appear in several arguments of function u^i).[15] This condition is a priori satisfied except fortuitously, and it is assumed. In the other case, the *redistributive regime*, $\lambda>0$. This may manifest a political or social power of the poor (votes or threat of social unrest); or a desire to redistribute more to the poor than manifested by functions u^i alone, for instance a desire of distributive justice elaborated at the level of society whereas the transfers induced by functions u^i alone would more be induced by compassion.[16] Finally, we assume $\lambda_i>0$ for all i

[14] Democracy tends to prevent that society be in a state such that possible others are preferred by everybody (with possible indifference for some). In particular, in an electoral democracy, the existence of such states means that a contending party can propose an alternative program that will carry the unanimity of expressed votes. The following results will derive from the sole Pareto efficiency property of government policy or of optimality. Insofar as government policy is the outcome of political life that produces a Pareto-efficient outcome, the fact that officials do not know the utility functions of the citizens is not relevant. At any rate, Pareto efficiency is defined for constraints including those concerning information. Coase's suggestion that Pareto efficiency always holds if all constraints of all types are taken into account is also relevant here.

[15] For a Cournot-Nash relation between the government policy and the givers, this is for given gifts $\{g_j\}$. The noted condition even needs to hold only at states that are Pareto efficient for the non-poor. From such a state, indeed, change the set of taxes $\{t_j\}$. Then a number of levels u^i change (at least one), since this includes at least those that correspond to a tax t_i that actually changes. All these changing u^i cannot all increase, from the definition of Pareto efficiency (for the non-poor). Hence, at least one decreases. But this decreasing u^i is also a decreasing member of the larger set encompassing all u^i and u. Hence, any possible change in the set of taxes t_i from the state in question makes one member of this larger set of the u^i and u decrease. Therefore, no possible change in the set of taxes t_i from this state makes all the u^i and u increase or not change with at least one increasing. Hence, by definition, the state in question is Pareto efficient for the whole population of the non-poor and of the poor. These properties are applications of general theorems (see appendix A).

[16] Almost all the literature on altruism and giving omits the case of the redistributive regime. Hence it bans the poor from the social welfare function and from the definition of Pareto efficiency, and considers them as altruists' "consumption" only. Exceptions in which the poor's welfare is an

although $\lambda_i=0$ is consistent with a Pareto efficient outcome, since this outcome would generally imply $x_i=0$ which is excluded.

II. WARM-GLOWS AND EFFICIENCY

II.1 The sacrifice and responsibility warm-glows

II.1.1 The contribution or sacrifice warm-glow

Individual *i*'s contribution c_i is the cost for her, her sacrifice, that benefits the poor. If this arouses a warm-glow for individual *i*,

$$u^i = u^i(x_i, x, c_i) \tag{2}$$

with $\partial u^i / \partial c_i = u^i_c > 0$. If individual *i* chooses a gift $g_i>0$,

$$du^i/dg_i = -u^i_1 + u^i_2 + u^i_c = 0. \tag{3}$$

If all individuals *j* may have a similar warm-glow ($u^j_c \geq 0$), the government's choice of tax t_i satisfies

$$dU/dt_i = \lambda_i \cdot (-u^i_1 + u^i_2 + u^i_c) + \Sigma_{j\neq i}\lambda_j u^j_2 + \lambda u' \leq 0 \tag{4}$$

with sign = if $t_i>0$. Conditions (3) and (4) together imply

$$\Sigma_{j\neq i}\lambda_j u^j_2 + \lambda u' \leq 0. \tag{5}$$

This condition does not contain u^i_c. The result is the same as if function u^i of form (2) did not contain c_i and were simply $u^i = u^i(x_i, x)$ (i.e. pure altruism only is possible for individual *i*). *The warm-glow has no effect on crowding out*. However, *the result depends on the regime*. In the *redistributive regime* ($\lambda>0$), $g_i=0$ for all *i*, *crowding out is complete*. In the *basically giving regime* ($\lambda=0$), $g_i>0$ implies $u^j_2=0$ for all $j\neq i$; hence, there is *at most one giver*; at the margin, there is also at most one altruist (the same person) and hence *x* is no longer an actual public good for the givers.

II.1.2 The responsibility warm-glow

---

argument of the government's maximand because of the receivers' political power are found in Roberts (1984), Becker (1978) and, somehow, Peltzman (1976).

However, individual *i* may experience a warm-glow not because of the cost for her that benefits the needy, but because of this cost and benefit she is responsible for. A priori, she does not choose the tax t_i she pays and hence she is not responsible for it. On the opposite, a priori she chooses her gift g_i and is responsible for it. One should thus distinguish the *sacrifice warm-glow* concerned with the cost to the person that benefits the needy (irrespective of who decides), and the *responsibility warm-glow* concerned with the part of this cost and benefit the person chooses and is responsible for. Note that responsibility requires a sacrifice-benefit on which it is applied. This is the gift g_i.[17]

Then, with a responsibility warm-glow only,

$$u^i = u^i(x_i, x, g_i). \tag{6}$$

with $\partial u^i/g_i = u^i_g > 0$. If individual *i* chooses to give $g_i > 0$,

$$du^i/dg_i = -u^i_1 + u^i_2 + u^i_g = 0. \tag{7}$$

If all individuals *j* may have a similar warm-glow ($u^j_g \geq 0$), the government's choice of tax t_i satisfies

$$dU/dt_i = \lambda_i \cdot (-u^i_1 + u^i_2) + \Sigma_{j \neq i} \lambda_j u^j_2 + \lambda u' \leq 0 \tag{8}$$

with sign = if $t_i > 0$. Conditions (7) and (8) together imply

$$\lambda_i u^i_g \geq \Sigma_{j \neq i} \lambda_j u^j_2 + \lambda u' \tag{9}$$

or, denoting

$$v = \Sigma_j \lambda_j u^j_2 + \lambda u' \tag{10}$$

the marginal social value of the poor's income *x*,

$$\lambda_i \cdot (u^i_2 + u^i_g) \geq v, \tag{11}$$

with sign = if $t_i > 0$.

II.1.3 Both warm-glows, the two formulations

However, one may have jointly and distinctly a warm-glow for sacrifice and one for responsibility. The effect of gift g_i cumulates both, but an effect of the distributive tax t_i

[17] The question of information and qualifications about responsibility may be relevant. See appendix B.

represents a pure sacrifice warm-glow. These effects can be represented in two ways in the utility functions. One can write

$$u^i = u^i(x_i, x, c_i, g_i) \tag{12}$$

where c_i represents the sacrifice warm-glow and the argument g_i the responsibility warm-glow only (the sacrifice effect of g_i is taken into account by its presence in $c_i=g_i+t_i$). At the margin, the sacrifice warm glow induces u_c^i, the responsibility warm-glow induces u_g^i, the total warm-glow effect of gift g_i is $u_c^i + u_g^i$, and the warm-glow effect of the tax t_i distributed to the poor is only the sacrifice effect inducing u_c^i.

In an alternative formulation,

$$u^i = u^i(x_i, x, t_i, g_i) \tag{13}$$

where t_i creates a pure sacrifice warm-glow (t_i is the tax, or the part of taxes, that is known to be used to help the poor), and g_i creates both a responsibility and a sacrifice warm-glow. Denote $u_t^i = \partial u^i/\partial t_i \geq 0$. At the margin, the pure sacrifice warm-glow induces u_t^i, hence the pure responsibility warm-glow effect induces $u_g^i - u_t^i = u_r^i$ by definition of this symbol. With formulation (12), $u_r^i = u_g^i$.

With formulation (12), if individual i chooses a gift $g_i>0$,

$$du^i/dg_i = -u_1^i + u_2^i + u_c^i + u_g^i = 0. \tag{14}$$

With similar utilities for all j, the government chooses tax t_i that satisfies

$$dU/dt_i = \lambda_i \cdot (-u_1^i + u_2^i + u_c^i) + \Sigma_{j\neq i}\lambda_j u_2^j + \lambda u' \leq 0 \tag{15}$$

with sign = if $t_i>0$. Then, conditions (14) and (15) imply

$$\lambda_i u_g^i \geq \Sigma_{j\neq i}\lambda_j u_2^j + \lambda u' \tag{16}$$

or

$$\lambda_i \cdot (u_2^i + u_g^i) \geq v \tag{17}$$

which are relations (9) and (11) with different functions.

With formulation (13), if individual i chooses a gift $g_i>0$,

$$du^i/dg_i = -u_1^i + u_2^i + u_g^i = 0. \tag{18}$$

With similar utility functions for all j, the government chooses tax t_i that satisfies

$$dU/dt_i=\lambda_i\cdot(-u_1^i+u_2^i+u_t^i)+\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u'\leq 0 \quad (19)$$

with sign = if $t_i>0$. Then, conditions (18) and (19) imply

$$\lambda_i\cdot(u_g^i-u_t^i)\geq\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u' \quad (20)$$

or

$$\lambda_i u_r^i\geq\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u', \quad (21)$$

that is

$$\lambda_i\cdot(u_2^i+u_r^i)\geq v. \quad (22)$$

This confirms that, in relations (16) and (17), u_g^i represents the responsibility warm-glow only and in nothing the sacrifice warm-glow.

Hence, the responsibility warm-glow offers the obtained possibility for $g_i>0$ (non-crowding out), and the sacrifice warm-glow has no effect.

These different warm-glow and their relations are neatly shown by the neuroeconomic experiments performed by Harbaugh, Mayr and Burghart (2007). The subjects respond to the transfer of some amount of money attributed to them to a food bank by neural excitements of the reward system, and more when it is a voluntary gift from them than when it is a forced transfer. All subjects manifest these effects.

II.2 Irrationality, immorality, and the moral and rational social efficiency

II.2.1 Irrationality of the responsibility warm-glow

The fact that the responsibility warm-glow alone is accountable for non-crowding out in conditions (16) or (21) is puzzling since this assumed sentiment is irrational.

Indeed, responsibility warm-glow is self-contradictory: one cannot give in order to be praiseworthy or praised as a compassionate altruist since this motive is not altruistic compassion. Sacrifice warm-glow, of a different and a priori milder type, avoids this inconsistency since it is not the result of a choice, but it cannot explain giving, as we have seen. The warm-glow objective of giving may then be to deceive others by making them believe that one is a compassionate altruist. This fraud is immoral, nothing to be proud of.

The effect may also be self-deception, leading one to have some impression of being the moral compassionate altruist that one is not, as psychoanalysis may be able to explain. This is an irrationality, however.[18]

II.2.2 Laundering preferences

Another important aspect is that warm-glow is often, in fact, vanity and vainglory, sometimes accompanied by a sentiment of superiority.

Warm-glow thus tends to be immoral and irrational, with important consequences.

The social criterion may have not to respect individuals' immoral social sentiments (should someone be deprived of something because some other people envy her?).[19] It may also have to discard contradictory aspects of individual preferences, but we shortly see that this has no consequence on the non-crowding out condition.

The method for laundering preferences for the effects of some variables, with preferences of general form, consists of assuming that these variables have some fixed level. In general, this level matters for the result. If there is no a priori given natural level for this purpose, the consistent and rational solution consists of choosing the level that would result from choices in which its variability has no effect. In the present problem, this means deleting the effects of the corresponding variables (g, c, or t) in the conditions of the government's choice of t_i (conditions (15) or (19)). New conditions (16) or (20) are then obtained, and the t_i and g_i result from the solution of these 2 n conditions (where n is the number of non-poor i). The effect of the variable in question is erased, and the choosen level of the variable is that consistent with the whole situation. The conditions of the individual choice of the free gifts g_i do not see their form affected since the individuals are free and the present issue is not moral education.

---

[18] The non-altruistic giver may also give because she wants to be a (compassionate) altruist and knows that a classical way to try to have a sentiment is to act as if one had it (with the help of dissonance reduction). This is probably still more praiseworthy than being altruistic.

[19] If Pareto efficiency results from political life, people may impose the government to respect their full preferences, including their vices. However, they may also agree, in the public discussion or in a collective agreement, to discard these immoral aspects for the social moral choice. They may even enjoy that the government discards these regrettable aspects of their preferences that they do not have the willpower to abandon by themselves.

Conditions (15) and (19) do not contain u_g^i. Therefore, laundering preferences for effects due to the responsibility warm-glow has no consequence. This is remarkable since it is precisely the responsibility warm-glow that, in the end, provide the non-crowding out conditions (16) and (21). Moreover, the noted irrationality (contradiction) concerns the responsibility warm-glow. Laundering the effects of the sacrifice warm-glow (for instance because it would be vanity and vainglory) consists of deleting terms u_c^i and u_t^i in relations (15) and (19) respectively. In both cases, the result is

$$dU/dt_i=\lambda_i\cdot(-u_1^i+u_2^i)+\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u'\leq 0 \qquad (23)$$

with sign = if $t_i>0$.

In the first formulation (utility of form (12)), condition (23) with condition (14) give

$$\lambda_i\cdot(u_c^i+u_g^i)\geq\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u' \qquad (24)$$

or

$$\lambda_i\cdot(u_2^i+u_c^i+u_g^i)\geq v. \qquad (25)$$

In the second formulation (utility of form (13)), condition (23) with condition (18) give

$$\lambda_i u_g^i=\lambda_i\cdot(u_t^i+u_r^i)\geq\Sigma_{j\neq i}\lambda_j u_2^j+\lambda u' \qquad (26)$$

or

$$\lambda_i\cdot(u_2^i+u_g^i)=\lambda_i\cdot(u_2^i+u_t^i+u_r^i)\geq v. \qquad (27)$$

This motivation that produces this condition, creating $u_c^i+u_g^i$ in the first formulation and $u_r^i+u_t^i$ or u_g^i in the second, is a *responsible sacrifice warm-glow*, corresponding to both effects of the gift g_i.

This result is important and paradoxical. Conditions (24) and (26) tend a priori to be more easily satisfied than conditions (16) and (21) since a term which can be positive (and is

non-negative) u_c^i and u_t^i respectively, is added in the left-hand side.[20] Psychologically, this means that the sacrifice warm-glow is added to the responsibility warm-glow. The paradox is that by erasing the effect of warm-glow in the objective function, this effect is reinforced as a result, since the sacrifice warm-glow is now added to the responsibility warm-glow in the final conditions for non-crowding out. More precisely, the laundering happens to bear on the sacrifice warm-glow only, and it is the effect of this sentiment that is now added to the non-crowding out conditions.

II.3 The basic efficiency condition

The basic efficiency condition for public goods with warm-glows permits one to see simply some important properties. With the first formulation for instance (function (12)), condition (17) for $g_i>0$ implies

$$\lambda_i \cdot (u_2^i + u_g^i) \geq v \geq \Sigma_{j \in G} \lambda_j u_2^j + \lambda u' \qquad (28)$$

where $G=\{i:g_i>0\}$ is the set of actual givers. Assume that the poor's income is socially valuable, $v>0$, which implies $\lambda>0$ or $u_2^j>0$ for at least one j. Condition (28) then implies for $i \in G$, $u_2^i + u_g^i > 0$, and $\lambda_i \geq v/(u_2^i + u_g^i)$. Then (28) for all $i \in G$ implies

$$\Sigma_{i \in G}[u_2^i/(u_2^i + u_g^i)] + \lambda u'/v \leq 1.^{21} \qquad (29)$$

This confirms that with $u_g^i=0$ for all i, there can be no giver in the redistributive regime ($\lambda>0$) and at most one in the basically giving regime ($\lambda=0$). Another crucial consequence is noted shortly.

For the morally laundered Pareto-efficient fiscal policy, with the first formulation, a similar derivation from condition (25) gives the condition

$$\Sigma_{i \in G}[u_2^i/(u_2^i + u_g^i + u_c^i)] + \lambda u'/v \leq 1, \qquad (30)$$

with similar conclusions with reference to both contribution and responsibility warm-glows (produced by gifts g_i).

II.4 Consequences of the large number

[20] However, the variables in all the terms of the conditions no longer have the same value, and hence an opposite conclusion is a priori possible with some form of the utility functions.
[21] With sign = if $t_i g_i>0$ for all i (everybody pays the tax and gives).

II.4.1 Vanishing individual altruism

When *n* becomes large, the non-crowding out conditions (9), (16), (20), (24) and (26), with limited u_g^i, u_c^i, and u_2^i (or with at least two givers), imply that $\Sigma\lambda_j u_2^j$ remains finite, and hence that average u_2^j vanishes: $\bar{u}_2 = (1/n)\sum u_2^j \to 0$. This implies $u_2^j \to 0$ for all *j* except possibly for a vanishing fraction of them. That is, in the limit, if any one gift is not crowded out, almost all non-poor individuals think that the poor have enough. This does not seem to be the case. That is a priori the basic obstacle to the explanation of non-crowding out of gifts to fight poverty by the consideration of warm-glows in the strict sense described as a preference for one's gift in itself.

II.4.2 Relative motives

More precisely, the non-crowding out conditions demand that $\bar{u}_2$ for large *n* has an order of magnitude at most 1/*n* times that of u_g^i (or $u_g^i + u_c^i$). Condition (29) implies a related result. If $\Gamma=|G|$ denotes the number of givers, it implies that, on the average, u_g^i / u_2^i for $i \in G$ has at least the order of magnitude of Γ. That is: on the average, *the last gifts are given at least* Γ *times more for the glory of the giver than for the relief of poverty, where* Γ *is several or many millions*. Condition (30) implies a similar result for morally laundered fiscal policy.

II.4.3 The possible effects of the large number

When $n \to \infty$, $\sum_{j \neq i} u_2^j$, $\sum u_2^j$ and $n\bar{u}_2$ become infinite, finite or zero according as $\bar{u}_2$ decreases more slowly than, as fast as, or faster than 1/*n*, that is, as average u^i increases faster than, as, or more slowly than Log *x* (i.e. average compassion diminishes faster than, as fast as, or more slowly than the logarithm of the relief of poverty). The first case implies crowding-out of gifts. The other two may prevent it.

In particular, with the third case $\sum_{j \neq i} u_2^j \to 0$ by positive values, hence decreasing. Therefore, if u_g^i is bounded from below by a positive value, a sufficiently large *n* entails that

the non-crowding out conditions are satisfied for λ=0 (the "practically giving" regime): the large number is favourable to non-crowding out, contrary to a common view. This happens with both an average compassion decreasing more slowly than the logarithm of the relief of poverty and a non-vanishing warm-glow.[22]

II.4.5 Altruism-based warm-glows

The cause of warm-glows should be considered, however. The standard warm-glow is the altruism-based one. It may vanish with vanishing altruism (both for marginal gifts). However, individuals *i* whose u_2^i does not vanish may for this reason have a non-vanishing own altruism-based warm-glow and hence warm-glow u_g^i. Yet these individuals constitute a vanishing fraction of the large number. For the others, the altruism-based warm-glow that may cause a non-vanishing u_g^i is a praise warm-glow due to other people's praise of individual *i*'s gift because they are altruistic and enjoy a higher *x*. The average u_2^j of these other people vanishes, but their number increases. The result depends on how individual *i* evaluates others' opinions. If this individual focusses on the average of their view, perhaps if she considers G.E. Mead's "generalized other", then only the average of these u_2^j, practically $\bar{u}_2$, matters for her. And since it vanishes, so does the u_g^i they induce. This jeopardizes the non-crowding out conditions if $n\bar{u}_2$ does not vanish and, at any rate, if λ>0. When $n\bar{u}_2$ vanishes and λ=0, a priori it decreases more slowly than u_g^i induced by $\bar{u}_2$, and the effect of the large number favourable to non-crowding out does not hold. However, individual *i*'s sensitivity to the altruistic praises may depend not only on $\bar{u}_2$ but also on the number of praisers which a priori increases with *n*.

One may, then, explicitely write, for these marginal values,

$$u_g^i = \Sigma_{j \neq i} \, \pi_j^i \, u_2^j \qquad (31)$$

[22] Ribar and Wilhelm (2002) point out the possibility of complete non-crowd out with an exogenous positive lower bound on u_g^i.

where π^i_j is individual *i*'s satisfaction due to individual *j*'s praise motivated by a unit of her satisfaction due to the marginal increase in *x* caused by individual *i*'s gift. Then, condition (16) writes

$$\Sigma_{j\neq i}(\lambda_i \pi^i_j - \lambda_j)\ u^j_2 \geq \lambda u' . \qquad (32)$$

This implies $\lambda_i \pi^i_j - \lambda_j > 0$ and sufficiently large for a sufficient number of *j*. For seeing the meaning of this condition, assume that all individuals *i* are identical, hence $\lambda_i = \tilde{\lambda}$ and $u^i_2 = \bar{u}_2$ are the same for all *i*, and denote $\pi^i_j = \pi$. Condition (32) becomes

$$(n-1)\tilde{\lambda}\bar{u}_2(\pi - 1) \geq \lambda u' \qquad (33)$$

which implies, for $\bar{u}_2 > 0$, $\pi \geq 1$ (and $\pi > 1$ if $\lambda > 0$). Hence, one also has $\pi u^j_2 \geq u^i_2$ for all *i* and *j*. This means that when any individual gives an extra dollar, she derives more (at least as much) satisfaction from the praise of *each* of the very numerous others than from her own altruism. This seems unlikely.

These two hypotheses about altruism-based warm-glow are limiting cases. Hence it seems that altruism-based warm-glows lead to large-number crowd-out of almost all gifts.

II.5 Shallow warm-glows

These difficulties in explaining large-number giving with altruism-based warm-glow leads one to consider the other types of warm-glow. In particular, shallow warm-glow have no further justification. Notably, they are not motivated by the needy's benefits. They constitute a heterogeneous set including norms of giving per se, tradition, custom, habit and simple imitation. Such norms can be motivated by other people's praise for following them or blame for failing to – not motivated by altruism here – and they may be internalized. Demands, injunctions or praise of moral or social institutions have a role here.[23] These norms are for a "moral behaviour" but they are not intrinsically moral, although people may feel them as purely social or as moral (hence as possibly inducing shame or guilt, respectively, when they are not followed). The amounts of the gifts are sometimes determined by the norm or the

[23] However if, or insofar as, these institutions encourage giving in order to keep or obtain the support of altruists, they are just intermediaries in praise altruism-based warm-glow. Moral demands also aim at sentiments by demanding people to be altruists and not only to give.

custom, rather than by marginal conditions. Such views bear on gifts g_i but they may also bear on contributions c_i and, then, individual *i* adjusts her gift to the distributive tax t_i she pays. If such actions are praised by other people, u_g^i (or u_c^i) may depend on the praises.

II.6 "External glows": paradoxes and possible effect

II.6.1 External preferences

If an individual *i* values g_j or c_j for some $j \neq i$, and hence praises individual *j* for this gift or values her sacrifice, because she (individual *i*) is altruistic by valuing *x*, this cause is manifested by $u_2^i > 0$ (at the margin) and not by a direct preference about g_j or c_j. However, there are a number of possible reasons, shortly noted, for *i* to value directly g_j or c_j (although these reasons ultimately rest on some altruism except in cases of shallow warm-glows). This has two possible types of effects. First, it may influence directly the conditions for non-crowding out. Second, these preferences of individual *i* may lead her to praise higher or lower g_j or c_j and this may influence u_g^j or u_c^j.

Individuals may have preferences about others' contributions or gifts for various reasons other than directly and solely their contribution to the poor's welfare. Praises inducing shallow warm-glows manifest such preferences. An individual may also feel a warm-glow because some member of a community she belongs to gives or contributes for whatever motive ("justified" or not) – she may feel proud of it, or ashamed if the other fails in this respect. A number of such preferences may derive from comparisons between gifts or contributions. This may result, for instance, from inequality-aversion or other sentiment of comparative fairness in giving, contributing or sharing the burden of aid. The comparison may be particularly influential when it involves the evaluator's own contribution or gift. She may feel proud or superior when she provides more, or ashamed, inferior, envious or jealous when she provides less, and different in both cases (which may be regretted or favoured by desires for conforming or for distinction). These comparisons may be qualified for characteristics of the individuals (wealth, social proximity or status, etc.). The sentiments induced tend to increase with the extent of the differences. They may influence the person's gifts when either gifts or contributions (given taxes) are compared. This includes, for instance,

"keeping up" with others, competitive giving or contributing, providing one's fair share insofar as others provide theirs (lateral reciprocity), avoiding shame or seeking pride, and shunning or seeking conformity or originality.

These preferences lead to introducing other people's gifts or contributions in utility functions. If $c_{-i}=\{c_j\}_{j\neq i}$ and $g_{-i}=\{g_j\}_{j\neq i}$ denote the set of contributions and gifts of individuals $j\neq i$, individual i's utility function writes, with the formulation corresponding to form (12),

$$u^i = u^i(x_i, x, c_i, g_i, c_{-i}, g_{-i}). \tag{34}$$

II.6.2 The non-crowding out condition

These influences can lead to dynamics, equilibria, and possibly cooperation, in giving (more directly than through effects on x). Given g_{-i} and the set of taxes $t=\{t_j\}$, each individual i has a preferred gift $g_i=g_i(g_{-i}, t)$. If she takes g_{-i} and t as given, her choice of $g_i>0$ implies

$$du^i/dg_i = -u_1^i + u_2^i + u_c^i + u_g^i = 0. \tag{35}$$

If the government chooses t_i given the g_j, then, denoting $u_{c_j}^i = \partial u^i/\partial c_j$ and $u_{g_j}^i = \partial u^i/\partial g_j$, this implies

$$dU/dt_i = \lambda_i\cdot(-u_1^i + u_2^i + u_c^i) + \Sigma_{j\neq i}\,\lambda_j\cdot(u_2^j + u_{c_i}^j) + \lambda u' \leq 0 \tag{36}$$

with sign= if $t_i>0$. Then, conditions (35) and (36) imply

$$\lambda_i u_g^i \geq \Sigma_{j\neq i}\,\lambda_j\cdot(u_2^j + u_{c_i}^j) + \lambda u' \tag{37}$$

or

$$\lambda_i u_g^i \geq v + \Sigma_{j\neq i}\,\lambda_j u_{c_i}^j\,. \tag{38}$$

This non-crowding out condition for g_i includes two surprises: the $u_{g_i}^j$ are *not* in it and $u_{c_i}^j>0$ *worsens* the chances for the condition to be satisfied (for given values of the other variables in the condition). *The chances for an individual's gift to be crowded out are not changed by others' appreciation of her decision, and are worsened by their appreciation of her contribution*.

The conditions (37) and (38) for the non-crowding out of g_i are *improved* by $u^j_{c_i}<0$ for $j\neq i$. This may result from the noted comparative sentiments applied to contributions. Individual j may prefer a lower c_i because this reduces her envy, jealousy, resentment or sense of inferiority if $c_i>c_j$, because it augments her pride or sense of superiority if $c_i<c_j$, or for the other noted comparisons. These comparisons may be qualified for characteristics of the individuals. The overall effect of all these comparisons is likely to favour $u^j_{c_i}<0$ more when c_i is higher.

Therefore, the effects of preferences about an individual's gift or contribution – that is, her responsibility or sacrifice for helping the poor –, for reasons that are not directly altruistic, are strikingly opposed when these preferences are those of this individual or of other people. For own judgements, the contribution has no effect and the gift has one, whereas for judgements of other people the gift has no effect and the contribution has one. Moreover, the chances of non-crowding out are augmented by a preference of the giver for a higher gift and of others for a lower contribution.

II.6.3 The large number

Since the large number essentially destroys the possibility of non-crowding out due to altruism-based (and even shallow) warm-glows, the "external glow" effects of $u^j_{c_i}<0$ arouses hope because they intervene in condition (37) by their sum $\Sigma_{j\neq i}\,\lambda_j u^j_{c_i}$ as the u^j_2 do. A priori, $\Sigma_{j\neq i}\,\lambda_j\cdot(u^j_2+u^j_{c_i})$ may remain limited even if $\bar{u}_2$ does not vanish. However, this implies that, on average, people dislike an extra contribution by others about as much as they altruistically approve its contribution to the poor's relief. Although the importance of sentiments of envy, inferiority or superiority in society should not be underestimated, it seems rather unlikely that they could have this effect. One reason is that, for each person, such comparisons are often limited to persons of some group for both reasons of estimated relevance and information, and this group tends to be small compared to the large number of people. However, from a moral point of view for the choice of optimum taxes t_i, the information issue may be irrelevant, and this may also be the case of the sentiment of relevance of comparisons insofar as it also

depends on information about other people. Yet moral may rather demand erasing the effects of most of these sentiments rather than extending them.

II.6.4 Moral efficiency

Sentiments of envy or superiority may have to be laundered from individual preferences defining the policy's objectives. The method is the one discussed above about warm-glows. It leads to deleting $u_{c_i}^j$ for all *j* in condition (36) ($u_{g_i}^j$ is not in it). Then, however, this condition takes form (15) and condition (37) takes form (16). If the warm-glow effects are also laundered out, then condition (36) takes form (23) and condition (37) takes form (24). That is, this laundering of sentiments about other people's contributions (or gifts) gives to the condition for g_i>0 the form it has when these effects do not exist (with different functions, however). This also contrasts with the case of warm-glows (section II.2.2).

II.6.5 Praise (or blame) glows

Moreover, individuals' preferences about others' contributions or gifts by themselves imply approval, disapproval, or more elaborate judgements about them, which may influence the giver. $u_{g_i}^j$ >0 for whatever reason may induce a non-altruistic praise warm-glow augmenting u_g^i; and the noted effects, notably the comparative ones, about gifts and responsibility may produce $u_{g_i}^j$ <0, and such preferences may induce individual *i* to give less by lowering u_g^i by fear of reproaches, accusations of showing-off, etc. By contrast, similar effects of $u_{c_i}^j \neq 0$ on u_c^i do not have the same consequences because a sacrifice warm-glow does not produce a condition for g_i>0. However, such effects on u_c^i have a consequence for a morally laundered public objective leading to non-crowding out condition (24): then, a higher or lower $u_{c_i}^j$

inducing a similar variation in u_c^i by praise or blame is favourable or unfavourable to non-crowding out of g_i, respectively (this opposes the noted direct effect of $u_{c_i}^j$).[24]

II.6.6 The general form

Condition (37), corresponding to the most general form considered here expressed by the form (34) of utility functions, contains all the previous ones as particular cases. In addition, it includes the cases in which previous situations concern only some of the individuals, with important consequences.

In particular, if $u_{c_i}^j$ =0 for all j for some individual i, condition (37) takes form (21). This form implies notably that if g_i>0 (not crowded out) for large n, $\bar{u}_2$ vanishes, that is, in the limit, almost all individuals i think that the poor have enough. The sensitivity of all individuals to all the variables except the noted ones can be anything.

If, for some individual i, u_g^i =0 and $u_{c_i}^j$ =0 for all j, then condition (37) takes form (5). Gift g_i=0 (crowded out) if λ>0. If λ=0, g_i>0 implies u_2^j =0 for all $j\neq i$ (individual i is the only

---

[24] All these differences in effects are confirmed by considering the formulation corresponding to form (13) for utilities, that is, with these external effects,

$$u^i = u^i(x_i, x, g_i, t_i, g_{-i}, t_{-i}) \tag{39}$$

where $t_{-i}=\{t_j\}_{j\neq i}$ is the set of distributive taxes t_j for all $j\neq i$. Then, if individual i chooses g_i>0 for given g_{-i} and t,

$$du^i/dg_i = -u_1^i + u_2^i + u_g^i = 0. \tag{40}$$

The government's choice of tax t_i for given g_j for all j implies, denoting $\partial u^j/\partial t_i = u_{t_i}^j$,

$$dU/dt_i = \lambda_i\cdot(-u_1^i + u_2^i + u_t^i) + \Sigma_{j\neq i}\,\lambda_j\cdot(u_2^j + u_{t_i}^j) + \lambda u' \leq 0 \tag{41}$$

with sign = if t_i>0. Conditions (40) and (41) imply

$$\lambda_i u_r^i = \lambda_i\cdot(u_g^i - u_t^i) \geq \Sigma_{j\neq i}\,\lambda_j\cdot(u_2^j + u_{t_i}^j) + \lambda u' \tag{42}$$

where $u_r^i = u_g^i - u_t^i$ is individual i's (marginal) warm-glow for her responsibility in helping the poor (value of extra gift minus value of the contribution in itself it includes, i.e. marginal value of tax t_i). Condition (42) also writes

$$\lambda_i\cdot(u_2^i + u_r^i) \geq v + \Sigma_{j\neq i}\,u_{t_i}^j . \tag{43}$$

Then, the remarks of the text about laundering immoral sentiments and induced warm-glows can be carried on by replacing u_g^i by u_r^i and u_c^i by u_t^i.

possible altruist). This individual *i* may be motivated to give by possible $u_2^i>0$ or $u_c^i>0$ (yet without altruism-based praise warm-glow), but the effect of u_c^i is cancelled out by the policy (except in case of moral laundering). All the other sensitivities of all individuals to all variables may be present. Any other individual $j\neq i$ may give because $u_g^j>0$ (and $u_c^j>0$ with moral laundering).

II.7 Rebate and matching-grant neutrality or dual effects (cost or benefit)

In many places and cases, philanthropy is subsidized by tax exemptions or rebates, or encouraged by matching grants. The basic thing about these policies is that, a priori, they have no effect, if all is considered by the analysis and the agents, including with all kinds of warm-glows, external effects, etc.[25] Notably, the financing of the cost of these policies should not be forgotten. Other things equal, they are financed by taxes. This product could have been directly provided to the poor, that is, what the poor receive from taxes is diminished by this amount. Their income is in this way diminished by the matching grant they receive. Or it is diminished by the rebate or subsidy received by the giver, and the gift minus the rebate is both the cost for the giver and the *final* receipt of the poor for which the giver's choice is responsible. Hence, in all cases, when the giver chooses her gift by balancing the cost for her and the benefit for the poor, both are equal, and this amount is also what the giver or other people may directly value as her gift or as a part of her contribution.

In all the foregoing models, including with all the possibilities of warm-glows, external effects – utility functions with a priori the most general form (34) – and laundering, if the gift g_i of giver *i* is augmented by the matching grant $m_i(g_i)$ (with $m_i(0)=0$), the poor receive $g_i+m_i(g_i)$, but the taxes Σt_i finance $m_i(g_i)$ and are diminished by this amount when transferred to the poor. Hence, the poor receive $\Sigma[g_i+m_i(g_i)]+\Sigma t_i-\Sigma m_i(g_i)=\Sigma g_i+\Sigma t_i$. For rebates or subsidies, if the giver *i*, giving g_i, receives a rebate or subsidy of $r_i(g_i)$ (with $r_i(0)=0$), this is financed from the taxes Σt_i (perhaps, for tax rebates, by a transfer to the income tax fund for leaving other things equal), this amount Σt_i is diminished by this amount $r_i(g_i)$ when it is transferred to the poor, and the poor's benefit due to the gift g_i is only $g_i-r_i(g_i)$, which is the

[25] This conclusion in the presence of warm-glows is at odds with the views of Bernheim (1986) and Andreoni (1990).

cost to giver *i*. The poor receive, on the whole, $\Sigma g_i+\Sigma t_i-\Sigma r_i(g_i)=\Sigma[g_i-r_i(g_i)]+\Sigma t_i$. All is identical to giver *i* deciding to give $g'_i=g_i-r_i(g_i)$. In all cases, since the taxes do not change, the same result holds if they are not lump-sum.[26]

Of course, if grants, rebates or subsidies are financed, in total or in part, from outside this system, and one forgets about their cost, or if the givers suffer from "gift illusion" and forget about this financing and its effects, other results obtain, with generally increases in the gifts.[27] Then, such a given amount generally enriches the receivers more when it is used for financing matching grants, rebates or subsidies increasing with the gift. In these cases, the cost for the giver differs from the corresponding benefit for the receivers. This raises, for concerns about gift or contribution in themselves, the problem of whether what matters is the giver's actual sacrifice, or the increase in the poor's benefit due to her action, or both, or some combination of both. This choice may more or less differ according as whether the issue is the gift g_i or the contribution c_i.[28] It may also depend on who evaluates (the giver herself or someone else – relevant for induced warm-glows and Pareto efficiency). The results may also depend on the hypotheses about the origin of the funds (possibly part exogenous and part endogenous, etc.). They include the determination of the optimal subsidy or matching-grant schedules. The same remarks hold for moral efficiency (along the lines of sections II.2 and II.6.4).[29]

III. SOCIAL-MORAL REASONS

[26] This is the reason of the result of Bernheim (1986) for the case of "pure altruism". Andreoni (for lump-sum taxes and proportional subsidies) sees well the general logic for the case of "pure altruism", in 1988, but obtains in 1990 a different result for the general "impure altruism" case because he writes (p. 469) that the warm-glow is concerned with the individual gift g_i rather than with $g_i\cdot(1-s_i)$ – where s_i is the subsidy rate for individual *i* –, which is both the cost for the giver and the benefit for the receivers if the subsidy s_ig_i is financed from taxes and hence deduced from the government's transfers to the poor. This assumption probably results from the three hypotheses that the individual thinks that: the poor will receive g_i, the subsidy $s_i\,g_i$ is given from outside as manna from heaven, and the relevant base for warm glow is the poor's benefit g_i (hence not reduced by the payment of $s_i\,g_i$ by taxes) and not the sacrifice the individual incurs for it $(1-s_i)\cdot g_i$ – for the items the individual is responsible for (i.e. not t_i). This is at odds with the assumptions of both the article of 1988 for pure altruism (concerning the financing of the subsidy), and a note mentioning a warm-glow for total sacrifice $g_i\cdot(1-s_i)+t_i$, with the neutrality resulting from the presence of the tax t_i.

[27] The givers do not "see through" the government budget in the expression of Boadway, Pestieau and Wildasin (1989).

[28] For instance, more weight may be put on the cost for the giver for the contribution $c_i=g_i+t_i$ than for the gift g_i by itself, because this cost is emphasized when the relevance of the contribution is justified by the argument that the tax paid should be included.

[29] The effects of all these questions are shown in Kolm (2008b).

III.1 General issues

III.1.1 The solutions

People commonly give to fight poverty for reasons somewhat elaborate, of a moral or social nature, often referring to hypothetical counterfactual situations, that make them avoid the public good problem (this differs from a direct reference to the satisfaction of some preference). These reasons are considered in this section which presents four types of them: putative reciprocities, "Kantian" reasons, lateral reciprocities or moral matching, and social contracts. Putative reciprocities apply to giving only (not to other public goods), essentially discard issues of cooperation, and, indeed, are causes of a duty or propriety warm-glow and of altruism. The other three types are reasons given for cooperating and apply to all public goods, although characteristics of the poor's welfare intervene in their application (importance, both private and public provision, or large number of givers). They also lead to warm-glows of giving or contributing due to duty, propriety or avoidance of guilt or shame. Each type has subtypes (generally two). They belong to the family of social moral "mechanisms" that aim at transforming uncoordinated individual behaviour inducing inefficiency into Pareto-efficient unanimity, studied by philosophers as with Kant's "categorical (i.e. unconditional) imperative", Hume's "conventions" (with a clear view of the public good problem), and Rousseau's "social contract" which transmutes individuals into citizens implementing the "general will".[30] Remarks about the sequential aspect will be added.

Paradoxes continue to abound, such as the four following ones.

1) *The punishment paradox.* An explanation of cooperation could be sequential giving, which is the actual situation. Givers punish an altruistic giver who gives less than at some "folk-theorem" equilibrium by giving less next time, that is, in fact they punish the poor who receive still less.[31] Moreover, this punishment affects all other altruistic givers for this non-

[30] See Kant (1785) and a perceptive overview by Nagel (1991).

[31] For example, most people are sorry for the suffering of the poor whatever their nationality, but there is an implicit agreement that, for the main part, each nation takes care of its own poor. Then, if China neglects its poor, the US can retaliate by cutting all welfare programs and banning charities in America.

excludable public good, and the action of any small giver relative to the large number is not actually felt.

2) *The lateral reciprocity paradox*. Lateral reciprocity is reciprocity with the other contributors or givers. It is expressed as: "I give or contribute given that the others give or contribute; I provide my fair share, given that the others provide theirs". However, the only way to be sure that the others give or contribute is that they be forced to. Then, to begin with, these contributions cannot be gifts, from the definition of a gift. Moreover, if all participants have the same motive, the constraint on them is not actually binding because they know the others contribute. Yet it is reached, and it is necessary in order for the others to be sure that each contributes. These transfers are both forced and free. They are jointly gifts and taxes.

3) *The Kantian paradox.* The Kant-like meta-rules are: "act as if others acted the same" (folk Kantianism) or "follow a rule such that you could want the consequences of everybody following it" (Kant). If people choose in this way by maximizing their own different utility functions, a priori this leads them to choose different acts (even depending on individuals' specific characteristics) or rules. An individual's action and the assumptions of each other about it are, a priori, all different and inconsistent.[32] The outcome is generally not Pareto efficient.

4) *The large number free-riding paradox.* The absence of free-riding is favoured by a larger number of beneficiary contributors in an important class of cases – contrary to common views. Indeed, no matter how small a moral motive for giving such as the shame of shirking, a sufficiently large number of givers makes it effective when average compassion decreases faster than the logarithm of the relief of poverty.

The Kantian paradox has two families of solutions, consistent teams and consistent rules. The fully Kantian solution, teams, is first considered. We then present briefly the theory of rules, which also applies to lateral reciprocity (and to pricing or taxes). Then we consider the social contract ethics, an application of the theory of the core for non-excludable public goods.[33] Putative reciprocities are finally discussed and remarks about sequential giving are presented.

---

[32] Moreover, some purists would regret that, then, the meta-rule does not apply to itself (a self-referential inconsistency): an individual who considers being such a Kantian while considering that everybody is assumes that individuals have both the same and different acts or rules.

[33] Kolm (1987). The Foley core is a priori for excludable public goods (or is the very particular α-core).

III.2 Kantian teams

"I do this because imagine that nobody does it" is a very common "reason" provided for contributing to a large spectrum of public goods. This is, for instance, the main reason people give when they are asked why they vote in large elections (thus, it permits democracy) or abstain from polluting public places. It is also common for helping. It amounts to acting as if everybody imitates you. This counterfactual and strictly irrational reason is a main basic mental structure that permits societies to stand up. This "generalization principle" was hypostasiated by Kant into the meta-rule of his "categorical imperative": "Act as if you could want the principle (rule, maxim) of your action to be followed by everybody". Helping the poor, a major moral objective and a contribution to a universal public good is certainly an object of this injunction. Indeed, Kant provides explicitly the example of helping people in need. A rule may a priori make individual gifts or contributions depend on characteristics of the giver or contributor.

The noted Kantian paradox implies that the interpretation of the Kantian principles requires the a priori introduction of more unanimity. This can be done by demanding a common evaluation or a common rule. This will provide efficiency.[34]

The standard meaning of utility functions is that they represent individual tastes, and Kant insists that moral conduct should not depend on the particular tastes of the actor. Even if the preferences they describe are also altruistic, this, for Kant, also belongs to the category of tastes ("inclinations") and not to rational morality. Kant's expression that "you could want the result" also suggests that this result is not the one that one prefers with one's utility function since it seems a priori that "could", in this context, should refer to a moral-social evaluation. In fact, it would seem inconsistent that Kantian actors, who act according to a moral-social reason, would not also evaluate according to a moral-social value. The most direct such consideration is that they judge with the same social welfare function, in the present case of

[34] Rules or acts defined with discreet parameters with few values may avoid the paradox. An instance is given by the choice of voting or abstaining (if a rule also includes the nature of the vote, the Kantian ethic demands voting non-strategically). Kant's own examples are of this kind, for instance the choice of not lying or lying (without statement of conditions or circumstances) or of aiding other people in need or not. But this often does not suffice to determine the precise action – for instance the amount given.

form (1) with individual utility functions u^i which are a priori of the most general form (34).[35,36,37]

The relevant aspects of a rule are its consequences, i.e. the resulting choice of gift g_i for each individual *i* here. Then, the best rule, as judged by all such Kantian individuals, is the (or a) set of g_i that maximizes *U*. Each individual chooses her own g_i, but the g_i of the set that maximizes *U* are determined jointly. However, the g_i are independent variables, and the solution can be obtained as the (relevant) equilibrium of a Cournot-Nash game in which each individual chooses her g_i that maximizes *U* for given values of the g_j for $j \neq i$. Given the structure of function *U*, this satisfies Pareto efficiency.

However, distributive taxes t_i can also be chosen. Noting $U_{g_i} = dU/dg_i$ and $U_{t_i} = dU/t_i$, the forms of u^i and *U* entail, with $c_i = g_i + t_i$:

$$U_{g_i} = U_{t_i} + \alpha_i \tag{44}$$

with

$$\alpha_i = \lambda_i u^i_g + \Sigma_{j \neq i} \lambda_j u^j_{g_i} = \partial U / \partial g_i. \tag{45}$$

For the g_i and t_i that maximize *U*, with the foregoing assumptions,

$U_{g_i} \leq 0$ with sign = if $g_i > 0$,

$U_{t_i} \leq 0$ with sign = if $t_i > 0$.

If the u^i_g and $u^j_{g_i}$ are not assumed all zero, for the set of g_j and t_j that maximize *U*, $\alpha_i \neq 0$ except fortuitously. Then U_{g_i} and U_{t_i} cannot both be zero. Hence of the gift g_i and the tax t_i, *if one exists it crowds the other out* at the highest *U*. Specifically, $t_i = 0$ if $\alpha_i > 0$ and $g_i = 0$ if

[35] A set of individuals with the same objective function has been called a "team" by Roy Radner.
[36] Tastes intervene, then, but not in a self-centered way. In an elaborate work, Bordignon (1990) considers in particular individuals who evaluate with their own different social evaluations, each of which assumes that other people have the evaluator's utility function (tastes) and is, then, utilitarian (it could also be a more general aggregation function, for instance a maximin which would demand ordinal utilities only). The outcome is not Pareto efficient (it is compared with an inefficient political provision). Brekke, Kverndokk and Nyborg suggest evaluation with "social welfare as I perceive it" without further precision or conclusion.
[37] Then, the Kantian principle demands that individuals reveal their preferences to the others.

$\alpha_i<0$. The case $\alpha_i>0$ occurs in particular if $u_g^i>0$ and $u_{g_i}^j=0$ for all $j\neq i$; this is another aspect of $u_g^i>0$ permitting $g_i>0$. If $\alpha_i=0$, which can be seen as non-fortuitous only under the "classical" assumptions implying $u_g^i=u_{g_i}^j=0$ for all j, g_i and t_i may both be positive; in fact, they are substitutable in all respects (only $c_i=g_i+t_i$ intervenes); this amounts to individual i freely paying her distributive tax. In all these conditions, u_c^i and $u_{c_i}^j$ play no role.

III.3 Rules for Kantianism, matching or taxes or prices[38]

III.3.1 The logic of rules

Individuals' choices that lead to Pareto-inefficient solutions need to be constrained by rules, followed voluntarily or by force, that make these individual actions produce a Pareto-efficient result. Since the choice between the Pareto-efficient outcomes is that of a distribution between the individuals, the rules in question should be about the distribution. The comparison is basically between individuals two by two, but this set of pairwise comparisons and the rule-constrained individual choices should be consistent. This leads to the theory of consistent rules.

Very generally, denote as $z_i\in\tilde{Z}$ something which is chosen to apply to individual i, and $C_i\in\tilde{C}$ a sufficient set of characteristics of individual i, for each of n individuals. For simplicity (sufficient here) the functions introduced are considered to be one-to-one. A *point-rule* for choosing the n z_i is a set of n functions $z_i=r(C_i)$, one for each i. A (binary) *comparative rule* says that for a given z_i, there is a corresponding $z_j=R(z_i, C_i, C_j)=\rho_j^i(z_i)$. For instance, if z_i holds, then it is fair that z_j has this value, for some comparison described by this

[38] The determination of Pareto-efficient rules for contributions to public goods is analyzed in Kolm (1970a, 1970b) for application to taxes and free contributions (with emphasis on linear rules), and by Bilodeau and Gravel (2004) as a theory of (half-) Kantian ethics. Lateral reciprocity is analyzed by Sugden(1984) and Kolm (1984). The former proposes and studies an elaborate rule of individual "effort" depending on others' efforts, with a result which is not non-fortuitously Pareto-efficient ("a most unlikely coincidence"), basically because the definition of the comparison of efforts cannot adjust for this purpose (the rule may be fair but is not for a fair sharing of all the surplus). The comparison with consistent rules is an instance of the great divide of principles of fairness: those that generally prevent Pareto efficiency and those that are consistent with it (and often permit it by solving the question of the opposition of interests).

rule of comparative fairness.[39] Such a comparative rule defined for all pairs *i, j,* is *logically consistent* if, for all *i, j, k,* denoting as 1 the identity function, $\rho_i^i = 1$ and $\rho_j^i \circ \rho_k^j = \rho_k^i$ (transitivity), which implies $\rho_j^i \circ \rho_i^j = 1$ (symmetry). Then, for each z_i, there is a set $z=\{z_j\}$ of *n* z_j corresponding to it by this rule. This set is equivalently parametrized by any of its z_j, or by the $\zeta \in \tilde{Z}$ of an added hypothetical individual of characteristics $C \in \tilde{C}$. In all cases, since $\zeta = z_j$ and $C = C_j$ for any *j* is possible, one can write $z_i = R(\zeta, C, C_i) = \rho_i(\zeta)$ for all *i*. Denote as $\rho = \{\rho_i\}$ the set of the *n* functions ρ_i. For any given ζ, the point-rule $z = \rho(\zeta)$ is a *specification* of rule ρ. If individual *i* has a preference over the set *z* described for instance by a utility function $U_i(z)$, then the (or a) specification of the rule ρ that maximizes U_i, defined by $\zeta_i = \arg \max_{\zeta} U_i[\rho(\zeta)]$, is individual *i*'s *preferred specification of the rule*. A rule is *socially consistent* if all the individuals have the same preferred specification, $\zeta_i = \zeta^*$ for all *i*. Such a rule, both logically and socially consistent, is simply *consistent*.

III.3.2 Three applications

This has, in particular, three types of applications. (1) The z_i can be allocations, taxes, etc., imposed to individuals *i* according to some rule (for instance of fairness). (2) In lateral reciprocity, if each individual *i* chooses the $z_i = r(C_i)$ of a given point-rule or her preferred $z_i = \rho_i(\zeta^*)$ of a given consistent comparative rule if all others do the same, these point-rules may hold (given the noted particularities in implementation). (3) With half-Kantian individuals who choose by maximizing their utilities, the Kantian paradox remains if each chooses a point-rule or a comparative rule, but – it turns out –, for contributing to a public good, not if they choose their preferred specification of any given consistent rule (if the solution is unique).[40]

III.3.3 Rules for public goods

[39] Such a comparative rule commonly results from some kind of egalitarian comparison between some function of the pairs (z_i, C_i) and (z_j, C_j) ("justice is equality", and the concept of equality refers to a comparison between two individuals).

[40] In the few examples provided by Kant, the ultimate reference of an individual is in fact sometimes unambiguously her interest, for instance with the rule of aiding the needy because it implies that one would be helped when in need.

Applying these concepts to the financing of a public good, $z_i \in \mathbb{R}_+$ is individual i's contribution, gift or tax (depending on application), and $Z=\Sigma z_i$ is the cost of the public good, taken as the public good itself (*w.ℓ.g.*). Consider a logically consistent rule $z=\rho(\zeta)$. The $\zeta \in \mathbb{R}$ can be in particular any of the z_i (then ρ_i for this i is the identity function). Functions ρ^i_j are normally increasing, which we assume. Then, functions $\rho_i(\zeta)$ can be taken as increasing and parameter ζ can be replaced by any increasing function of itself with a corresponding contravariant change of functions $\rho_i(\zeta)$. We have $Z=\Sigma z_i=\Sigma\rho_i(\zeta)$. Inversing gives the increasing function $\zeta(Z)$. Then, Z can be taken as a particular ζ, and functions $s_i(Z)= \rho_i [\zeta (Z)]$ are *sharing functions* that give each individual i's contribution to a given level of the public good Z if the comparative rule ρ_i is followed. The comparative rule between contributions z_i is thus translated into the corresponding *sharing rule* of the cost Z of the public good for any Z. With $\bar{z}=Z/n$ denoting the average contribution, functions $\sigma_i(\bar{z})=s_i(n\bar{z})$ use homogeneous variables and provide the *deviations from average* of each contribution for each average, $\delta_i(\bar{z})=\sigma_i(\bar{z})-\bar{z}$ with $\Sigma\delta_i(\bar{z})=0$. The rest of the section considers the case of a public good, which can in particular be the poor's income.

III.3.4 Pareto efficiencies

Consider increasing utility functions of individuals i, $u^i(x_i, x)$, with $x_i=y_i-z_i$ and $x=y+Z$, $Z=\Sigma z_i$, a logically consistent rule $z=\rho(\zeta)$, and given y_i and y.[41] Denote $v^i(x_i, x)= u^i_2 / u^i_1$, $V^i(\zeta)=v^i[y_i-\rho_i(\zeta), y+\Sigma\rho_j(\zeta)]$, and ζ_i the ζ preferred by individual i, which satisfies, with a differentiable rule $\rho(\zeta)$ and for an interior solution,

$$-u^i_1\, \rho'_i(\zeta_i)+ u^i_2 \Sigma_j \rho'_j(\zeta_i)=0$$

or

$$V^i(\zeta_i)= \rho'_i(\zeta_i)/\Sigma_j \rho'_j(\zeta_i). \qquad (46)$$

$Z_i=\Sigma_j\rho_j(\zeta_i)$ is the level of Z preferred by individual i for the rule $\rho(\zeta)$ (Z can also be taken as parameter ζ, with $\rho_j(\zeta)=s_j(Z)$).

---

[41] The presence of y is for application to the poor's income. For a standard public good, a priori $y=0$. In the various interpretations, each ζ_i may be a tax (without gifts, and y_i and y are the initial incomes); or it may be a contribution $\zeta_i=c_i$ and then $y_i=X_i$, $y=X$, and individual i freely provides $g_i=c_i-t_i$ if there are given taxes t_i; or it may be such a gift g_i and then $y_i=X_i-t_i$ and $y=X+\Sigma t_i$ if there are given taxes t_i.

The condition for Pareto efficiency for interior solutions is

$$\Sigma v_i(y_i - z_i,\ y + \Sigma z_j) = 1 \qquad (47)$$

for the actual choices of the z_i. It is in particular satisfied by the maximization of all u^i under the rule $\rho(\zeta)$ in the two following cases.

1) *The rule is consistent* (Bilodeau and Gravel, 2004, for "Kantian rules").

Then, indeed, $\zeta_i = \zeta^*$, the same for all i, $z_i = \rho_i(\zeta^*)$ for all i, $v^i = V^i(\zeta^*)$, and, adding conditions (46) for all i, condition (47). Therefore, if all individuals agree on (or are imposed) a rule of fairness such that, when this rule is followed, their self-interest leads them to prefer the same outcome, or to choose individual actions that correspond to one another according to this rule, the result is Pareto efficient.

2) *The linear case*.

That is, the two following properties hold:

(1) The rule is linear,

$$\rho_i = a_i\zeta + b_i \qquad (48)$$

with constant $a_i > 0$ and b_i for all i (this includes, in particular, the *proportional* rule where $b_i = 0$ for all i, that is, the z_i are in given proportions – for example the z_i are proportional to some income of the individuals i, the y_i or others –, and in particular *equality* or *duplication* in which all the z_i are equal).

(2) All utility functions are quasi-linear in x, i.e. have a specification of the form $u^i = \varphi_i(x_i) + x$.

Then, indeed,

$$v^i = 1/\varphi'_i(x_i) = 1/\varphi'_i(y_i - a_i\zeta - b_i), \qquad (49)$$

condition (46) writes

$$V^i(\zeta_i) = 1/\varphi'_i(y_i - a_i\zeta_i - b_i) = a_i/\Sigma a_j, \qquad (50)$$

and, adding conditions (50) for all i,

$$\Sigma V^i(\zeta_i) = 1, \qquad (51)$$

the condition for Pareto efficiency of the $z_i = y_i - a_i\zeta_i - b_i$. Note that, a priori, these ζ_i are different for the different i and these z_i do not correspond to one another according to the rule (the result is not a specification of the rule).

*

The consistent rules ρ for given functions u^i have to satisfy the n equations

$$V^i(\zeta^*)=\rho'_i(\zeta^*)/\Sigma\rho'_j(\zeta^*). \quad (52)$$

These n equations can in general determine (uniquely or not) n real number parameters. By symmetry, each is a parameter $\gamma_i\in\mathbb{R}$ of one function $\rho_i(\zeta)$. However, ζ^* can a priori be chosen arbitrarily and hence this set of γ_i is determined up to a parameter. Therefore, the consistent rules ρ are of the form $\rho_i=f(\gamma_i, \zeta)$ for an arbitrarily chosen two-variables function f, and the n functions (52) determine the c_i.

Any Pareto efficient solution can be obtained by consistent rules. If z is this state, such a rule satisfies, in particular, $\rho(\zeta^*)=z$ and the $\rho'_i(\zeta^*)$ are proportional to the $V^i(z)$.

III.3.5 Consistent linear rules

The simplest rules are linear,

$$\rho_i=a_i\zeta+b_i \quad (48)$$

for all i with constant $a_i>0$ and b_i. Then, equations (52) become

$$v^i(y_i-a_i\zeta^*-b_i, y+\zeta^*\Sigma a_j+\Sigma b_j)=a_i/\Sigma a_j. \quad (53)$$

This implies

$$a_i\zeta^*=v_i\Sigma a_j\zeta^* \quad (54)$$

which means that the sharing of the amount $\zeta^*\Sigma a_j$ follows the Lindahl rule. The total amount $Z^*=\zeta^*\Sigma a_i+\Sigma b_i$ is divided into two parts, one divided into arbitrary fixed amounts b_i (but a priori $b_i\gtrless 0$), and the other allocated according to the Lindahl rule. However, there are two typical cases in which, respectively, the a_i or the b_i are given, and the n others are determined by the n equations (53). Denote $a=\{a_i\}$ and $b=\{b_i\}$ the vectors of the a_i and b_i respectively. These two cases are denoted by functions $a(b)$ and $b(a)$ where, respectively, b or a are given and a or b is a function of it given by equations (53). Lindahl solutions are $a(0)$, with given $b=0$, hence a *proportional rule* $z=a\zeta$ in which the $z_i=a_i\zeta$ are in the same proportions a_i determined by these equations.[42]

[42] A Lindahl solution z^* is reached by the proportional rule with $a_i=z_i^*$ and $\zeta=1$.

The choice of the given b or a depends on the specific problem. The indeterminacies due to the a priori choice of any ζ^* are that, for solutions $a(b)$, the a_i vary inversely proportionally to ζ^*, and, for solutions $b(a)$, if ζ^* is augmented by any number β each b_i is diminished by βa_i. Given a_i or b_i can be related to characteristics of the individuals i, or be equal. For instance, the given a_i may be some income of the individuals, y_i possibly augmented or diminished by some other payment, and the Lindahl part of the contributions are proportional to it. They may be equal and provide an equal Lindahl part of the payments. The given b_i may be equal (of any sign), giving the "equally augmented Lindahl solutions." For n=2, any linear rule with unequal a_i is identical to a rule of this category.

One can also consider linear sharing rules

$$z_i = \rho_i = s_i(Z) = \alpha_i Z + \beta_i \tag{55}$$

with given α_i (a priori >0) and β_i, and, since $\Sigma z_i = Z$, $\Sigma \alpha_i = 1$ and $\Sigma \beta_i = 0$. For a consistent such rule all individuals i prefer $Z=Z^*$ such that

$$v^i(y_i - \alpha_i Z^* - \beta_i, y + Z^*) = \alpha_i, \tag{56}$$

and hence $\Sigma v_i = 1$. The financial transfers consist of two parts. First, there is a balanced redistribution of income in which each individual i receives the amount $\beta_i \gtrless 0$ (receives $\beta_i > 0$ or yields $-\beta_i$ if $\beta_i < 0$) with $\Sigma \beta_i = 0$. Second, each individual contributes to the financing of Z^* with $\alpha_i Z^* = v^i Z^*$ i.e. according to the Lindhal rule. Equations (56) can determine Z^* and, for instance, the α_i for given β_i or the β_i for given α_i.[43] Lindahl solutions correspond to given $\beta_i = 0$ for all i. A priori equal α_i give the β_i that permit equal Lindahl payments. Given α_i may be proportional to some characteristics of individuals i.

Consistent linear rules are the consistent rules with the smallest number of parameters ($2n$ in general) that permit reaching any Pareto efficient solution with quasi-concave preferences. For such a state z, it suffices to choose a consistent linear rule with $\rho(\zeta^*) = z$ and a_i proportional to the $V^i(\zeta^*)$. In particular, these rules can be sharing rules (with the α_i proportional to the $V^i(\zeta^*)$ and $2n$-2 independent parameters).

---

[43] $\Sigma \alpha_i = 1$, $\Sigma \beta_i = 0$ and the n equations (56) related a priori by $\Sigma v_i = 1$ constitute n+1 a priori independent equations for determining X^* and either the α_i given the β_i or the β_i given the α_i.

Since Lindhal solutions are often considered, it is possible to present consistent rules as a generalization of them. Lindahl solutions correspond to proportional consistent rules: $\rho_i=a_i\zeta$ or α_i Z for all *i*. There are in fact successive levels of generalization: consistent linear sharing rules with possibly non-zero β_i, a priori given or more generally; consistent linear rules with *a*(*b*) which provide, in a sense, a Lindahl solution from any given point *b* rather than from the particular *b*=0 only; consistent linear rules in general (notably with *a*(*b*) or *b*(*a*)); consistent sharing rules or consistent rules in general. All these cases lead to Pareto-efficient unanimous choices. The drawback of Lindahl solutions is that they impose a particular distribution (or particular distributions). By contrast, as we have noted, any Pareto-efficient solution can be reached by consistent rules or sharing rules, and by linear such rules with quasi-concave utilities.

III.3.6 Particular structures

When the a priori given structure of the rule ρ has less than *n* independent parameters to be determined, the rule cannot be consistent in general. This includes, for instance, equal $z_i=\zeta$, equal final private income obtained with rule $z_i=y_i-\zeta$, contributions proportional to given incomes $z_i=y_i\zeta$ or to any other characteristics. This is a drawback since such rules are common.

However, similarities in the utility functions open possibilities. If the functions u^i are ordinally the same, denoting as *u* a common specification,

$$u^i=u(x_i, x)=u(y_i-z_i, y+Z), \tag{57}$$

equations (52) or (53) can determine the value ζ^* of a parameter ζ if the *n* equations are the same. With such same u^i, the only remaining difference is that of the y_i. This difference may be eliminated in three ways, two of which with more specific given structures. Denote $v=v^i$ for all *i*.

1) The chosen contributions equalize the remaining incomes, $z_i=\rho_i=y_i-\zeta$ for all *i*, and

$$v(\zeta^*, Y-n\zeta^*)=1/n \tag{58}$$

where $Y=\Sigma y_i+y$ is total income.

2) If the y_i happen to be the same, $y_i=\eta$, equal $z_i=\zeta$ provide the solution, with

$$v(\eta-\zeta^*, y+n\zeta^*)=1/n \tag{59}$$

(Laffont, 1975).

3) If function u is quasilinear, $u=x_i+w(x)$, $v=w'(x)$ and an additive rule $z_i=b_i+\zeta$ gives

$$w'(y+\Sigma b_i+n\zeta^*)=1/n. \qquad (60)$$

With a strictly quasi-concave increasing function u, equations (58), (59), (60) have a unique solution ζ^* (equations (58) and (59) amount to maximizing this function under the linear constraint $nx_i+x=Y$).

III.3.7 The three applications and the public-good specific unanimity-efficiency implication

If the z_i are taxes or tariffs for financing the public good, presenting the rule in the form of the sharing function, $z_i=s_i(Z)$, shows that consistent rules are payments that lead all individuals to prefer the same level of the public good. Consistent rules constitute the general form of the spirit of Lindahl pricing (which is the proportional consistent rule). They permit reaching all the Pareto-efficient solutions (whereas the Lindahl rule determines the distribution(s)). Linear rules are sufficient for this and constitute the set of rules that permit it with the smallest number of parameters. For lateral reciprocity (matching) or half-Kantian conducts, the rule may be a social moral norm or convention; there may be given taxes but, with consistent rules, they are not necessary for Pareto efficiency.

Consistent rules for contributing to a public good are the social mechanisms that associate Pareto-efficiency and unanimity about the choice of the good: with such a rule unanimity entails Pareto efficiency and, conversely, any Pareto-efficient outcome can result from unanimity under such rules. Consistent rules do this by endorsing the distributive question which makes individuals disagree about the choice of a Pareto-efficient solution, by the comparative fairness they imply.

Moreover, consistent rules are the mechanisms associating Pareto-efficiency and unanimity in this way that is *specific to the public-good structure*. Indeed, assume that each individual utility depends on the z_j not necessarily through their sum $Z=\Sigma z_j$ but possibly more generally, as $U^i(x_i,z)$ with $x_i=y_i-z_i$ and $z=\{z_j\}$. Denote $U^i_{x_i}=\partial U^i/\partial x_i$, $U^i_j=\partial U^i/\partial z_j$ and

$V_j^i = U_j^i / U_{x_i}^i$. Let $\rho(\zeta)$ be a consistent rule with a unique unanimously preferred $\zeta=\zeta^*$. Then, with $z=\rho(\zeta)$, for each i and interior solutions,

$$\Sigma_j V_j^i \, \rho'_j(\zeta^*) = \rho'_i(\zeta^*). \qquad (61)$$

However, each z_j is a public good and the corresponding condition for Pareto efficiency is

$$\Sigma_j V_j^i = 1. \qquad (62)$$

Conditions (61) imply non-fortuitously conditions (62) only when, for each i, the V_j^i are the same for all j, $V_j^i = V^i$. Then, indeed, conditions (61) imply $\Sigma V^i = 1$, that is, conditions (62). This implies that, at least marginally, the U^i depend on the z_j by their sum $Z=\Sigma z_j$, and hence this externality has the structure of a public good.

III.3.8 Ruled deviations

For z_i that are quantities, a rule $\rho(\zeta)$ is *deviational* and denoted as $\tilde{\rho}(\tilde{\zeta})$ when $\tilde{\rho}(0)=0$.

A state z' is a ruled deviation of state z for deviational rule $\tilde{\rho}(z, \tilde{\zeta})$ when $z'=z+\tilde{\rho}(z, \tilde{\zeta})$ for some $\tilde{\zeta}$.

A basic property is: *if a state z of individual contributions z_i to a public good is unanimously (weakly) preferred to all ruled deviations from it for a given deviational rule, it is Pareto efficient*. This holds whatever this given deviational rule is.

Indeed, let z and $\tilde{\rho}$ denote this state and the given deviational rule. For each i

$$u^i = u^i [y_i - z_i - \tilde{\rho}_i(z, \tilde{\zeta}), \, y + \Sigma z_j + \Sigma \tilde{\rho}_i(z, \tilde{\zeta})]$$

is maximum at $\tilde{\zeta}=0$ if, for an interior solution and denoting $\tilde{\rho}'_i(z, \tilde{\zeta}) = \partial \tilde{\rho}_i(z, \tilde{\zeta}) / \partial \tilde{\zeta}$,

$$-u_1^i \cdot \tilde{\rho}'_i(z,0) + u_2^i \cdot \Sigma \tilde{\rho}'_j(z,0) = 0$$

or, if $\Sigma \tilde{\rho}'_i(z,0) \neq 0$,

$$v^i(x_i, x) = \tilde{\rho}'_i(z,0) / \Sigma \tilde{\rho}'_j(z,0)$$

for $x_i = y_i - z_i$ and $x = y + \Sigma z_j$. Summing up for i gives

$$\Sigma v^i(x_i, x) = 1,$$

the condition for Pareto efficiency.

The concept of ruled deviation, and this result, applies to the fields of half-Kantianism and of lateral reciprocity. It demands one to choose a state such that nobody prefers its variations according to some given rule. Then each individual *i* provides her z_i of this set *z*.

The rule $\tilde{\rho}(z, \tilde{\zeta})$ can be, for instance, proportional $\tilde{\rho}_i = \alpha_i \tilde{\zeta}$ with *n* numbers α_i and $v^i = \alpha_i / \Sigma\alpha_i$. These α_i may be some income of individuals *i*, $\alpha_i = y_i$, x_i, X_i (or t_i). They may be equal, corresponding to equal deviations ρ_i, with $v^i = 1/n$. They may also be z_i, corresponding to proportional deviations $\rho_i = \tilde{\zeta} z_i$, and to $z_i = v^i \Sigma z_j$ which is the Lindahl rule.

III.4 Social contracts and shame of shirking

A social contract is the technical name for an implicit unanimous agreement the realization of which is a moral demand or requirement because of the implicit freedom in the choice. Doing one's part of this contract may also be supported as a Kantian rule (or as lateral reciprocity). This agreement may be implicit for various reasons, one of which is the large number of contractors. This applies in particular for contributing to a non-excludable public good concerning many people, such as the poor's welfare. Nevertheless, abstaining to contribute saves this amount. However, in a full rational theory of this (implicit) agreement, the advantage of free-riding depends also on the other people's reaction. Yet a relatively small individual in a large number certainly thinks that the others continue to cooperate and contribute, and she free-rides their provision. Nevertheless, two effects may check this advantage of abstaining. First, the total amount provided may decrease, even given all adjustments to this situation. Second, there may also be some psychic cost of shirking caused by the shame or guilt of not abiding by the social or moral norm of cooperating (possibly also favoured by other people's expressed or imagined judgement) – especially for such an important and moral objective.

The issue is individuals' free choice, and their gifts. An idea of the result can be obtained simply by the consideration of *n* individuals identical for the relevant purpose. With full cooperation, each gives $g(n)$ and the total amount is $G(n) = ng(n)$. They are assumed to have quasi-linear utility functions which can be taken as

$$u=v(G)-g, \tag{63}$$

with a smooth function v, $v'>0$ and $v''<0$.[44] If all n individuals choose g cooperatively, this g maximizes u or $n\,u$ and hence satisfies

$$n\,v'-1=0 \tag{64}$$

if $g>0$ or $n\,v'-1\leq 0$ if $g=0$. Since function v' is decreasing, such a $G=ng>0$ is an increasing function of n.

If an individual decides to stop cooperating, first, she gains $g(n)$. Her relation with the others become non-cooperative. A result of the theory of the core for non-excludable public goods is that if the givers are partitioned into a set of groups cooperating within themselves (coalitions) but not with the others, then, with quasi-linear utilities, at a Cournot-Nash or Stackelberg (with a leading group) equilibrium only one group gives and it gives the amount it would give if it were alone, and this is the group with the largest such amount in the Cournot-Nash case.[45] With large n, an individual's defection probably leaves the $n-1$ others cooperating between themselves (and, in a non-repeated situation, they have no interest in giving less for the purpose of punishing the dissenter). Then, since, if $G(n-1)>0$ and $n>2$, $G(n-1)>G(1)$, the dissenting individual does not give (she is a full free rider) and the others give $G(n-1)$. The total amount given passes from $G(n)$ to $G(n-1)$. A choice to free ride is meaningful only if $g(n)>0$, which implies $v'=1/n$ from (64). This creates, for the free rider and large n, a loss of

$$v[G(n)]-v[G(n-1)]\simeq v'\,G'=G'/n=(g+n\,g')/n=g/n+g'. \tag{65}$$

The advantage of free riding is, therefore,

$$r=g-g/n-g'-s=(1-1/n)g-g'-s$$

where s is the psychic cost of the shame, guilt or reproach of shirking. When $n\to\infty$,

$$r=g-g'-s. \tag{66}$$

The case $g\to\infty$ is excluded (would it only be because of givers' incomes). Hence $g(\infty)=\gamma\geq 0$. This implies $g'(\infty)=0$ and, therefore, $r(\infty)=\gamma-s$. If $\gamma=0$, $r(\infty)<0$ for any $s>0$: whatever the positive moral-social cost of shirking, no matter how small, it prevents free

[44] Function v is defined as $v(G)=V(X+G)$ where X is the poor's income without these gifts. Moreover, if there are given distributive taxes, these g may represent individuals' contributions and the gifts are these amounts minus the taxes. These taxes may no longer be necessary for achieving Pareto efficiency.

[45] Kolm (1987).

riding for a sufficiently large n. However, a normal case is $\gamma>0$. Then, for large n, there is free riding or not according as $\gamma \gtrless s$. In this case, condition (64) writes, when $n\to\infty$,

$$v'(n\gamma)=1/n \quad \text{or} \quad v'(G)=\gamma/G,$$

which integrates as

$$v(G)=\gamma \operatorname{Log} G+c \tag{67}$$

for a constant c. This means that, for a large number of givers, compassion diminishes like the logarithm of the relief of poverty, as a kind of Weber-Fechner law of charity. A general result is that when compassion diminishes faster than the logarithm of the relief of poverty, any morality, no matter how small, suffices to induce people to give if their number is sufficiently large.

More precisely, deriving equation (64) for n gives

$$(E^g+1)E^{v'}+1=0 \tag{68}$$

where $E^g=n\,g'/g$ and $E^{v'}=G\,v''/v'$ are the of elasticities of functions $g(n)$ and $v'(G)$ respectively. Since $E^{v'}<0$, one should have $E^g>-1$ or $g+n\,g'>0$. Denote as $a=\lim_{n\to\infty} E^{v'}$ and $b=\lim_{n\to\infty} E^g$. Then,

$$1+(1+b)a=0. \tag{69}$$

with $a\leq 0$. Therefore, for large n, functions v and g behave respectively as functions

$$v=\alpha G^{1+a}+\beta \text{ with } -1<a<0 \tag{70}$$

or

$$v=\gamma' \operatorname{Log} G + c \text{ if } a=-1, \tag{71}$$

and

$$g=\gamma' n^b \tag{72}$$

with constant $\alpha>0$, $\gamma'>0$, β and c.

The case $b=0$ corresponds to $a=-1$ with $\gamma'=\gamma>0$. The condition $g(\infty) < \infty$ implies $b\leq 0$, and $b<0$ implies $g(\infty)=0$.

III.5 Reciprocities, putative reciprocities (an overview)

Other explanations are specific to giving (rather than applying to any public good).[46] Sentiments in the family of reciprocity provide some of the most important motives and reasons for giving in general. They intervene here in three forms. Lateral reciprocities are particular in that they are between co-givers. Reciprocity is in general between givers and receivers. Yet it may be actual or, importantly, putative, that is, by imagining gifts from the receivers. A basic fact for the present purpose is that there are two genuine reciprocitarian motives (that is, apart from simple sequential exchange). The *balance* or *matching* motive induces a return-gift that matches the gift or establishes a balance between both in some sense. This motive is non-altruistic and hence creates a warm-glow supported by a sense of duty or propriety, as a norm, with possible praise or blame of other people, which can a priori be a cause of non-crowding out of gifts. By contrast, by the *liking* motive the receiver comes to like the benevolent giver and gives her for this reason: this is simply creating altruism. Moreover, simple reciprocity extends into *generalized* reciprocity by which one tends to help if one has been helped even by other people, and into *reverse* reciprocity by which helpers tend to be helped even by people other than those they have helped.

Helping the needy for a reason of actual reciprocity occurs in non-static situations: one has been helped in the past, someone presently poor has helped, or one wishes to be helped in the future if circumstances come to require it. Putative reciprocity is more important.

A number of explanations of giving are based on a giver's experience when she imagines she is in the poor's situation. The most straightforward consists of empathies, which are a source of altruism.[47] This substitution is also implied in the standard rule "Do to others as you would like others do to you" which, if it is applied to demand helping the poor, implies "if you were poor" and, perhaps, if the poor were not. A further elaboration is based on the giver's imagination of the permutation of the situation of the two persons. This permits sentiments of reciprocity although one of the situations of giving is just imagined. The reason becomes "I give to her because she would have given to me if our situations were reversed". It is not unfrequently heard, and still more so if one includes the two cases of *extended* reciprocity. Applied putatively, they give the reasons: "I help her because I would have been helped (by anybody) if I needed it", or "because she would have helped someone in need (possibly not myself) if she could".All the properties of the relations of reciprocity also apply

[46] See Kolm (1984, 2008a).
[47] On the various types of empathy, see Kolm (2006, p. 58).

to putative reciprocities. The motives can be described by the structure of utility functions. Reciprocity is concerned with gifts but, in the present question, this can be either g_i or the full contribution $c_i=g_i+t_i$ (then the actual gift g_i adjusts to the taxes).

In putative balance reciprocity, gift g_i is determined as a return-gift to the gift γ_i received in the imaginary reversed situation, and γ_i is considered to be symmetrically chosen (or similarly for contributions c_i).[48] The solutions of this "reciprocity game" can be "dominations" in which one of the parties is a Stackelberg leader. Yet, the spirit of "balance" may prefer the "equilibrium" solution in which it makes no difference whether one giver or the other is the first or the second to give, which implies a solution of the Cournot-Nash form. The sentiment of this reciprocity can be represented by the structure of the agent's utility function, and the choice described by the maximization of this function. This is done by writing the putatively received gift γ_i as a parameter of this function, alongside the variable of the chosen gift g_i, and this function can also have all the other arguments previously considered (x_i, x, c_i, g_j, c_j). In the "equilibrium" solution, individual i chooses g_i while assuming γ_i to be fixed. Therefore, all the consequences of individual i having a direct preference about her gift g_i previously discussed apply to this case. Putative balance reciprocity reinforces the various motives for caring directly about one's gift by adding praise or praiseworthiness for providing a matching gift, with possible aspects of fairness, norm following or gratitude, and aversion to blame, blameworthiness, guilt or shame for failing in this respect (this can also concern contribution c_i, but we have seen that this is less interesting for explaining giving).

Hence, putative liking reciprocity creates or reinforces the public good problem, whereas putative balance reciprocity is based on an interpersonal relation that a priori does not raise this problem. However, the assistances that are balanced may be relative to the parties' needs, and the poor's needs depend on aid received from other people or fiscal transfers, which reintroduces the public good problem. Gratitude has aspects of both types of reciprocity: it induces a sentiment of the liking kind, but a return-gift that it may induce has also an aspect of balance or matching. It can also have some place in the putative relationship.

III.6 Sequential giving

[48] The complete model, discussion and results for ordinary reciprocities are presented in Kolm 2008, part IV.

Giving for fighting poverty extends in time, both overall and in specific instances of cumulative contributions to a particular aid. This can have important consequences on its realization and possibilities. Three types of issues are raised. First, we have noted the handicaps of sequential collective giving due to the problems of punishing a co-giver by giving less: it punishes the poor (and the other co-givers) and it has no practical effect with a larger number of givers. Second, in shallow warm-glows, sequential giving is necessary for traditions, habits, imitation and example, and following social standards and observed norms of behaviour. Third, sequential giving associates with a number of the motives presented into important giving processes. Each gift can be influenced by observed previous gifts, and take its influence on later gifts into account, for comparative giving (such as conforming or keeping up with others), in lateral reciprocity and matching, by example, imitation or norm-building, or for abiding sequentially by an implicit free agreement (with the possible help of imitation or lateral reciprocity about this agreement). In all these cases, the observation bears on some of the others' acts only. Sequential giving permits lateral reciprocity or matching in the cases in which individuals give even when a fraction of the others only do (and some may be unconditional givers). Guilt or shame of not matching previous gifts may induce giving. However, in cumulative contributions to specific aid giving diminishes the remaining need and giving less may induce others to give more for this reason. These dynamic effects combine in various possible ways which are not presented further here.[49]

III.7 Solidarity and sense of community

Propensity to help, in particular durably, is strongly fostered by a sentiment of solidarity with the beneficiary. This sentiment is closely related to the sense of belonging to a same community of one kind or another as the beneficiary of the gift and to the corresponding "social distance". There is in fact a spectrum of more or less close communities of various sizes, which induce more or less helping, form intra-family support to assistance in local, professional or cultural communities, fiscal redistribution mostly in national communities, and helping a fellow human member of the community of mankind. This has major consequences. Alesina and Glaeser (2004) attribute the lower level of transfers of all types in

[49] The basis of sequential contribution in Kolm (1987) and Varian (1994) is followed by fuller analyses by Admati and Perry (1991), Fershtan and Nitzan (1991), Marx and Matthews (2000) and Masclet, Willinger and Figuières (2007).

the US than in Europe to a lower sense of belonging to the same community as the bulk of the poor.[50] Solidarity may entail preference for helping or duty to help. It may enhance altruism or the various types of warm-glows. It tends to introduce a discrimination between the poor for each potential giver, depending on the communities to which they belong. However, the poor's wellbeing remains a public good, and the same solidarity for the same poor may a priori concern many people beyond families and very local communitites.

Conclusion

Motives of givers and voters are the necessary basis of the explanation of the fight against poverty and of its possibilities, even though its manifestations also depend on institutions with politics and charitable organizations, and on history.[51] Explaining the gifts observed in the presence of fiscal transfers is both important in itself and the touchstone of the understanding of these motives and of their effects and possibilities. The very simple device of comparing the marginal conditions permits a discriminating analysis of the relevantly different varieties of possible "warm-glows" and of their various effects. Along with the other motives, notably those leading to implicit cooperations, this shows a number of puzzling results. Whereas moral sentiments (altruism) induce somehow immoral results (no gift), immoral sentiments (vainglory, envy) induce moral acts (charity), but only if they crowd out moral sentiments, which undermines their own reason. The intrinsic valuing of one's sacrifice has no effect; that of one's gift is contradictory. A person ends up giving more if other people want her to contribute less. Discarding people's vainglory from the policy choice ends up extending its influence. Obeying injunctions or traditions is genuinely neither moral nor rational and also precludes altruism. Rebates, subsidies and matching grants have no effect on informed givers. Giving if others give leads to taxing everybody. The large number of givers may help checking free riding in various ways. Punishing failing co-givers punishes the poor. The paragon of social and moral rationality, Kantianism, a priori yields incoherence and inefficiency. When it does not, it is based, at any rate, on an a priori irrational rationale. Like putative reciprocity and social contracts, it relies on counterfactual reasoning.

[50] A main obstacle to European integration is that the sense of community, and the common history that have built it, is at national levels, and hence transfers of fiscal responsibility to European institutions would induce lower redistribution, which is strongly resisted.
[51] Bilodeau and Steinberg (2006) provide an extensive survey of the role of charitable organizations.

In the end, the explanation of gifts by the motive of "warm-glow" for being praiseworthy or praised is handicapped, in the presence of fiscal transfers which a priori tend to be Pareto-efficient, by the very large number of people who a priori care about painful poverty: it implies, indeed, that almost everybody think that the poor have enough. Moreover, sacrifice warm-glows are a priori powerless; altruism-based warm-glows make the noted difficulty still worse; shallow and external warm-glows and moral efficiency improve the case but do not suffice; and responsibility warm-glows imply an intrinsic contradiction. Hence, explanation by warm-glows seems to require fiscal, government and political Pareto-inefficiency and be limited to its scope (if any). Shallow warm-glows inducing specific aid may play a role, with their low level of both rationality (non autonomous and alienated choices) and morality (not altruistic). Putative (and other) balance reciprocity applies to aid only and provides a part of the explanation, but certainly with a rather limited scope. Folk Kantianism can be present for all public goods and is for aid, in spite of its peculiar rationality. Rebates, subsidies and matching grants have actual effects due to gift illusion. Lateral reciprocity plays a role either with forced-free tax-gifts or in sequential giving with givers demanding less than full contributions of the others. The shame of shirking an implicit agreement also has a place (with the possible help of the large number of givers). The theory of consistent rules of comparative fairness helps explaining efficient norms, conventions or taxation.

The public good problem with standard motives has been pointed out for long and only confirmed by later studies. Free riding, however, is not the case in many instances. The explanation has to rest on the acknowledgement that people are also motivated by different rationales. The possible reasons, particularly with large numbers and for helping, are peculiar and sometimes strange from an individualistic point of view, and are often of a social nature (Kantianisms, lateral reciprocity, social contracts, shallow warm-glows, putative reciprocity). Explaining them would have to consider education and cultural formation and selection, building on some general genetic background possibility formed, notably, by group selection.[52]

[52] Group selection offers more possibilities than is usually said if it is not a priori assumed that individual selection is infinitely faster (i.e. if it is a non-adiabatic process, see Weibull and Salomonsson, 2006).

Appendix A - Pareto efficiency for sub-populations

The relation between Pareto efficiency in the two regimes is a particular case of more general properties. Let *z* denote a state, *Z* the set of possible states, $U^i(z)$ the utility function of *any* individual *i*, and *I*, I' and I'' sets of individuals *i*. Say that $z=z^*$ is *strictly Pareto efficient* for the set *I* of individuals *i* if $z^* \in Z$ and, for any $z' \in Z/\{z^*\}$, $U^i(z) < U^i(z^*)$ for at least one $i \in I$. Strict Pareto efficiency implies ordinary Pareto efficiency. Then, if z^* is strictly Pareto efficient for the population I', it is also strictly Pareto efficient for any population $I'' \supset I'$, and therefore it is Pareto efficient for this population. Note that if $W(\{U^i\}_{i \in I'})$ is a strictly increasing function which has a unique maximum on *Z* at z^*, then z^* is strictly Pareto efficient for population I', and therefore for any larger population $I'' \supset I'$, and it is also Pareto efficient for these populations. In particular, if $|I'|=1$ and I' is made of a single individual *i*, both strict Pareto efficiency for I' and this unique maximum mean a unique maximum of function U^i. This implies strict and usual Pareto efficiency for any population including individual *i*. This can result from individual *i*'s choice of z^* in the set *Z*.

Appendix B – Responsibility and information

In the real world, however, the responsibility issue may not be so clear-cut, and questions of information may play a role. Social pressures and even interiorized strong norms of giving may attenuate the person's responsibility for her gift. Moreover, a person might sometimes be considered having some responsibility for the distributive taxes she pays. This happens if these taxes have to result from a collective unanimous agreement (each person's veto gives her full responsibility for the whole of the outcome). One principle of public finance ("liberal social contracts"), in particular for financing public goods, consists of imposing the outcome from such a hypothetical collective agreement. The taxpayer's responsibility is lower if she is only a voter in a vote requiring lower unanimity, but it comes back if she fully endorses this system.

Moreover, there may be differences in information about g_i and c_i, for individual *i* and for other people whose opinion influences her warm-glow. Differences in information may not be relevant from a normative point of view, but they are for actual preferences and

actions. The giver generally knows her gift g_i. She knows the distributive taxes she pays if they are separated from the rest of taxes. If not, she knows her direct taxes, may estimate her indirect taxes, and may derive t_i from an information about the share of the budget used to help the poor (including by public education, subsidized health care, or other programs). For the effects of the praise or blame of other people on the person's warm-glow, they may just be imagined by her, in particular for the case in which they knew what she knows. Other people may also estimate the person's gifts (she may boast about them), and the distributive tax she pays from some idea about her general taxes (perhaps from her lifestyle) and about the share of the public budget used for helping the poor.

References

Admati, A., and M.Perry. 1991. "Joint projects without commitment." *Review of Economic Statistics*, 58: 259-76.
Alesina, A., and E. Glaeser. 2004. *Fighting Poverty in the US and in Europe. A World of Difference*. Oxford: Oxford University Press.
Andreoni, J. 1988. "Privately provided public goods in a large economy: the limits of altruism." *Journal of Public Economics*, 35: 57-73.
Andreoni, J. 1989. "Giving with impure altruism: Applications to charity and Ricardian equivalence." *Journal of Political Economy*, 97: 1447-58.
Andreoni, J. 1990. "Impure altruism and donations to public goods: a theory of warm-glow giving." *Economic Journal*, 100: 464-77.
Andreoni, J., and T. Bergstrom. 1996. "Do government subsidies increase the private supply of public goods?" *Public Choice*: 88, 295-308.
Arrow, K.J. 1972. "Gifts and Exchanges." *Philosophy and Public Affairs*, 4: 343-62.
Batson, C. 1998. "Altruism and prosocial behavior." In *The Handbook of Social Psychology*, ed. D. Gilbert et al., 282-316. Oxford: Oxford University Press.
Becker, G.S. 1974. "A Theory of Social Interactions." *Journal of Political Economy*, 82: 1063-93.
Becker, G.S. 1978. "A positive theory of the redistribution of income." Working paper n° 006. Chicago: University of Chicago, Center Study Econ. and State.
Bilodeau, M., and N. Gravel. 2004. "Voluntary provision of public good and individual morality." *Journal of Public Economics*, 88: 645-66.
Bilodeau, M., and R. Steinberg. 2006. "Donative non-profit organization." In ed. S.-Ch. Kolm and J. Mercier Ythier, 1271-1333.
Bergstrom, T., L. Blume, and H. Varian. 1986. "On the private provision of public goods." *Journal of Public Economics*, 29: 25-49.
Bernheim, D. 1986. "On the voluntary and involuntary provision of public goods." *American Economic Review*, 76 (4): 789-93.
Boadway, R., P. Pestieau, and D. Wildasin. 1989. "Tax-transfer policies and the voluntary provision of public goods." *Journal of Public Economics*, 39: 157-76.
Bordignon, M. 1990. "Was Kant right? Voluntary provision of public goods under the principle of unconditional commitment." *Economic Notes*, 3: 342-72.
Brekke, K., S. Kverndokk, and K. Nyborg. 2003. "An economic model of moral motivation." *Journal of Public Economics*, 87: 1967-83.

Clotfelter, C. 1980. “Tax incentives and charitable giving: Evidence from a panel of taxpayers.” *Empirical Economics*, Springer, 13(3): 319-40.
Clotfelter, C. 1985. *Federal Tax Policy and Charitable Giving*. Chicago: University of Chicago Press.
Cornes, R., and T. Sandler. 1984a. “Easy Riders, Joint Production, and Public Goods.” *Economic Journal*, 94: 580-98.
Cornes, R., and T. Sandler. 1984b. “The comparative static property of the impure public good model.” *Journal of Public Economics*, 54: 403-21.
Cornes, R., and Sandler. 1986. *The Theory of Externalities*, *Public Goods and Club Goods*. Cambridge: Cambridge University Press.
Fershtan, C., and S. Nitzan. 1991. “Dynamic voluntary provision of public goods.” *European Economic Review*, 35: 1057-67.
Glazer, A., and K. Konrad. 1996. “A signaling explanation for charity.” *American Economic Review*, 86: 1019-28.
Harbaugh, W. 1998a. “The prestige motive for making charitable transfers.” *American Economic Review (Papers and Proceedings*), 88: 277-82.
Harbaugh, W. 1998b. “What do donations buy? A model of philanthropy based on prestige and warm glow.” *Journal of Public Economics*, 67: 269-84.
Harbaugh, W.T., U. Mayr, and D.R. Burghart. 2007. “Neural responses to taxation and voluntary giving reveal motives for charitable donations.” *Science*, June, vol. 316, n°5831: 1622-25.
Holländer, H. 1990. “A social exchange approach to voluntary cooperation.” *American Economic Review*, 80: 1157-67.
Kahneman, D., and J. Knetsch. 1992. “Valuing public goods: the purchase of moral satisfaction.” *Journal of Environmental Economics and Management*, 22: 57-70.
Kant, I. [1785] 1907. *Fundamental Principles of the Metaphysics of Ethics*. 3rd ed. London: Longmans, (English Translation by T.K. Abott).
Kolm, S.-Ch. 1966. “The optimal production of social justice.” In International Economic Association Conference on Public Economics, Biarritz, proceedings ed. by H. Guitton and J. Margolis. *Economie Publique*, Paris: CNRS, 1968, pp.109-77. *Public Economics*, London: Mac Millan, 1969, pp. 145-200. Reprinted in *Landmark Papers in General Equilibrium Theory, Social Choice and Welfare, The Foundations of 20th Century Economics*, selected by K.J. Arrow and G. Debreu, 2001, Cheltenham: Edward Elgar; pp.606-653.
Kolm, S.-Ch. 1970a. *La Valeur Publique*. Paris: CNRS-Dunod.
Kolm, S.-Ch. 1970b. *L’Etat et le Système des Prix*. Paris: CNRS-Dunod.
Kolm, S.-Ch. 1971. *Le Service des masses.* Paris: CNRS-Dunod.
Kolm, S.-Ch. 1984. *La Réciprocité Générale – La Bonne Economie*. Paris: Presses Universitaires de France.
Kolm, S.-Ch. 1987. *Freedom, cores and public goods*. Paris: CERAS, n°66.
Kolm, S.-Ch. 2006. “Introduction to the economics of giving, altruism and reciprocity.” In ed. S.-Ch. Kolm and J. Mercier Ythier, 1-122.
Kolm, S.-Ch. 2008a. *Reciprocity, The Economics of Social Relations*. Cambridge: Cambridge University Press.
Kolm, S.-Ch. 2008b. “Giving with subsidy illusion.”
Kolm, S.-Ch., and J. Mercier Ythier, ed. 2006. *Handbook in the Economics of Giving, Altruism, and Reciprocity*. Amsterdam: Elsevier North-Holland.
Konow, J. 2007. “Mixed feelings: Theories and evidence of warm glow and altruism.” MPRA Paper n°2727, November.
Laffont, J.J. 1975. “Macroeconomic constraints, economic efficiency and ethics: An introduction to Kantian economics.” *Economica*, 42 (168): 430-37.

Lindbeck, A., and J. Weibull. 1988. “Altruism and time consistency: The economics of the fait accompli.” *Journal of Political Economy*, 96: 1165-82.
Marx, L., and S. Matthews. 2000. “Dynamic voluntary contribution to a public project.” *Review of Economic Studies*, 64: 327-58.
Masclet, D., M. Willinger, and C. Figuières. 2007. “The economics of telethon: leadership, reciprocity and moral motivation.” Montpellier: Lameta, DR n° 2007-08.
Nagel, T. 1991. *Equality and Partiality*. Oxford: Oxford University Press.
Olson, M. 1965. *The Logic of Collective Action*. Cambridge MA: Harvard University Press.
Pareto, V. 1913. “Il massimo di utilità per una collettività.” *Giornale degli Economisti*, 3: 337-41. Reprinted as a long footnote in *Treatise of General Sociology* (and in *Mind and Society*).
Peltzman, S. 1976. “Toward a more general theory of regulation.” *Journal of Law and Economics*, 19: 211-40.
Rege, M., and K. Telle. 2004. “The impact of social approval and framing on cooperation in public good situations.” *Journal of Public Economics*, 88 (7-8): 1625-44.
Ribar, C.D., and M.O. Wilhelm. 2002. “Altruistic and joy-of-giving motivations in charitable behavior.” *Journal of Political Economy*, 110: 425-57.
Roberts. R. 1984. “A positive model of private charity and public transfers.” *Journal of Political Economy*, 92: 136-48.
Roberts, R. 1987. “Financing public goods.” *Journal of Political Economy*, 95: 420-37.
Schokkaert, E. 2006. “The empirical analysis of transfer motives.” In ed. S.-Ch. Kolm and J. Mercier Ythier, 127-81.
Schockkaert, E., and L. Van Ootegem. 2000. “Preference variation and private donations.” In *The Economics of Reciprocity, Giving, and Altruism*, ed. L.A. Gérard-Varet, S.-Ch. Kolm and J. Mercier Ythier, 78-95. London: Macmillan.
Spash, C. 2000. “Ethical motives and charitable contributions in contingent valuation: empirical evidence from social psychology and economics.” *Environmental Values*, 9: 453-79.
Steinberg, R. 1986. “Charitable Giving as a Mixed Public/Private Good: Implications for Tax Policy.” *Public Finance Quarterly*, 14: 415-31.
Sugden R. 1982. “On the economics of philanthropy.” *Economic Journal*, 92: 341 -50.
Sugden, R. 1984. “Reciprocity: The supply of public goods through voluntary contributions.” *Economic Journal*, 94: 772-87.
Varian, H. 1994. “Sequential contributions to public goods.” *Journal of Public Economics*, (February): 165-86.
Warr, P. 1982. “Pareto optimal redistribution and private charity.” *Journal of Public Economics*, 19: 131-38.
Weibull, J., and M. Salomonsson. 2006. “Natural selection and social preferences.” *Journal of Theoretical Biology*.
Woodward, R. 1984. “An empirical analysis of philanthropic motivations.” Department of Economics, Middlebury.
Yamagishi, T. 1986. “Motivational bases of the public good problem.” *Journal of Personality and Social Psychology*, 50(1): 87-73.